

$$y^2 - x^2 = 0$$

Identify the graph of the equation

### EXAMPLE 6 IDENTIFYING THE CONIC SECTION

Since the sum of two squares is nonnegative, this equation has no graph.

$$\begin{aligned}x^2 - 2)^2 + (y - 1)^2 &= -4 \\(x^2 - 4x + 4) + (y^2 - 2y + 1) &= -9 + 4 + 1\end{aligned}$$

and complete the square in both  $x$  and  $y$ .

$$(x^2 - 4x) + (y^2 - 2y) = -9$$

We rewrite the equation as

### SOLUTION

$$x^2 - 4x + y^2 - 2y + 9 = 0$$

Identify the graph of the equation

### EXAMPLE 5 IDENTIFYING THE CONIC SECTION

a pair of lines, or no graph.

is a conic section or a degenerate form of a conic section, such as a point, a line,

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

It can be shown that the graph of the equation

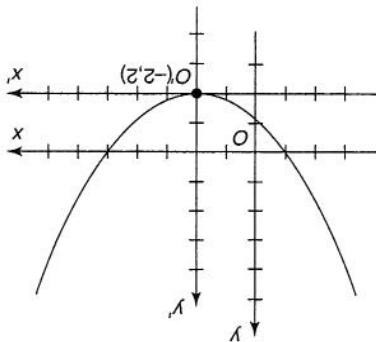
is a parabola.

$$y^2 + 4y - 6x + 22 = 0$$

Show that the graph of the equation

✓ Progress Check

FIGURE 37 Graph of  $x^2 - 4x - 4y - 4 = 0$



**SOLUTION**

Solving for  $y$ ,

$$y = \pm x$$

These are the equations of a pair of intersecting lines.

**✓ Progress Check**

Identify the conic section.

- a.  $\frac{x^2}{5} - 3y^2 - 2x + 2y - 4 = 0$
- b.  $x^2 - 2y - 3x = 2$
- c.  $x^2 + y^2 - 4x - 6y = -11$
- d.  $4x^2 + 3y^2 + 6x - 10 = 0$
- e.  $x^2 - 2x - 3 = 0$

Answers

- |                           |             |
|---------------------------|-------------|
| a. hyperbola              | b. parabola |
| c. circle                 | d. ellipse  |
| e. pair of parallel lines |             |

We present some facts in Table 3 that may help to identify the various conic sections.

**TABLE 3 Identifying Conic Sections**

$Ax^2 + Cy^2 + Dx + Ey + F = 0$ Conic Section	Remarks
$A = 0$ or $C = 0$	Parabola Second degree in one variable, first degree in the other variable
$A = C \neq 0$	Circle Coefficients $A$ and $C$ are the same. <i>Caution:</i> Complete the square to obtain the standard form and check that $r > 0$ .
$A \neq C, AC > 0$	Ellipse A and $C$ are unequal but have the same sign. <i>Caution:</i> Complete the square and check that the right-hand side is a positive constant.
$AC < 0$	Hyperbola A and $C$ have opposite signs.

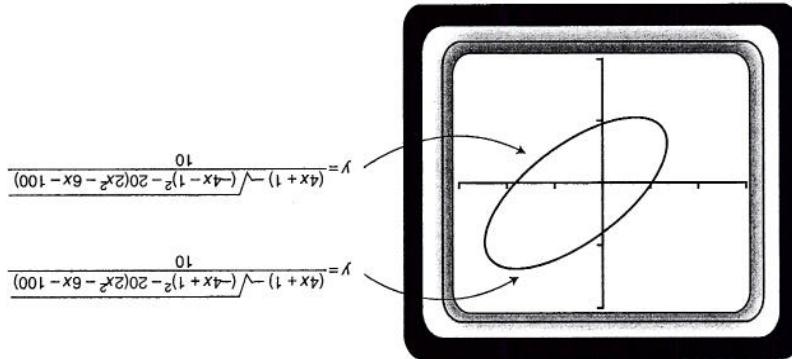
**Graphing Calculator Power User's Corner****Graphing General Conic Sections**

The most general equation for a conic section is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

**Graphing Calculator Challenge**  
 Write a program that will prompt for the values A, B, C, D, E, and F; and then graph the conic section  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ . Do not forget to allow for the special case when  $C = 0$ .

FIGURE 38 Graph of  $2x^2 - 4xy + 5y^2 - 6x - y - 100 = 0$



rectangle does not necessarily guarantee an undistorted graph. In the viewing rectangle that is three times the EQUAL viewing rectangle, the shape of an ellipse or hyperbola may be distorted if the viewing angle is not proportionally scaled. Unfortunately, a proportionally scaled viewing rectangle does not necessarily guarantee an undistorted graph.

$$y = \frac{10}{4x + 1 - \sqrt{(-4x - 1)^2 - 20(2x^2 - 6x - 100)}}$$

and

$$y = \frac{10}{4x + 1 + \sqrt{(-4x + 1)^2 - 20(2x^2 - 6x - 100)}}$$

As shown in Figure 38, we GRAPH

$$y = \frac{10}{(4x + 1) \pm \sqrt{(-4x - 1)^2 - 20(2x^2 - 6x - 100)}}$$

and solve for  $y$

$$5y^2 + (-4x - 1)y + (2x^2 - 6x - 100) = 0$$

rewrite the equation as

$$2x^2 - 4xy + 5y^2 - 6x - y - 100 = 0$$

For example, to display the graph of the conic section

(This is of the form  $ay^2 + by + c = 0$  with  $a = C$ ,  $b = Bx + E$ , and  $c = Ax^2 + Dx + F$ .) We can use the quadratic formula to solve this equation for  $y$ .

$$Cy^2 + (Bx + E)y + (Ax^2 + Dx + F) = 0$$

this equation to see that it is a quadratic in  $y$ :

The  $Bxy$  term causes a rotation of the graph of the conic section. We can rewrite

## Exercise Set 5.5

In Exercises 1–4, the origin  $O'$  of the  $x'y'$ -coordinate system is at  $(-1, 4)$ . Find the  $x'y'$ -coordinates of the point whose  $xy$ -coordinates are given.

- |             |               |
|-------------|---------------|
| 1. $(0, 0)$ | 2. $(-2, 1)$  |
| 3. $(4, 3)$ | 4. $(-6, -2)$ |

In Exercises 5–8, the origin  $O'$  of the  $x'y'$ -coordinate system is at  $(-3, 4)$ . Find the  $xy$ -coordinates of the point whose  $x'y'$ -coordinates are given.

- |             |               |
|-------------|---------------|
| 5. $(0, 0)$ | 6. $(-2, 1)$  |
| 7. $(4, 3)$ | 8. $(-6, -2)$ |

In Exercises 9–18, sketch the graph of the given equation. Then, determine appropriate WINDOW values, and check your answer using your graphing calculator.

9.  $36x^2 - 100y^2 + 216x + 99 = 0$
10.  $x^2 - 4y^2 + 10x - 16y + 25 = 0$
11.  $x^2 + 4x - y + 5 = 0$
12.  $2x^2 - 12x + y + 21 = 0$
13.  $y^2 + 2x + 15 = 0$
14.  $16x^2 + 4y^2 + 12y - 7 = 0$
15.  $x^2 + 4y^2 + 10x - 8y + 13 = 0$
16.  $9x^2 + 25y^2 - 36x + 50y - 164 = 0$
17.  $x^2 + 9y^2 - 54y + 72 = 0$
18.  $x^2 - y^2 + 4x + 8y - 11 = 0$

In Exercises 19–42, identify the conic section.

19.  $y^2 - 8x + 6y + 17 = 0$
20.  $4x^2 + 4y^2 - 12x + 16y - 11 = 0$
21.  $4x^2 + y^2 + 24x - 4y + 24 = 0$
22.  $4x^2 - y^2 - 40x - 4y + 80 = 0$
23.  $x^2 - y^2 + 6x + 4y - 4 = 0$
24.  $2x^2 + y^2 - 4x + 4y - 12 = 0$
25.  $25x^2 - 16y^2 + 210x + 96y + 656 = 0$

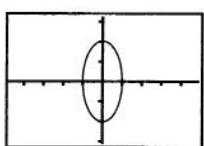
26.  $x^2 - 10x + 8y + 1 = 0$
27.  $2x^2 + y - x + 3 = 0$
28.  $4y^2 - x^2 + 2x - 3y + 5 = 0$
29.  $4x^2 + 4y^2 - 2x + 3y - 4 = 0$
30.  $3x^2 + 6y^2 - 2x + 8 = 0$
31.  $36x^2 - 4y^2 + x - y + 2 = 0$
32.  $x^2 + y^2 - 6x + 4y + 13 = 0$
33.  $16x^2 + 4y^2 - 2y + 3 = 0$
34.  $2y^2 - 3x + y + 4 = 0$
35.  $x^2 + y^2 - 4x - 2y + 8 = 0$
36.  $x^2 + y^2 - 2x - 2y + 6 = 0$
37.  $4x^2 + 9y^2 - x + 2 = 0$
38.  $3x^2 + 3y^2 - 3x + y = 0$
39.  $4x^2 - 9y^2 + 2x + y + 3 = 0$
40.  $x^2 + y^2 + 6x - 2y + 10 = 0$
41.  $x^2 + y^2 - 4x + 4 = 0$
42.  $4x^2 + y^2 = 32$



In Exercises 43–48, use your graphing calculator to GRAPH the following conic sections in the indicated viewing rectangle:

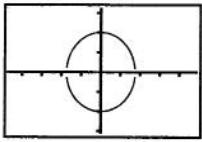
43.  $x^2 + xy + y^2 - 1 = 0$   
 $-2 \leq X \leq 2$     $-2 \leq Y \leq 2$
44.  $12x^2 + 21xy - 15y^2 - 3x + 12y + 7 = 0$   
 $-10 \leq X \leq 10$     $-10 \leq Y \leq 10$
45.  $-25x^2 + 21xy - 15y^2 - 3x + 12y + 7 = 0$   
 $-1 \leq X \leq 1$     $-1 \leq Y \leq 2$
46.  $8x^2 + 24xy + 18y^2 + 50x + 37 = 0$   
 $-15 \leq X \leq 5$     $-5 \leq Y \leq 15$
47.  $2x^2 - 3xy - y^2 + 4x - y - 1 = 0$   
 $-10 \leq X \leq 10$     $-10 \leq Y \leq 10$
48.  $2x^2 - 3xy + y^2 + 4x - y - 1 = 0$   
 $-25 \leq X \leq 30$     $-25 \leq Y \leq 30$

$X_{SCL} = 0.5$ ,  $Y_{SCL} = 0.5$   
rectangle.



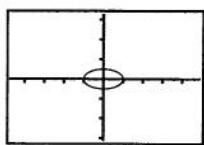
$$15. \quad \frac{x^{\frac{3}{2}}}{x^2} + \frac{x^{\frac{9}{2}}}{x^2} = 1; \left(0, \pm \frac{3\sqrt{2}}{4}\right), \left(\mp \frac{1}{2}, 0\right)$$

**EQUAL** viewing rectangle



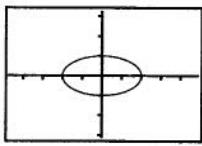
$$13. \quad \frac{x^3}{y^2} + \frac{4}{y^2} = 1; (0, \pm 2), (\pm \sqrt{3}, 0)$$

EQUAL viewing rectangle



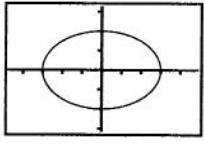
$$11. \quad \frac{1}{x^2} + \frac{4}{y^2} = 1; \left(0, \pm \frac{1}{2}\right), (\pm 1, 0)$$

**EQUAL** viewing rectangle  
 $X_{SCL} = 1, Y_{SCL} = 1$



$$9. \quad \frac{4}{x^2} + \frac{1}{y^2} = 1; (0, \pm 1), (\pm 2, 0)$$

**EQUAL** viewing rectangle



$$7. \quad \frac{9}{x^2} + \frac{y^2}{4} = 1; (0, \pm 2), (\pm 3, 0)$$

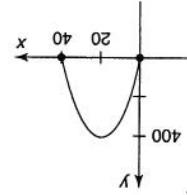
43. vertex:  $(-1, 0)$ ; axis:  $y = 0$ ; direction: left

XMIN = -20, XMAX = 0, XSCL = 1  
YMIN = -10, YMAX = 10, YSCL = 1

ou .q

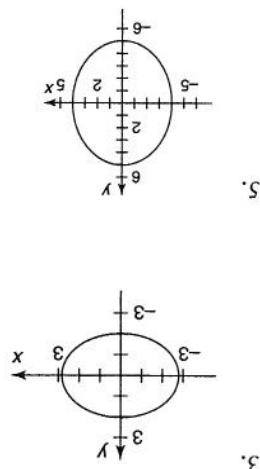
b. 31 inches

47. a. 20 $\sqrt{6}$  inches



**X** and **Y** values are from 2 times the EQUAL viewing rectangle

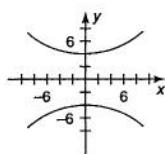
$\text{XSCL} = 2, \text{YSCL} = 2$



### **Exercise Set 5.4**

**XSCl** and **YSCl** values are from 2 times the EQUAL viewing rectangle.

17.

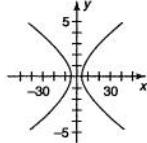


$$y\text{-intercepts: } (0, \pm 4)$$

X and Y values are from 4 times the EQUAL viewing rectangle.

XSCL = 4, YSCL = 4

19.

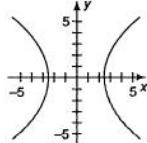


$$x\text{-intercepts: } (\pm 6, 0)$$

X and Y values are from 3 times the EQUAL viewing rectangle.

XSCL = 3, YSCL = 3

21.

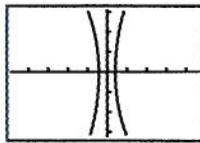


$$x\text{-intercepts: } (\pm \sqrt{6}, 0)$$

X and Y values are from 5 times the EQUAL viewing rectangle.

XSCL = 5, YSCL = 5

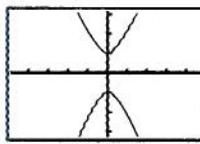
$$23. \frac{x^2}{4} - \frac{y^2}{64} = 1; (\pm 2, 0)$$



X and Y values are from 5 times the EQUAL viewing rectangle.

XSCL = 5, YSCL = 5

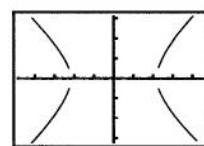
$$25. \frac{y^2}{4} - \frac{x^2}{\frac{1}{4}} = 1; \left(0, \pm \frac{1}{2}\right)$$



EQUAL viewing rectangle

XSCL = 1, YSCL = 1

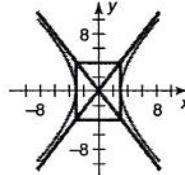
$$27. \frac{x^2}{5} - \frac{y^2}{4} = 1; (\pm \sqrt{5}, 0)$$



EQUAL viewing rectangle

XSCL = 1, YSCL = 1

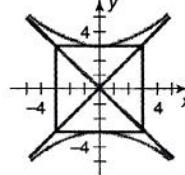
29.



X and Y values are from 2 times the EQUAL viewing rectangle.

XSCL = 2, YSCL = 2

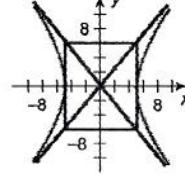
31.



X and Y values are from 3 times the EQUAL viewing rectangle.

XSCL = 3, YSCL = 3

33.



X and Y values are from 3 times the EQUAL viewing rectangle.

XSCL = 3, YSCL = 3

$$37. \frac{1}{2}\sqrt{299} \text{ inches}$$

$$39. \frac{80\sqrt{6}}{7} \text{ feet}$$

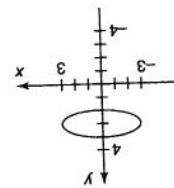
$$41. \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$43. \frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$45. \frac{x^2}{9} - y^2 = 1$$

XSCl = 2, YSCl = 2  
rectangle.

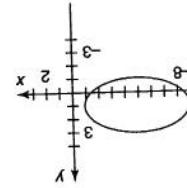
X and Y values are from 2 times the EQUAL viewing



17.

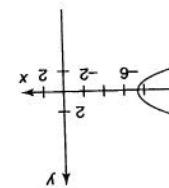
XSCl = 2, YSCl = 2  
rectangle.

X and Y values are from 2 times the EQUAL viewing



15.

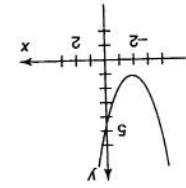
YMIN = -10, YMAX = 10, XSCl = 1  
XMIN = -20, XMAX = 0, XSCl = 1



13.

XSCl = 3, YSCl = 3  
rectangle.

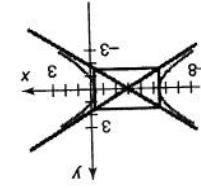
X and Y values are from 3 times the EQUAL viewing



11.

XSCl = 3, YSCl = 3  
rectangle.

X and Y values are from 3 times the EQUAL viewing



9.

- |               |               |               |
|---------------|---------------|---------------|
| 1. (1, -4)    | 3. (5, -1)    | 5. (-3, 4)    |
| 23. parabola  | 21. ellipse   | 25. hyperbola |
| 25. hyperbola | 27. parabola  | 29. circle    |
| 31. hyperbola | 33. no graph  | 35. no graph  |
| 37. no graph  | 39. hyperbola | 41. point     |

### Exercise Set 5.5

Appendix I ■ Answers to Selected Odd-Numbered Exercises, Review Exercises, and Review Tests