Graphing Calculator Power User's Corner

Graphing Parabolas

To display the parabola $(x-h)^2 = 4p(y-k)$ on your graphing calculator, GRAPH $y = k + \frac{(x-h)^2}{4p}$. To display the parabola $Ax^2 + Dx + Ey + F = 0$ on your graphing calculator, GRAPH $y = -\frac{Ax^2 + Dx + F}{E}$.

To display the parabola $(y-k)^2 = 4p(x-h)$ on your graphing calculator, GRAPH $y = \sqrt{4p(x-h)} + k$ and $y = -\sqrt{4p(x-h)} + k$. To display the parabola $Cy^2 + Dx + Ey + F = 0$ on your graphing calculator, use the quadratic formula to solve for y and GRAPH

$$y = \frac{-E + \sqrt{E^2 - 4C(Dx + F)}}{2C}$$

and

$$y = \frac{-E - \sqrt{E^2 - 4C(Dx + F)}}{2C}$$

For example, to display $y^2 - 2y - 2x - 5 = 0$, GRAPH

$$y = \frac{2 + \sqrt{4 - 4(-2x - 5)}}{2}$$

and

$$y = \frac{2 - \sqrt{4 - 4(-2x - 5)}}{2}$$

Compare the graphs of this parabola in the default viewing rectangle and the EQUAL viewing rectangle.

Exercise Set 5.3

In Exercises 1–8, determine the focus and directrix of the given parabola, and sketch the graph.

1. $x^2 = 4y$
2. $x^2 = -4y$
3. $y^2 = 2x$
4. $y^2 = -\frac{3}{2}x$
5. $x^2 + 5y = 0$
6. $2y^2 - 3x = 0$
7. $y^2 - 12x = 0$
8. $x^2 - 9y = 0$

In Exercises 9–20, determine the equation of the parabola that has its vertex at the origin and satisfies the given conditions.

9. Focus at $(1, 0)$
10. Focus at $(0, -3)$
11. Directrix $x = -\frac{3}{2}$
12. Directrix $y = \frac{5}{2}$
13. Axis is the y-axis, and the parabola passes through the point $(4, -2)$.
14. Axis is the x-axis, and the parabola passes through the point $(2, 1)$.
15. Axis is the x-axis and $p = -\frac{5}{4}$.
16. Axis is the y-axis and $p = 2$.
17. Focus at $(-1, 0)$ and directrix $x = 1$.

18. Focus at $(0, -\frac{5}{2})$ and directrix $y = \frac{5}{2}$.

19. Axis is the x-axis, and the parabola passes through the point (4, 2).

20. Axis is the y-axis, and the parabola passes through the point (2, 4).

In Exercises 21–24, determine in which direction each parabola opens.

21. $4x^2 + y = 0$

22. $4x^2 - y = 0$

23. $2x + y^2 = 0$

24. $2x - 5y^2 = 0$

In Exercises 25–38, write the equation in standard form. Determine the vertex, axis, and the direction in which each parabola opens. Determine appropriate WINDOW values, and GRAPH the parabola on your graphing calculator.

25. $x^2 - 2x - 3y + 7 = 0$

26. $x^2 + 4x + 2y - 2 = 0$

27. $y^2 - 8y + 2x + 12 = 0$

28. $y^2 + 6y - 3x + 12 = 0$

29. $x^2 - x + 3y + 1 = 0$

30. $y^2 + 2y - 4x - 3 = 0$

31. $y^2 - 10y - 3x + 24 = 0$

32. $x^2 + 2x - 5y - 19 = 0$

33. $x^2 - 3x - 3y + 1 = 0$

34. $y^2 + 4y + x + 3 = 0$

35. $y^2 + 6y + \frac{1}{2}x + 7 = 0$

36. $x^2 + 2x - 3y + 19 = 0$

37. $x^2 + 2x + 2y + 3 = 0$

38. $y^2 - 6y + 2x + 17 = 0$

In Exercises 39–44, determine the vertex, axis, and the direction in which each parabola opens. Sketch the graph. Determine appropriate WINDOW values, and check your answer using your graphing calculator.

39. $x^2 - 4x - 2y + 2 = 0$

40. $y^2 + 2x - 4y + 6 = 0$

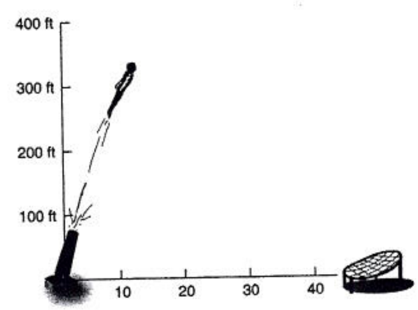
41. $2x^2 + 16x + y + 34 = 0$

42. $2x^2 - y + 3 = 0$

43. $y^2 + 2x + 2 = 0$

44. $y^2 + 3x - 2y - 5 = 0$

45. A stuntman has agreed to perform his famous human cannonball routine for the "Greatest Show in the Galaxy" carnival. The stuntman determines that his path will follow the parabola $y = 40x - x^2$, with the units measured in feet. His assistant has a 30 square foot circular net to catch his landing. The assistant places the net so that its center is 45 feet from the cannon.



a. Sketch the path the stuntman will travel.

b. Determine if the net will "catch" the stuntman.

46. Consider a large parabolic reflector in the shape of a headlight used by a construction crew late at night. The lamp is designed in such a way that its light source is placed at the focus of the parabola, which lies $3\frac{1}{4}$ inches from its vertex. Suppose the lamp is to be $16\frac{1}{8}$ inches deep.

a. Sketch a graph of the parabola.

b. How wide will the reflector be?

c. How far will the rim of the lamp be from the light source?

47. A police helicopter has a searchlight that is designed so that its light source lies at the focal point of a parabolic reflector that is 6 inches from its vertex. Suppose the searchlight is designed to be 25 inches deep.

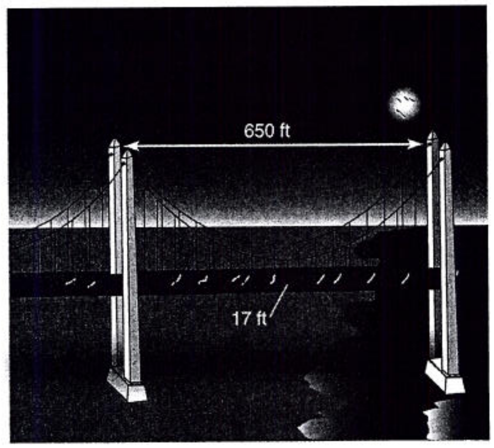
a. Find the width of the searchlight.

b. How far will the light source be from the rim of the searchlight?

48. A cable from a suspension bridge in the shape of a parabola hangs from two 75-feet towers, which are 650 feet apart. The lowest part of the cable hangs 17 feet from the roadway of the bridge.

a. Assume that the bridge cable will be supported by vertical beams. How long should such a support beam be if it is located 125 feet from the center of the span of the bridge?

b. Determine the length of vertical supporting beams for the bridge cable if they are to be placed (i) 75 feet, (ii) 200 feet, or (iii) 300 feet from the center of the cable.



5.4 The Ellipse and Hyperbola

The Ellipse

The geometric definition of an ellipse is as follows:

Definition of an Ellipse

An **ellipse** is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Each fixed point is called a **focus** of the ellipse.

An ellipse may be constructed in the following way. Place a thumbtack at each of the foci F_1 and F_2, and attach one end of a string to each of the thumbtacks. Hold a pencil tightly against the string, as shown in Figure 24, and move the pencil. The point P describes an ellipse since the sum of the distances from P to the foci is always a constant, the length of the string.