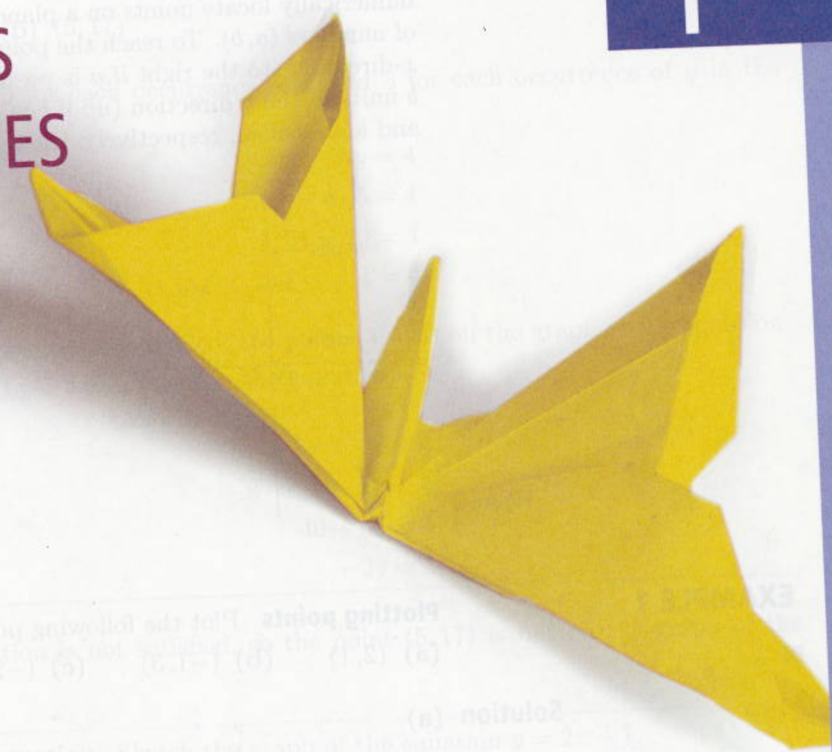
CHAPTER 1

LINEAR EQUATIONS AND STRAIGHT LINES



Many applications considered later in this text involve linear equations and their geometric counterparts—straight lines. So let us begin by studying the basic facts about these two important notions.

1.1 Coordinate Systems and Graphs

Often we can display numerical data by using a **Cartesian coordinate system** on either a line or a plane. We construct a Cartesian coordinate system on a line by choosing an arbitrary point O (the **origin**) on the line and a unit of distance along the line. We then assign to each point on the line a number that reflects its directed distance from the origin. Positive numbers refer to points on the right of the origin, negative numbers to points on the left. In Fig. 1 we have drawn a Cartesian coordinate system on the line and have labeled a number of points with their corresponding numbers. Each point on the line corresponds to a number (positive, negative, or zero).

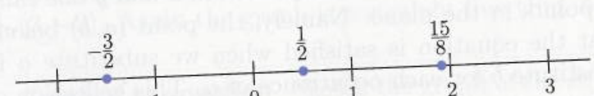


Figure 1

1.2 Linear Inequalities

In this section we study the properties of inequalities. Start with a Cartesian coordinate system on the line (Fig. 1). Recall that it is possible to associate with each point of the line a number; and conversely, with each number (positive, negative, or zero) it is possible to associate a point on the line.

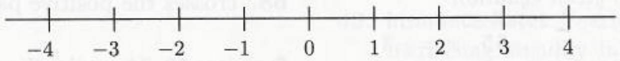


Figure 1

Let a and b be any numbers. We say that ***a* is less than *b*** if a lies to the left of b on the line. When a is less than b, we write $a < b$ (read: a is less than b). Thus, for example, $2 < 3$, $-1 < 2$, $-3 < -1$, and $0 < 4$. When a is less than b, we also say that ***b* is greater than *a*** ($b > a$). Thus, for example, $3 > 2$, $2 > -1$, $-1 > -3$, and $4 > 0$. Sometimes it is convenient to have a notation that means that a is no larger than b. The notation used for this is $a \leq b$ (read: a is less than or equal to b). Similarly, the notation $a \geq b$ (read: a is greater than or equal to b) means that a is no smaller than b. The symbols $>$, $<$, $\geq$, and $\leq$ are called **inequality signs.** It is easiest to remember the meaning of these various symbols by noting that the symbols $>$ and $<$ always point toward the smaller of a and b.

An inequality expresses a relationship between the quantities on both of its sides. This relationship is very similar to the relationship expressed by an equation. And just as some problems require solving equations, others require solving inequalities. Our next task will be to state and illustrate the arithmetic operations permissible in dealing with inequalities. These permissible operations form the basis of a technique for solving the inequalities occurring in applications.

Inequality Property 1 Suppose that $a < b$ and that c is any number. Then $a + c < b + c$ and $a - c < b - c$. In other words, the same number can be added or subtracted from both sides of the inequality.

For example, start with the inequality $2 < 3$ and add 4 to both sides to get

$$2 + 4 < 3 + 4.$$

That is,

$$6 < 7,$$

a correct inequality.

EXAMPLE 1 **Solving a linear inequality** Solve the inequality $x + 4 > 3$. That is, determine all values of x for which the inequality holds.

Solution We proceed as we would in dealing with an equation. Isolate x on the left by subtracting 4 from both sides, which is permissible by Inequality Property 1:

$$x + 4 > 3$$
$$(x + 4) - 4 > 3 - 4$$
$$x > -1.$$

That is, the values of x for which the inequality holds are exactly those x greater than -1. ■

In dealing with an equation, both sides may be multiplied or divided by a number. However, multiplying or dividing an inequality by a number requires some care. The result depends on whether the number is positive or negative. More precisely,

Inequality Property 2

2A. If $a < b$ and c is positive, then $ac < bc$.

2B. If $a < b$ and c is negative, then $ac > bc$.

In other words, an inequality may be multiplied by a positive number, just as in the case of equations. But to multiply an inequality by a negative number, it is necessary to reverse the inequality sign. For example, the inequality $-1 < 2$ can be multiplied by 4 to get $-4 < 8$, a correct statement. But if we were to multiply by -4, it would be necessary to reverse the inequality sign, because -4 is negative. In this latter case we would get $4 > -8$, a correct statement.

NOTE Inequality Properties 1 and 2 are stated using only $<$. However, exactly the same properties hold if $<$ is replaced by $>$, $\leq$, or $\geq$.

EXAMPLE 2 **Solving a linear inequality** Solve the inequality $-3x + 2 \geq -1$.

Solution Treat the inequality as if it were an equation. The goal is to isolate x on one side. To this end, first subtract 2 from both sides (Property 1). This gives

$$(-3x + 2) - 2 \geq -1 - 2$$
$$-3x \geq -3.$$

Next, we multiply by $-\frac{1}{3}$. (This gives x on the left.) But $-\frac{1}{3}$ is negative, so by Property 2B we must reverse the inequality sign. Thus,

$$-\tfrac{1}{3}(-3x) \leq -\tfrac{1}{3}(-3)$$
$$x \leq 1.$$

Therefore, the values of x satisfying the inequality are precisely those values that are ≤ 1.

Now Try Exercise 7

The inequalities of greatest interest to us[1] are those in two variables, x and y, and having the form $cx + dy \leq e$ or $cx + dy \geq e$, where c, d, and e are given numbers, with c and d not both 0. We will call such inequalities **linear inequalities**. When $d \neq 0$ (that is, when y actually appears), the inequality can be put into one of the **standard forms** $y \leq mx + b$ or $y \geq mx + b$. When $d = 0$, the inequality can be put into one of the standard forms $x \leq a$ or $x \geq a$. The procedure for putting a linear inequality into standard form is analogous to that for putting a linear equation into standard form.

EXAMPLE 3 **Standard form of a linear inequality** Put the linear inequality $2x - 3y \geq -9$ into standard form.

Solution Since we want the y-term on the left and all other terms on the right, begin by subtracting $2x$ from both sides:

$$-3y \geq -2x - 9.$$

[1]Such inequalities arise in our discussion of linear programming in Chapter 3.

Next, multiply by $-\frac{1}{3}$, remembering to change the inequality sign, since $-\frac{1}{3}$ is negative:

$$y \le -\tfrac{1}{3}(-2x - 9)$$
$$y \le \tfrac{2}{3}x + 3.$$

Now Try Exercise 9 The last inequality is in standard form. ■

EXAMPLE 4 **Standard form of a linear inequality** Find the standard form of the inequality $\frac{1}{2}x \ge 4$.

Solution Note that y does not appear in the inequality. Just as was the case in finding the standard form of a linear equation when y does not appear, solve for x. To do this, multiply by 2 to get

$$x \ge 8,$$

Now Try Exercise 13 the standard form. ■

Graphing Linear Inequalities Associated with every linear inequality is a set of points in the plane, the set of all those points that satisfy the inequality. This set of points is called the **graph** of the inequality.

EXAMPLE 5 **Solution of a linear inequality** Determine whether or not the given point satisfies the inequality $y \ge -\frac{2}{3}x + 4$.

(a) $(3, 4)$ (b) $(0, 0)$

Solution Substitute the x-coordinate of the point for x and the y-coordinate for y, and determine if the resulting inequality is correct or not.

(a) $4 \ge -\frac{2}{3}(3) + 4$
$4 \ge -2 + 4$
$4 \ge 2$ (correct)

(b) $0 \ge -\frac{2}{3}(0) + 4$
$0 \ge 0 + 4$
$0 \ge 4$ (not correct)

Now Try Exercises 15 and 17 Therefore, the point $(3, 4)$ satisfies the inequality and the point $(0, 0)$ does not. ■

It is easiest to determine the graph of a given inequality after it has been written in standard form. Therefore, let us describe the graphs of each of the standard forms. The easiest to handle are the forms $x \ge a$ and $x \le a$.

A point satisfies the inequality $x \ge a$ if, and only if, its x-coordinate is greater than or equal to a. The y-coordinate can be anything. Therefore, the graph of $x \ge a$ consists of all points to the right of and on the vertical line $x = a$. We will display the graph by crossing out the portion of the plane to the left of the line (see Fig. 2). Similarly, the graph of $x \le a$ consists of the points to the left of and on the line $x = a$. This graph is shown in Fig. 3.

Here is a simple procedure for graphing the other two standard forms.

To graph $y \ge mx + b$ or $y \le mx + b$,

1. Draw the graph of $y = mx + b$.
2. Throw away, that is, "cross out," the portion of the plane not satisfying the inequality. The graph of $y \ge mx + b$ consists of all points above or on the line. The graph of $y \le mx + b$ consists of all points below or on the line.

The graphs of the inequalities $y \ge mx + b$ and $y \le mx + b$ are shown in Fig. 4. Some simple reasoning suffices to show why these graphs are correct. Draw the line

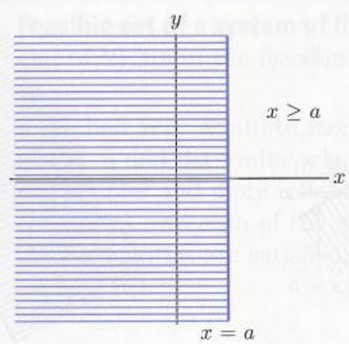


Figure 2

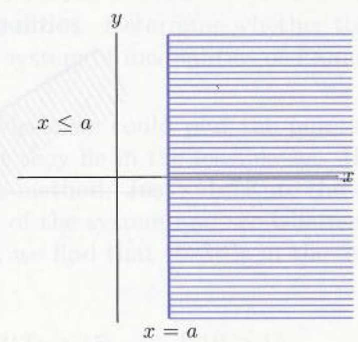


Figure 3

$y = mx + b$, as in Fig. 5, and pick any point P above the line. Suppose that P has coordinates (x, y). Let Q be the point on the line that lies directly below P. Then Q has the same first coordinate as P, that is, x. Since Q lies on the line, the second coordinate of Q is $mx + b$. Since the second coordinate of P is clearly larger than the second coordinate of Q, we must have $y \geq mx + b$. Similarly, any point below the line satisfies $y \leq mx + b$. Thus the two graphs are as given in Fig. 4.

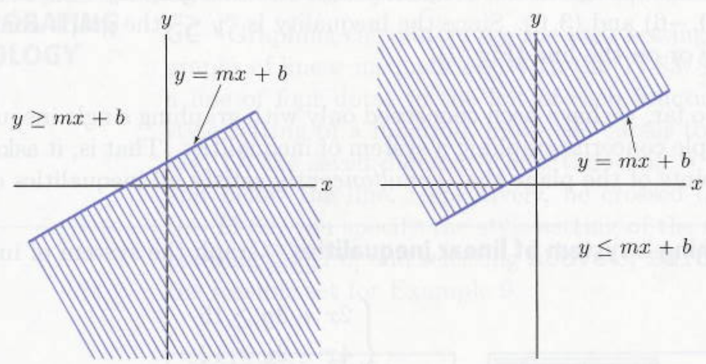


Figure 4

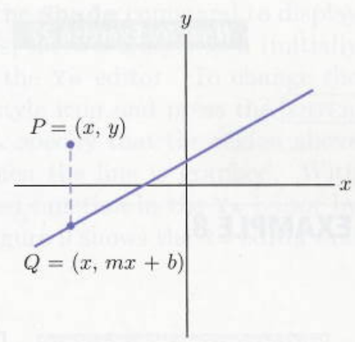


Figure 5

EXAMPLE 6 **Graphing a linear inequality** Graph the inequality $2x + 3y \geq 15$.

Solution In order to apply the aforementioned procedure, the inequality must first be put into standard form:

$$2x + 3y \geq 15$$

$$3y \geq -2x + 15 \quad \text{Subtract } 2x \text{ from both sides.}$$

$$y \geq -\tfrac{2}{3}x + 5 \quad \text{Divide both sides by 3.}$$

The last inequality is in standard form. Next, we graph the line $y = -\frac{2}{3}x + 5$. Its intercepts are $(0, 5)$ and $(\frac{15}{2}, 0)$. Since the inequality is "$y \geq$" we cross out the region below the line and label the region above with the inequality. The graph consists of all points above or on the line (Fig. 6 on the next page). ■

Now Try Exercise 31

EXAMPLE 7 **Graphing a linear inequality** Graph the inequality $4x - 2y \geq 12$.

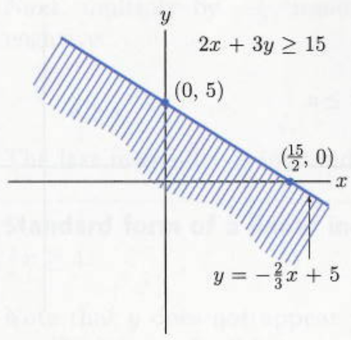


Figure 6

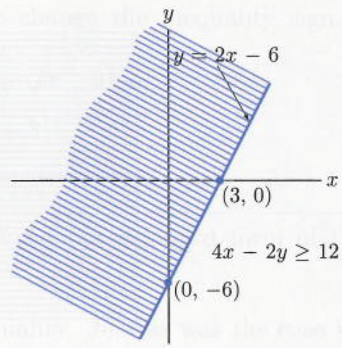


Figure 7

Solution First, put the inequality in standard form:

$$\begin{aligned} 4x - 2y &\geq 12 \\ -2y &\geq -4x + 12 \qquad \text{Subtract } 4x \text{ from both sides.} \\ y &\leq 2x - 6 \qquad \text{Divide both sides by } -2. \end{aligned}$$

(Note the change in the inequality sign!). Next, graph $y = 2x - 6$. The intercepts are $(0, -6)$ and $(3, 0)$. Since the inequality is "$y \leq$" the graph consists of all points below or on the line (Fig. 7).

Now Try Exercise 27

So far, we have been concerned only with graphing single inequalities. The next example concerns graphing a system of inequalities. That is, it asks us to determine all points of the plane that *simultaneously* satisfy all inequalities of a system.

EXAMPLE 8

Graphing a system of linear inequalities Graph the system of inequalities

$$\begin{cases} 2x + 3y \geq 15 \\ 4x - 2y \geq 12 \\ \qquad\; y \geq 0. \end{cases}$$

Solution The first two inequalities have already been graphed in Examples 6 and 7. The graph of $y \geq 0$ consists of all points above or on the x-axis. In Fig. 8, any point that is crossed out is *not* on the graph of at least one inequality. So the points that simultaneously satisfy all three inequalities are those in the remaining clear region and its border.

Now Try Exercise 41

REMARK At first, our convention of crossing out those points *not* on the graph of an inequality (instead of shading the points *on* the graph) may have seemed odd. However the real advantage of this convention becomes apparent when graphing a system of inequalities. Imagine trying to find the graph of the system of Example 8 if the points *on* the graph of each inequality had been shaded. It would have been necessary to locate the points that had been shaded three times. This is hard to do.

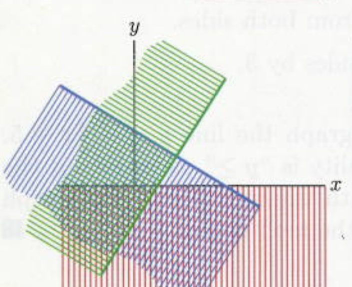


Figure 8

The graph of a system of inequalities is called a **feasible set**. The feasible set associated to the system of Example 8 is a three-sided, unbounded region.

Given a specific point, we should be able to decide whether or not the point lies in the feasible set. The next example shows how this is done.

EXAMPLE 9 **Feasible set of a system of linear inequalities** Determine whether the points (5, 3) and (4, 2) are in the feasible set of the system of inequalities of Example 8.

Solution If we had very accurate measuring devices, we could plot the points in the graph of Fig. 8 and determine whether or not they lie in the feasible set. However, there is a simpler and more reliable algebraic method. Just substitute the coordinates of the points into each of the inequalities of the system and see whether or not *all* of the inequalities are satisfied. So doing, we find that (5, 3) is in the feasible set and (4, 2) is not.

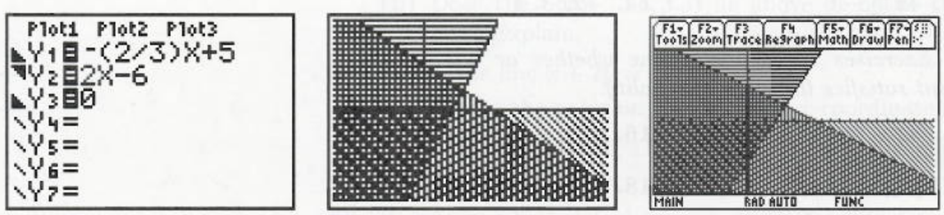
$$(5,3) \quad \begin{cases} 2(5)+3(3) \geq 15 \\ 4(5)-2(3) \geq 12 \\ (3) \geq 0 \end{cases} \quad \begin{cases} 19 \geq 15 \\ 14 \geq 12 \\ 3 \geq 0 \end{cases}$$

$$(4,2) \quad \begin{cases} 2(4)+3(2) \geq 15 \\ 4(4)-2(2) \geq 12 \\ (2) \geq 0 \end{cases} \quad \begin{cases} 14 \geq 15 \\ 12 \geq 12 \\ 2 \geq 0 \end{cases}$$

Now Try Exercises 45 and 47

INCORPORATING TECHNOLOGY

GC Graphing calculators use the *style* setting and the **Shade** command to display graphs of linear inequalities. With the TI-83/84 Plus, there is a style icon (initially a line of four dots) to the left of each function in the **Y=** editor. To change the style setting of a function, move the cursor to the style icon and press the [ENTER] key. Of the seven different icons, the icons ◥ and ◣ specify that the region above and below the line, respectively, be crossed out when the line is graphed. With the TI-89, you specify the style setting of the selected function in the **Y=** editor by pressing [2nd][F6] and selecting **Above** or **Below**. Figure 9 shows the **Y=** editor and the feasible set for Example 9.



(a) (b) TI-83/84 Plus (c) TI-89

Figure 9

Regions to the left and right, respectively, of the vertical line $x = k$ can be crossed out with the commands

`Shade(c-1,d+1,a,k,2,3)`	and	`Shade(c-1,d+1,k,b,2,3)`	[TI-83/84 Plus]
`Shade ymin,ymax,xmin,k`	and	`Shade ymin,ymax,k,xmax`	[TI-89]

where the window setting of the TI-83/84 Plus is $[a, b]$ *by* $[c, d]$. With the TI-83/84 Plus, the **Shade** command is invoked from the home screen with [2nd][DRAW]. With the TI-89, **Shade** can typed directly into the entry line or selected from the CATALOG menu.

Practice Problems 1.2

1. Graph the inequality $3x - y \geq 3$.

2. Graph the feasible set for the system of inequalities

$$\begin{cases} x \geq 0,\ y \geq 0 \\ x + 2y \leq \ \ 4 \\ 4x - 4y \geq -4. \end{cases}$$

EXERCISES 1.2

In Exercises 1–4, state whether the inequality is true or false.

1. $2 \leq -3$ **2.** $-2 \leq 0$ **3.** $7 \leq 7$ **4.** $0 \geq \frac{1}{2}$

In Exercises 5–7, solve for x.

5. $2x - 5 \geq 3$ **6.** $3x - 7 \leq 2$

7. $-5x + 13 \leq -2$

8. Which of the following results from solving $-x + 1 \leq 3$ for x?

(a) $x \leq 4$ (b) $x \leq 2$

(c) $x \geq -4$ (d) $x \geq -2$

In Exercises 9–14, put the linear inequality into standard form.

9. $2x + y \leq 5$ **10.** $3x + y \geq 1$

11. $5x - \frac{1}{3}y \leq 6$ **12.** $\frac{1}{2}x - y \leq -1$

13. $4x \geq -3$ **14.** $-2x \leq 4$

In Exercises 15–22, determine whether or not the given point satisfies the given inequality.

15. $3x + 5y \leq 12$, $(2, 1)$ **16.** $-2x + y \geq 9$, $(3, 15)$

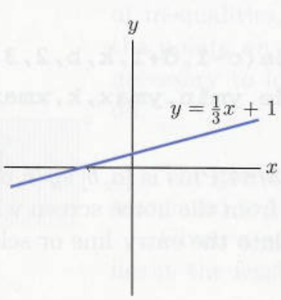
17. $y \geq -2x + 7$, $(3, 0)$ **18.** $y \leq \frac{1}{2}x + 3$, $(4, 6)$

19. $y \leq 3x - 4$, $(3, 5)$ **20.** $y \geq x$, $(-3, -2)$

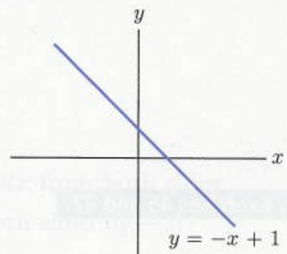
21. $x \geq 5$, $(7, -2)$ **22.** $x \leq 7$, $(0, 0)$

In Exercises 23–26, graph the given inequality by crossing out (i.e., discarding) the points not satisfying the inequality.

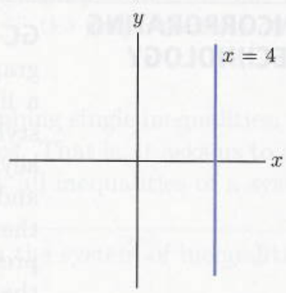
23. $y \leq \frac{1}{3}x + 1$



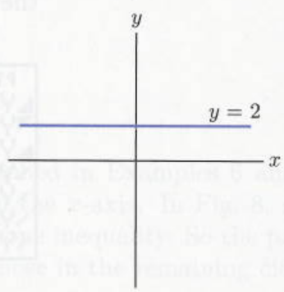
24. $y \geq -x + 1$



25. $x \geq 4$



26. $y \leq 2$



In Exercises 27–38, graph the given inequality.

27. $y \leq 2x + 1$ **28.** $y \geq -3x + 6$

29. $x \geq 2$ **30.** $x \geq 0$

31. $x + 4y \geq 12$ **32.** $4x - 4y \geq 8$

33. $4x - 5y + 25 \geq 0$ **34.** $.1y - x \geq .2$

35. $\frac{1}{2}x - \frac{1}{3}y \leq 1$ **36.** $3y + \frac{1}{2}x \leq 2y + x + 1$

37. $.5x + .4y \leq 2$ **38.** $y - 2x \geq \frac{1}{2}y - 2$

In Exercises 39–44, graph the feasible set for the system of inequalities.

39. $\begin{cases} y \leq 2x - 4 \\ y \geq 0 \end{cases}$ **40.** $\begin{cases} y \geq -\frac{1}{3}x + 1 \\ x \geq 0 \end{cases}$

41. $\begin{cases} x + 2y \geq 2 \\ 3x - \ \ y \geq 3 \end{cases}$ **42.** $\begin{cases} 3x + 6y \geq 24 \\ 3x + \ \ y \geq \ 6 \end{cases}$

43. $\begin{cases} x + 5y \leq \ 10 \\ x + \ \ y \leq 3 \\ x \geq 0, \ y \geq 0 \end{cases}$ **44.** $\begin{cases} x + 2y \geq 6 \\ x + \ \ y \geq 5 \\ x \geq 1 \end{cases}$

In Exercises 45–48, determine whether the given point is in the feasible set of this system of inequalities:

$$\begin{cases} 6x + 3y \leq 96 \\ x + \ \ y \leq 18 \\ 2x + 6y \leq 72 \\ x \geq 0, \ y \geq 0. \end{cases}$$

45. $(8, 7)$ **46.** $(14, 3)$ **47.** $(9, 10)$ **48.** $(16, 0)$

In Exercises 49–52, determine whether the given point is above or below the given line.

49. $y = 2x + 5$, $(3, 9)$ **50.** $3x - y = 4$, $(2, 3)$

51. $7 - 4x + 5y = 0$, $(0, 0)$ **52.** $x = 2y + 5$, $(6, 1)$

53. Give a system of inequalities for which the graph is the region between the pair of lines $8x - 4y - 4 = 0$ and $8x - 4y = 0$.

54. PE The shaded region in Fig. 10 is bounded by four straight lines. Which of the following is NOT an equation of one of the boundary lines?

(a) $y = 0$ (b) $y = 2$ (c) $x = 0$

(d) $2x + 3y = 12$ (e) $3x + 2y = 12$

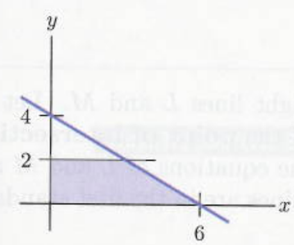


Figure 10

55. PE The shaded region in Fig. 11 is bounded by four straight lines. Which of the following is NOT an equation of one of the boundary lines?

(a) $x = 0$ (b) $y = x$ (c) $y = 5$

(d) $y = 0$ (e) $x + 2y = 6$

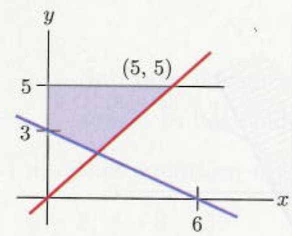


Figure 11

56. PE Which quadrant in Fig. 12 contains no points that satisfy the inequality $x + 2y \geq 6$?

(a) none (b) I (c) II (d) III (e) IV

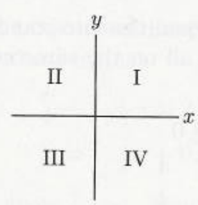


Figure 12

57. PE Which quadrant in Fig. 12 contains no points that satisfy the inequality $3y - 2x \geq 6$?

(a) none (b) I (c) II (d) III (e) IV

Use a graphing calculator to solve Exercises 58–61.

58. Graph the line $4x - 2y = 7$.

(a) Locate the point on the line with x-coordinate 3.6.

(b) Does the point $(3.6, 3.5)$ lie above or below the line? Explain.

59. Graph the line $x + 2y = 11$.

(a) Locate the point on the line with x-coordinate 6.

(b) Does the point $(6, 2.6)$ lie above or below the line? Explain.

60. If your graphing calculator can shade feasible sets of inequalities, display the feasible set in Exercise 39.

61. If your graphing calculator can shade feasible sets of inequalities, display the feasible set in Exercise 42.

Solutions to Practice Problems 1.2

1. Linear inequalities are easiest to graph if they are first put into standard form. Subtract $3x$ from both sides and multiply by -1:

$$3x - y \geq 3$$
$$-y \geq -3x + 3$$
$$y \leq 3x - 3.$$

Now graph the line $y = 3x - 3$ (Fig. 13 on the next page). The graph of the inequality is the portion of the plane below and on the line ("$\leq$" corresponds to below), so throw away (that is, cross out) the portion above the line.

(Continued)

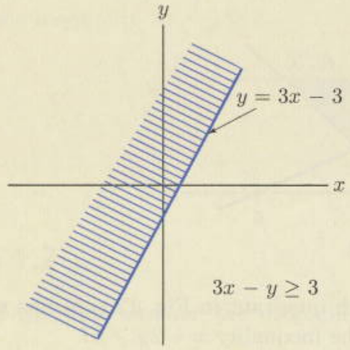


Figure 13

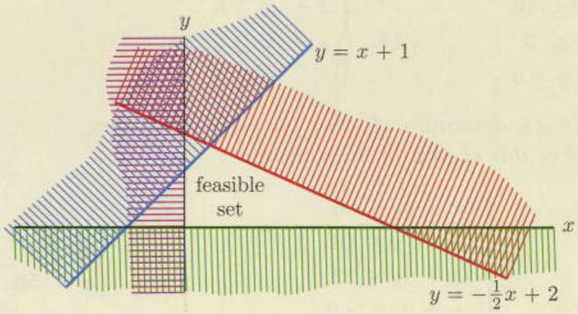


Figure 14

2. Begin by putting the linear inequalities into standard form and then graphing them all on the same coordinate system.

$$\begin{cases} x \geq 0,\ y \geq 0 \\ x + 2y \leq \ \ 4 \\ 4x - 4y \geq -4 \end{cases}$$

has standard form

$$\begin{cases} x \geq 0,\ y \geq 0 \\ y \leq -\frac{1}{2}x + 2 \\ y \leq \quad x + 1 \end{cases}$$

A good procedure to follow is to graph all of the linear equations and then cross out the regions to be thrown away one at a time (Fig. 14). The inequalities $x \geq 0$ and $y \geq 0$ arise frequently in applications. The first has the form $x \geq a$, where $a = 0$, and the second has the form $y \geq mx + b$, where $m = 0$ and $b = 0$. To graph them, just cross out all points to the left of the y-axis and all points below the x-axis, respectively.

1.3 The Intersection Point of a Pair of Lines

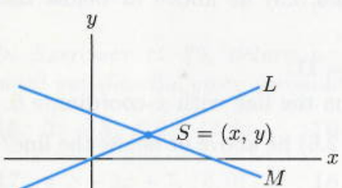


Figure 1

Suppose that we are given a pair of intersecting straight lines L and M. Let us consider the problem of determining the coordinates of the **point of intersection** $S = (x, y)$ (see Fig. 1). We may as well assume that the equations of L and M are given in standard form. First, let us assume that both lines are in the first standard form—that is, that the equations are

$$L\colon\ y = mx + b, \qquad M\colon\ y = nx + c.$$

Since the point S is on both lines, its coordinates satisfy both equations. In particular, we have two expressions for its y-coordinate:

$$y = mx + b = nx + c.$$

The last equality gives an equation from which x can easily be determined. Then the value of y can be determined as $mx + b$ (or $nx + c$). Let us see how this works in a particular example.

EXAMPLE 1 **Finding the point of intersection** Find the point of intersection of the two lines $y = 2x - 3$ and $y = x + 1$.

Solution To find the x-coordinate of the point of intersection, equate the two expressions for y and solve for x:

$$2x - 3 = x + 1$$
$$x - 3 = 1 \quad \text{Subtract } x \text{ from both sides.}$$
$$x = 4 \quad \text{Add 3 to both sides.}$$

To find the value of y, set $x = 4$ in either equation, say the first. Then

$$y = 2 \cdot 4 - 3 = 5.$$

Now Try Exercise 1 So the point of intersection is $(4, 5)$. ■

EXAMPLE 2 **Finding the point of intersection** Find the point of intersection of the two lines $x + 2y = 6$ and $5x + 2y = 18$.

Solution To use the method described, the equations must be in standard form. Solving both equations for y, we get the standard forms

$$y = -\tfrac{1}{2}x + 3$$
$$y = -\tfrac{5}{2}x + 9.$$

Equating the expressions for y gives

$$-\tfrac{1}{2}x + 3 = -\tfrac{5}{2}x + 9$$
$$\tfrac{5}{2}x - \tfrac{1}{2}x + 3 = 9 \quad \text{Add } \tfrac{5}{2}x \text{ to both sides.}$$
$$2x + 3 = 9 \quad \text{Combine } x \text{ terms.}$$
$$2x = 6 \quad \text{Subtract 3 from both sides.}$$
$$x = 3 \quad \text{Divide both sides by 2.}$$

Setting $x = 3$ in the first equation gives

$$y = -\tfrac{1}{2}(3) + 3 = \tfrac{3}{2}.$$

Now Try Exercise 3 So the intersection point is $(3, \frac{3}{2})$. ■

The preceding method works when both equations have the first standard form $(y = mx + b)$. In case one equation has the standard form $x = a$, things are much simpler. The value of x is then given directly without any work, namely $x = a$. The value of y can be found by substituting a for x in the other equation.

EXAMPLE 3 **Finding the point of intersection** Find the point of intersection of the lines $y = 2x - 1$ and $x = 2$.

Solution The x-coordinate of the intersection point is 2, and the y-coordinate is $y = 2 \cdot 2 - 1 = 3$. Therefore, the intersection point is $(2, 3)$. ■
Now Try Exercise 5

The method just introduced may be used to solve systems of two equations in two variables.

EXAMPLE 4 **Solving a system of equations** Solve the following system of linear equations:

$$\begin{cases} 2x + 3y = 7 \\ 4x - 2y = 9. \end{cases}$$

A Cartesian coordinate system may be used to numerically describe points on a line. In a similar fashion, we can construct a Cartesian coordinate system to numerically locate points on a plane. Each point of the plane is identified by a pair of numbers (a, b). To reach the point (a, b), begin at the origin, move a units in the x-direction (to the right if a is positive, to the left if a is negative), and then move b units in the y direction (up if b is positive, down if b is negative). The numbers a and b are called, respectively, the **x- and y-coordinates** of the point.

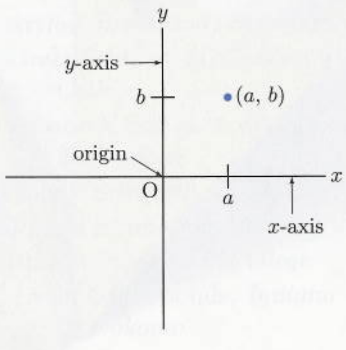


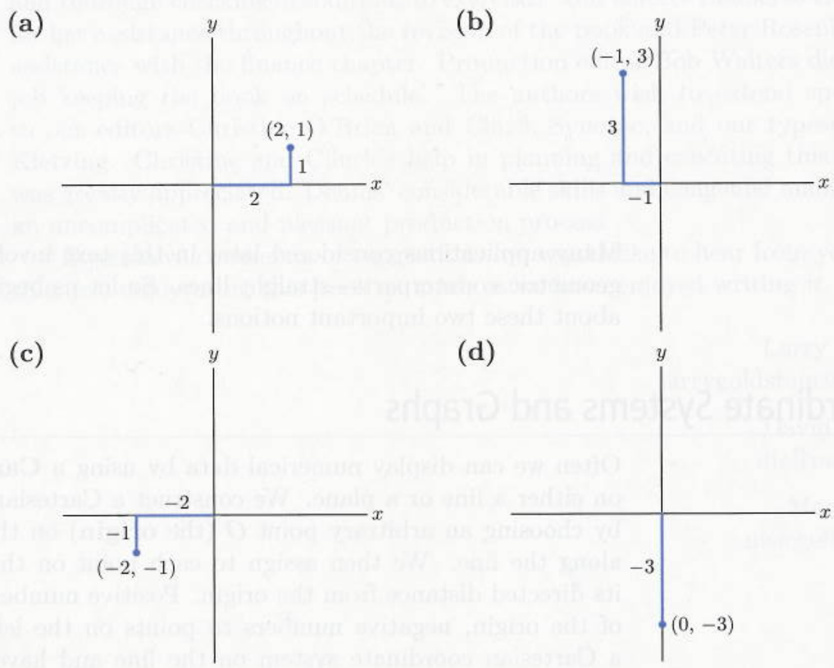
Figure 2

EXAMPLE 1

Plotting points Plot the following points.

(a) $(2, 1)$ **(b)** $(-1, 3)$ **(c)** $(-2, -1)$ **(d)** $(0, -3)$

Solution



Now Try Exercise 1 ■

In many applications one encounters equations expressing relationships between variables x and y. To any equation in x and y one can associate a certain collection of points in the plane. Namely, the point (a, b) belongs to the collection provided that the equation is satisfied when we substitute a for each occurrence of x and substitute b for each occurrence of y. This collection of points is usually a curve of some sort and is called the **graph of the equation**.

Solution First convert the equations to standard form:

$$2x + 3y = 7$$
$$3y = -2x + 7 \qquad \text{Subtract } 2x \text{ from both sides.}$$
$$y = -\tfrac{2}{3}x + \tfrac{7}{3} \qquad \text{Divide both sides by 3.}$$

$$4x - 2y = 9$$
$$-2y = -4x + 9 \qquad \text{Subtract } 4x \text{ from both sides.}$$
$$y = 2x - \tfrac{9}{2} \qquad \text{Divide both sides by } -2.$$

Now equate the two expressions for y:

$$2x - \tfrac{9}{2} = -\tfrac{2}{3}x + \tfrac{7}{3}$$
$$\tfrac{8}{3}x - \tfrac{9}{2} = \tfrac{7}{3} \qquad \text{Add } \tfrac{2}{3}x \text{ to both sides.}$$
$$\tfrac{8}{3}x = \tfrac{7}{3} + \tfrac{9}{2} \qquad \text{Add } \tfrac{9}{2} \text{ to both sides.}$$
$$\tfrac{8}{3}x = \tfrac{14}{6} + \tfrac{27}{6} = \tfrac{41}{6} \qquad \text{Add fractions on right.}$$
$$x = \tfrac{3}{8} \cdot \tfrac{41}{6} = \tfrac{41}{16} \qquad \text{Multiply both sides by } \tfrac{3}{8}.$$
$$y = 2x - \tfrac{9}{2} = 2\left(\tfrac{41}{16}\right) - \tfrac{9}{2} \qquad \text{Substitute value for } x \text{ into second equation.}$$
$$y = \tfrac{41}{8} - \tfrac{36}{8} = \tfrac{5}{8} \qquad \text{Perform arithmetic.}$$

Now Try Exercise 9 So the solution of the given system is $x = \frac{41}{16}$, $y = \frac{5}{8}$.

Supply and Demand Curves The price p that a commodity sells for is relate to the quantity q available. Economists study two kinds of graphs that expres relationships between q and p. To describe these graphs, let us plot *quantity* along the horizontal axis and *price* along the vertical axis. The first graph relating and p is called a **supply curve** (Fig. 2) and expresses the relationship between and p from a manufacturer's point of view. For every quantity q, the supply curv specifies the price p for which the manufacturer is willing to produce the quantit q. The greater the quantity to be supplied, the higher the price must be. So suppl curves rise when viewed from left to right.

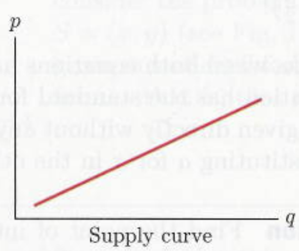


Figure 2

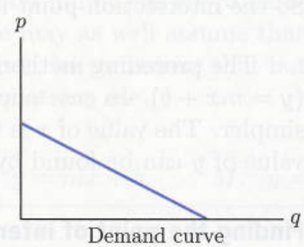


Figure 3

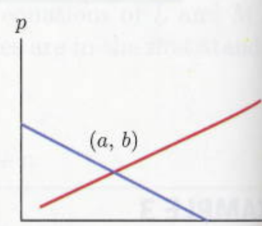


Figure 4

The second curve relating q and p is called a **demand curve** (Fig. 3) a expresses the relationship between q and p from the consumer's viewpoint. F each quantity q, the demand curve gives the price p that must be charged in ord for q units of the commodity to be sold. The greater the quantity that must b sold, the lower the price that consumers must be asked to pay. So demand curv fall when viewed from left to right.

Suppose the supply and demand curves for a commodity are drawn on a sing coordinate system (Fig. 4). The intersection point (a, b) of the two curves has economic significance: The quantity produced will stabilize at a units and the pr will be b dollars per unit.

EXAMPLE 5 **Applying the law of supply and demand** Suppose the supply curve for a certain commodity is the straight line whose equation is $p = .0002q + 2$ (p in dollars). Suppose the demand curve for the same commodity is the straight line whose equation is $p = -.0005q + 5.5$. Determine both the quantity of the commodity that will be produced and the price at which it will sell.

Solution We must solve the system of linear equations

$$\begin{cases} p = .0002q + 2 \\ p = -.0005q + 5.5 \end{cases}$$

$$.0002q + 2 = -.0005q + 5.5$$

$.0007q + 2 = 5.5$ Add $.0005q$ to both sides.

$.0007q = 3.5$ Subtract 2 from both sides.

$q = \frac{3.5}{.0007} = 5000$ Divide both sides by .0007.

$p = .0002(5000) + 2$ Substitute value for q into first equation.

$p = 1 + 2 = 3$ Perform arithmetic.

Thus 5000 units of the commodity will be produced and it will sell for \$3 per unit. ■

Now Try Exercise 27

INCORPORATING TECHNOLOGY

GC Graphing utilities have commands that find the intersection point of a pair of lines. Figure 5 shows the result of solving Example 4 on a TI-83/84 Plus with the intersect command of the CALC menu. Since the x-coordinate of the intersection point is assigned to ANS, the x-intercept can be converted to a fraction by pressing [MATH] 1 [ENTER] from the home screen. Figure 6 shows the result of solving Example 4 on a TI-89 with the Intersection routine in the menu resulting from pressing [F5]. ■

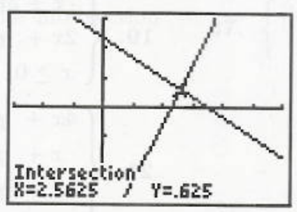


Figure 5. $[-3, 6]$ by $[-3, 3]$

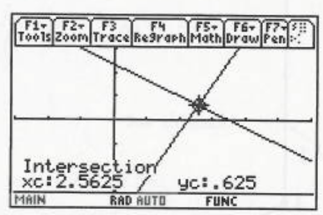


Figure 6. $[-3, 6]$ by $[-3, 3]$

Practice Problems 1.3

Figure 7 shows the feasible set of a system of linear inequalities; its four vertices are labeled A, B, C, and D.

1. Use the method of this section to find the coordinates of the point C.
2. Determine the coordinates of the points A and B by inspection.
3. Find the coordinates of the point D.

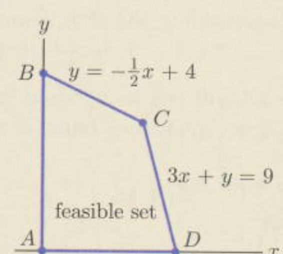


Figure 7

EXERCISES 1.3

In Exercises 1–6, find the point of intersection of the given pair of straight lines.

1. $\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$ **2.** $\begin{cases} y = 3x - 15 \\ y = -2x + 10 \end{cases}$

3. $\begin{cases} x - 4y = -2 \\ x + 2y = 4 \end{cases}$ **4.** $\begin{cases} 2x - 3y = 3 \\ y = 3 \end{cases}$

5. $\begin{cases} y = \frac{1}{3}x - 1 \\ x = 12 \end{cases}$ **6.** $\begin{cases} 2x - 3y = 3 \\ x = 6 \end{cases}$

7. Does $(6, 4)$ satisfy the following system of linear equations?

$$\begin{cases} x - 3y = -6 \\ 3x - 2y = 10 \end{cases}$$

8. Does $(12, 4)$ satisfy the following system of linear equations?

$$\begin{cases} y = \frac{1}{3}x - 1 \\ x = 12 \end{cases}$$

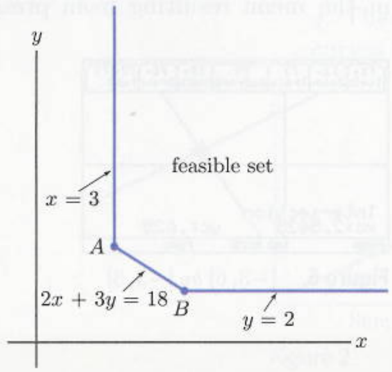
In Exercises 9–12, solve the systems of linear equations.

9. $\begin{cases} 2x + y = 7 \\ x - y = 3 \end{cases}$ **10.** $\begin{cases} x + 2y = 4 \\ \frac{1}{2}x + \frac{1}{2}y = 3 \end{cases}$

11. $\begin{cases} 5x - 2y = 1 \\ 2x + y = -4 \end{cases}$ **12.** $\begin{cases} x + 2y = 6 \\ x - \frac{1}{3}y = 4 \end{cases}$

In Exercises 13–16, find the coordinates of the vertices of the feasible sets.

13.



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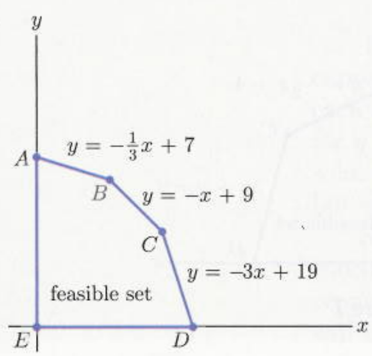

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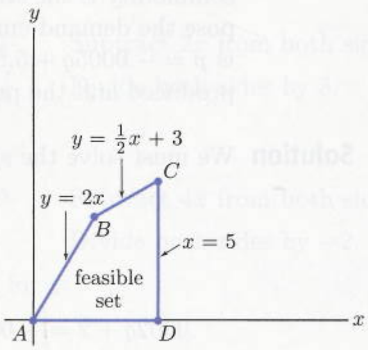

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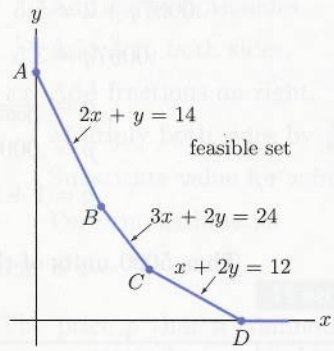


In Exercises 17–22, graph the feasible set for the system of inequalities and find the coordinates of the vertices.

17. $\begin{cases} 2y - x \le 6 \\ x + 2y \ge 10 \\ x \le 6 \end{cases}$ **18.** $\begin{cases} 2x + y \ge 10 \\ x \ge 2 \\ y \ge 2 \end{cases}$

19. $\begin{cases} x + 3y \le 18 \\ 2x + y \le 16 \\ x \ge 0,\ y \ge 0 \end{cases}$ **20.** $\begin{cases} 5x + 2y \ge 14 \\ x + 3y \ge 8 \\ x \ge 0,\ y \ge 0 \end{cases}$

21. $\begin{cases} 4x + y \ge 8 \\ x + y \ge 5 \\ x + 3y \ge 9 \\ x \ge 0,\ y \ge 0 \end{cases}$ **22.** $\begin{cases} x + 4y \le 28 \\ x + y \le 10 \\ 3x + y \le 24 \\ x \ge 0,\ y \ge 0 \end{cases}$

23. Supply Curve The supply curve for a certain commodity is $p = .0001q + .05$.

(a) What price must be offered in order for 19,500 units of the commodity to be supplied?

(b) What prices result in no units of the commodity being supplied?

24. Demand Curve The demand curve for a certain commodity is $p = -.001q + 32.5$.

(a) At what price can 31,500 units of the commodity be sold?

(b) What quantities are so large that many units of the commodity cannot possibly all be sold no matter how low the price?

25. Supply and Demand Suppose that supply and demand for a certain commodity are described by the supply and demand curves of Exercises 23 and 24. Determine the quantity of the commodity that will be produced and the selling price.

26. Temperature Conversion The precise formula for converting Celsius degrees to Fahrenheit degrees is $F = \frac{9}{5}C + 32$. An easier-to-use formula that approximates the conversion is $F = 2C + 30$.

(a) Compare the values given by the two formulas for a temperature of 5°C.

(b) Compare the values given by the two formulas for a temperature of 20°C.

(c) For what Celsius temperature do the two formulas give the same Fahrenheit temperature?

27. Supply and Demand A discount book seller has determined that the supply curve for a certain author's newest paperback book is $p = \frac{1}{300}q + 13$. The demand curve for this book is $p = -.03q + 19$. What quantity of sales would result in supply exactly meeting demand, and for what price should the book be sold?

28. Time Apportionment A plant supervisor must apportion her 40-hour workweek between hours working on the assembly line and hours supervising the work of others. She is paid \$12 per hour for working and \$15 per hour for supervising. If her earnings for a certain week are \$504, how much time does she spend on each task?

29. Calling Card Options A calling card offers two methods of paying for a phone call. Method A charges 1 cent per minute but has a 45-cent connection fee. Method B charges 3.5 cents per minute but has no connection fee. Write the equations that show the total cost, y, of a call of x minutes for methods A and B and determine their intersection point. What does the intersection point represent?

30. Weight Determination PE In a wrestling competition, the total weight of the two contestants is 700 pounds. If twice the weight of the first contestant is 275 pounds more than the weight of the second contestant, what is the weight (in pounds) of the first contestant?

(a) 275 (b) 300 (c) 325 (d) 350 (e) 375

31. Sales Determination PE An appliance store sells a 15″ TV for \$280 and a 19″ TV of the same brand for \$400. During a one-week period the store sold 5 more 19″ TVs than 15″ TVs and collected \$15,600. What was the total number of TV sets sold?

(a) 42 (b) 43 (c) 44 (d) 45 (e) 46

Exercises 32–37 require the use of a graphing calculator.

In Exercises 32–35, graph the lines and use the Intersect *command to estimate the point of intersection to two decimal places.*

32. $\begin{cases} y = .25x + 1.3 \\ y = -.5x + 4.1 \end{cases}$

33. $\begin{cases} y = -\frac{2}{3}x + 4.5 \\ y = 2x \end{cases}$

34. $\begin{cases} x - 4y = -5 \\ 3x - 2y = 4.2 \end{cases}$

35. $\begin{cases} 2x + 3y = 5 \\ -4x + 5y = 1 \end{cases}$

In Exercises 36 and 37, (a) graph the feasible set for the system of inequalities (use a window such as `ZDecimal` *or* `ZoomDec` *that gives nice values of the coordinates of points); (b) find the coordinates of the vertices of the feasible set; (c) locate the given point with the cursor; (d) determine whether or not the given point lies in the feasible set.*

36. $\begin{cases} -x + 3y \geq 3 \\ .4x + y \geq 3.2 \end{cases}$ (3.2, 2)

37. $\begin{cases} 2x + y \geq 5 \\ x - 2y \leq 0 \end{cases}$ (2.2, 1.4)

Solutions to Practice Problems 1.3

1. Point C is the point of intersection of the lines with equations $y = -\frac{1}{2}x + 4$ and $3x + y = 9$. To use the method of this section, the second equation must be put into its standard form $y = -3x + 9$. Now equate the two expressions for y and solve.

$$-\tfrac{1}{2}x + 4 = -3x + 9$$
$$\tfrac{5}{2}x = 5$$
$$x = \tfrac{2}{5} \cdot 5 = 2$$
$$y = -\tfrac{1}{2}(2) + 4 = 3$$

Therefore, $C = (2, 3)$.

2. $A = (0, 0)$, because the point A is the origin. $B = (0, 4)$, because it is the y-intercept of the line with equation $y = -\frac{1}{2}x + 4$.

3. D is the x-intercept of the line $3x + y = 9$. Its first coordinate is found by setting $y = 0$ and solving for x.

$$3x + (0) = 9$$
$$x = 3$$

Therefore, $D = (3, 0)$.

1.4 The Slope of a Straight Line

As we have seen, any linear equation can be put into one of the two standard forms, $y = mx + b$ or $x = a$. In this section, let us exclude linear equations whose standard form is of the latter type. *Geometrically, this means that we will consider only nonvertical lines.*

DEFINITION Given a nonvertical line L with equation $y = mx + b$, the number m is called the **slope** of L. That is, the slope is the coefficient of x in the standard form of the equation of the line.

EXAMPLE 1 **Finding the slope of a line from its equation** Find the slopes of the lines having the following equations.

(a) $y = 2x + 1$ (b) $y = -\frac{3}{4}x + 2$ (c) $y = 3$ (d) $-8x + 2y = 4$

Solution (a) $m = 2$.

(b) $m = -\frac{3}{4}$.

(c) When we write the equation in the form $y = 0 \cdot x + 3$, we see that $m = 0$.

(d) First, we put the equation in standard form:

$$\begin{aligned} -8x + 2y &= 4 \\ 2y &= 8x + 4 && \text{Add } 8x \text{ to both sides.} \\ y &= 4x + 2 && \text{Divide both sides by 2.} \end{aligned}$$

Now Try Exercise 1

Thus $m = 4$. ■

The definition of the slope is given in terms of the standard form of the equation of the line. Let us give an alternative definition.

DEFINITION **Geometric Definition of Slope** Let L be a line passing through the points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$. Then the slope of L is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}. \tag{1}$$

That is, the slope is the difference in the y-coordinates divided by the difference in the x-coordinates, with both differences formed in the same order.

Before proving this definition equivalent to the first one given, let us show how it can be used.

EXAMPLE 2 **Finding the slope of a line from two points** Find the slope of the line passing through the points $(1, 3)$ and $(4, 6)$.

Solution We have

$$m = \frac{[\text{difference in } y\text{-coordinates}]}{[\text{difference in } x\text{-coordinates}]} = \frac{6 - 3}{4 - 1} = \frac{3}{3} = 1.$$

Thus $m = 1$. [Note that if we reverse the order of the points and use formula (1) to compute the slope, then we get

$$m = \frac{3 - 6}{1 - 4} = \frac{-3}{-3} = 1,$$

which is the same answer. The order of the points is immaterial. The important concern is to make sure that the differences in the x- and y-coordinates are formed in the same order.

Now Try Exercise 5

The slope of a line does not depend on which pair of points is chosen as (x_1, y_1) and (x_2, y_2). Consider the line $y = 4x - 3$ and two points $(1, 1)$ and $(3, 9)$, which are on the line. Using these two points, we calculate the slope to be

$$m = \frac{9-1}{3-1} = \frac{8}{2} = 4.$$

Now let's choose two other points on the line, say $(2, 5)$ and $(-1, -7)$, and use these points to determine m. We obtain

$$m = \frac{-7-5}{-1-2} = \frac{-12}{-3} = 4.$$

The two pairs of points give the same slope.

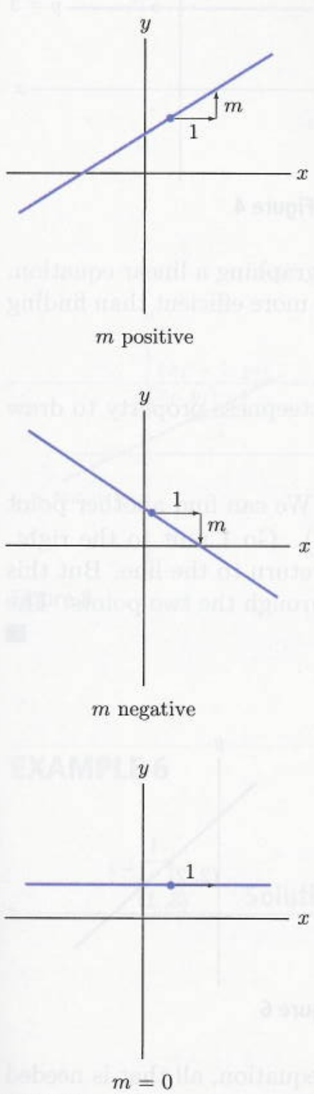


Figure 1

Justification of Formula (1) Since (x_1, y_1) and (x_2, y_2) are both on the line, both points satisfy the equation of the line, which has the form $y = mx + b$. Thus

$$y_2 = mx_2 + b$$
$$y_1 = mx_1 + b.$$

Subtracting these two equations gives

$$y_2 - y_1 = mx_2 - mx_1 = m(x_2 - x_1).$$

Dividing by $x_2 - x_1$, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

which is formula (1). So the two definitions of slope lead to the same number.

Let us now study four of the most important properties of the slope of a straight line. We begin with the **steepness property**, since it provides us with a geometric interpretation for the number m.

Steepness Property Let the line L have slope m. If we start at any point on the line and move 1 unit to the right, then we must move m units vertically in order to return to the line (Fig. 1). (Of course, if m is positive, then we move up; and if m is negative, we move down.)

EXAMPLE 3 **Steepness property of a line** Illustrate the steepness property for each of the lines.

(a) $y = 2x + 1$ (b) $y = -\frac{3}{4}x + 2$ (c) $y = 3$

Solution (a) Here $m = 2$. So starting from any point on the line, proceeding 1 unit to the right, we must go 2 units up to return to the line (Fig. 2 on the next page).

(b) Here $m = -\frac{3}{4}$. So starting from any point on the line, proceeding 1 unit to the right, we must go $\frac{3}{4}$ unit down to return to the line (Fig. 3).

(c) Here $m = 0$. So going 1 unit to the right requires going 0 units vertically to return to the line (Fig. 4).

Now Try Exercise 51

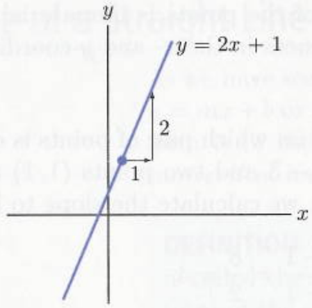


Figure 2

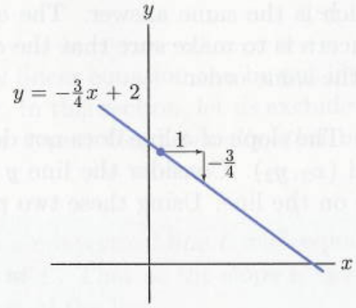


Figure 3

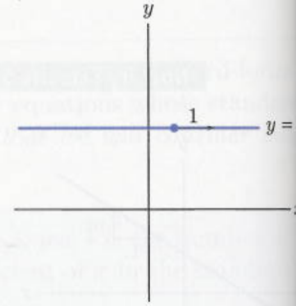


Figure 4

In the next example we introduce a new method for graphing a linear equatio
This method relies on the steepness property and is much more efficient than findin
two points on the line (e.g., the two intercepts).

EXAMPLE 4 **Using the steepness property to graph a line** Use the steepness property to dra the graph of $y = \frac{1}{2}x + \frac{3}{2}$.

Solution The y-intercept is $\left(0, \frac{3}{2}\right)$, as we read from the equation. We can find another poi on the line using the steepness property. Start at $\left(0, \frac{3}{2}\right)$. Go 1 unit to the righ Since the slope is $\frac{1}{2}$, we must move vertically $\frac{1}{2}$ unit to return to the line. But th locates a second point on the line. So we draw the line through the two points. Th entire procedure is illustrated in Fig. 5.

Now Try Exercise 11

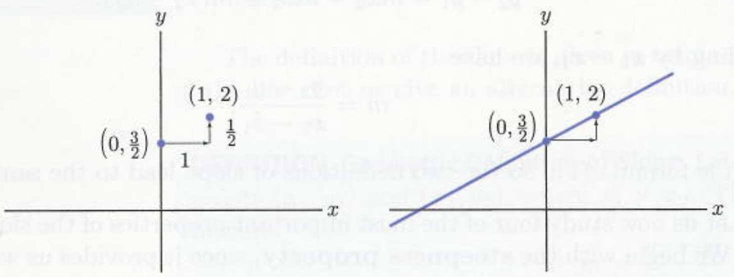


Figure 5

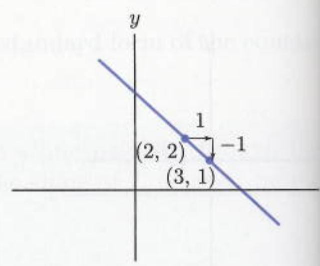


Figure 6

Actually, to use the steepness property to graph an equation, all that is neede is the slope plus *any* point (not necessarily the y-intercept).

EXAMPLE 5 **Using the steepness property to graph a line** Graph the line of slope −1 whic passes through the point (2, 2).

Solution Start at (2, 2), move 1 unit to the right and then −1 unit vertically, that is, 1 un down. The line through (2, 2) and the resulting point is the desired line (see Fig. 6

Now Try Exercise 25

Slope measures the steepness of a line. Namely, the slope of a line tells whethe it is rising or falling, and how fast. Specifically, lines of positive slope rise as w move from left to right. Lines of negative slope fall, and lines of zero slope sta level. The larger the magnitude of the slope, the steeper the ascent or descen These facts are directly implied by the steepness property (see Fig. 7).

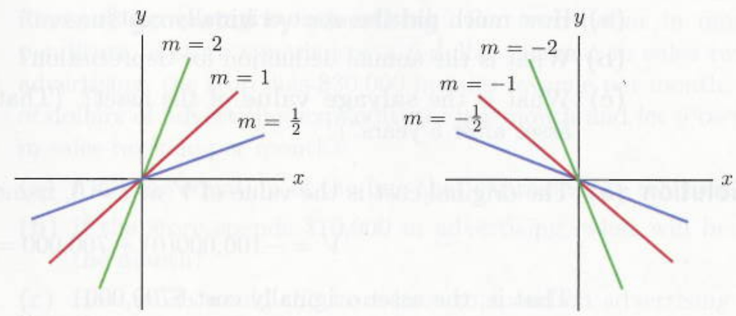


Figure 7

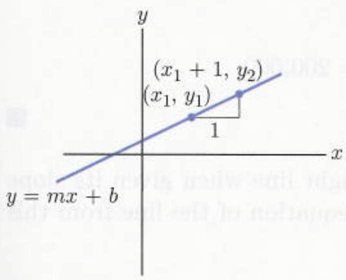


Figure 8

Justification of the Steepness Property Consider a line with equation $y = mx+b$, and let (x_1, y_1) be any point on the line. If we start from this point and move 1 unit to the right, the first coordinate of the new point is $x_1 + 1$ since the x-coordinate is increased by 1. Now go far enough vertically to return to the line. Denote the y-coordinate of this new point by y_2 (see Fig. 8). We must show that to get y_2, we add m to y_1. That is, $y_2 = y_1 + m$. By equation (1), we can compute m as

$$m = \frac{[\text{difference in } y\text{-coordinates}]}{[\text{difference in } x\text{-coordinates}]} = \frac{y_2 - y_1}{1} = y_2 - y_1.$$

In other words, $y_2 = y_1 + m$, which is what we desired to show. ■

Often the slopes of the straight lines that occur in applications have interesting and significant interpretations. An application in the field of economics is illustrated in the next example.

EXAMPLE 6 **Slope of the cost line** Suppose a manufacturer finds that the cost y of producing x units of a certain commodity is given by the formula $y = 2x + 5000$. What interpretation can be given to the slope of the graph of this equation?

Solution Suppose that the firm is producing at a certain level and increases production by 1 unit. That is, x is increased by 1 unit. By the steepness property, the value of y then increases by 2, which is the slope of the line whose equation is $y = 2x + 5000$. Thus each additional unit of production costs $2. (The graph of $y = 2x + 5000$ is called a **cost curve**. It relates the size of production to total cost. The graph is a straight line, and economists call its slope the **marginal cost of production**. The y-coordinate of the y-intercept is called the **fixed cost**. In this case the fixed cost is $5000, and it includes costs such as rent and insurance which are incurred even if no units are produced.) ■

Now Try Exercise 27

In applied problems having time as a variable, the letter t is often used in place of the letter x. If so, straight lines have equations of the form $y = mt + b$ and are graphed on a ty-coordinate system.

EXAMPLE 7 **Straight-line depreciation** The federal government allows businesses an income tax deduction for the decrease in value (or **depreciation**) of capital assets (such as buildings and equipment). One method of calculating the depreciation is to take equal amounts over the expected lifetime of the asset. This method is called **straight-line depreciation**. For tax purposes the value V of an asset t years after purchase is figured according to the equation $V = -100{,}000t + 700{,}000$. The expected life of the asset is 5 years.

(a) How much did the asset originally cost?

(b) What is the annual deduction for depreciation?

(c) What is the **salvage value** of the asset? (That is, what is the value of the asset after 5 years?)

Solution (a) The original cost is the value of V at $t = 0$, namely

$$V = -100{,}000(0) + 700{,}000 = 700{,}000.$$

That is, the asset originally cost \$700,000.

(b) By the steepness property, each increase of 1 in t causes a decrease in V of 100,000. That is, the value is decreasing by \$100,000 per year. So the depreciation deduction is \$100,000 each year.

(c) After 5 years, the value of V is given by

$$V = -100{,}000(5) + 700{,}000 = 200{,}000.$$

The salvage value is \$200,000.

We have seen in Example 5 how to sketch a straight line when given its slope and one point on it. Let us now see how to find the equation of the line from this data.

Point-Slope Formula The equation of the straight line passing through (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$.

EXAMPLE 8 **Finding the equation of a line from its slope and a point on the line** Find the equation of the line that passes through (2, 3) and has slope $\frac{1}{2}$.

Solution Here $x_1 = 2$, $y_1 = 3$, and $m = \frac{1}{2}$. So the equation is

$$y - 3 = \tfrac{1}{2}(x - 2)$$

$$y - 3 = \tfrac{1}{2}x - 1 \qquad \text{Perform multiplication on right side.}$$

$$y = \tfrac{1}{2}x + 2 \qquad \text{Add 3 to both sides.}$$

Now Try Exercise 41

EXAMPLE 9 **Finding the equation of a line** Find the equation of the line through the points (3, 1) and (6, 0).

Solution We can compute the slope from equation (1):

$$[\text{slope}] = \frac{[\text{difference in } y\text{-coordinates}]}{[\text{difference in } x\text{-coordinates}]} = \frac{1-0}{3-6} = -\frac{1}{3}.$$

Now we can determine the equation from the point-slope formula with $(x_1, y_1) = (3, 1)$ and $m = -\frac{1}{3}$:

$$y - 1 = -\tfrac{1}{3}(x - 3)$$

$$y - 1 = -\tfrac{1}{3}x + 1 \qquad \text{Perform multiplication on right side.}$$

$$y = -\tfrac{1}{3}x + 2 \qquad \text{Add 1 to both sides.}$$

Now Try Exercise 47

[*Question:* What would the equation be if we had chosen $(x_1, y_1) = (6, 0)$?]

EXAMPLE 10 **Revenue generated by advertising** For each dollar in monthly advertising expenditure, a store experiences a 6-dollar increase in sales revenue. Even without advertising, the store has $30,000 in sales revenue per month. Let x be the number of dollars of advertising expenditures per month and let y be the number of dollars in sales revenue per month.

(a) Find the equation of the line that expresses the relationship between x and y.

(b) If the store spends $10,000 in advertising, what will be the sales revenue for the month?

(c) How much would the store have to spend on advertising to attain $150,000 in sales revenue for the month?

Solution (a) The steepness property tells us that the line has slope $m = 6$. Since $x = 0$ (no advertising expenditures) yields $y = \$30{,}000$, the y-intercept of the line is $(0, 30{,}000)$. Therefore, the standard form of the equation of the line is

$$y = 6x + 30{,}000.$$

(b) If $x = 10{,}000$, then $y = 6(10{,}000) + 30{,}000 = 90{,}000$. Therefore, the sales revenue for the month will be $90,000.

(c) We are given that $y = 150{,}000$, and we must find the value of x for which

$$150{,}000 = 6x + 30{,}000.$$

Solving for x, we obtain $6x = 120{,}000$, and hence $x = \$20{,}000$. To attain $150,000 in sales revenue, the store should invest $20,000 in advertising. ■

Now Try Exercise 37

Verification of the Point-Slope Formula Let (x, y) be any point on the line passing through the point (x_1, y_1) and having slope m. Then, by equation (1), we have

$$m = \frac{y - y_1}{x - x_1}.$$

Multiplying through by $x - x_1$ gives

$$y - y_1 = m(x - x_1). \quad (2)$$

Thus every point (x, y) on the line satisfies equation (2). So (2) gives the equation of the line through (x_1, y_1) and having slope m. ■

The next property of slope relates the slopes of two perpendicular lines.

Perpendicular Property When two lines are perpendicular, their slopes are negative reciprocals of one another. That is, if two lines with slopes m and n are perpendicular to one another, then[1]

$$m = -\frac{1}{n}.$$

Conversely, if two lines have slopes that are negative reciprocals of one another, they are perpendicular.

A proof of the perpendicular property is outlined in Exercise 81. Let us show how it can be used to help find equations of lines.

[1]If $n = 0$, this formula does not say anything, since $1/0$ is undefined. However, in this case, one line is horizontal and one is vertical, the vertical one having an undefined slope.

EXAMPLE 2 **Solution of an equation** Are the following points on the graph of the equation $8x - 4y = 4$?

(a) $(3, 5)$ (b) $(5, 17)$

Solution (a) Substitute 3 for each occurrence of x and 5 for each occurrence of y in the equation.

$$8x - 4y = 4$$
$$8 \cdot 3 - 4 \cdot 5 = 4$$
$$24 - 20 = 4$$
$$4 = 4$$

Since the equation is satisfied, the point $(3, 5)$ is on the graph of the equation.

(b) Replace x by 5 and y by 17 in the equation.

$$8x - 4y = 4$$
$$8 \cdot 5 - 4 \cdot 17 = 4$$
$$40 - 68 = 4$$
$$-28 = 4$$

The equation is not satisfied, so the point $(5, 17)$ is *not* on the graph of the equation.

Now Try Exercises 11 and 13

EXAMPLE 3 **Graph of an equation** Sketch the graph of the equation $y = 2x - 1$.

Solution First find some points on the graph by choosing various values for x and determining the corresponding values for y:

x	y
-2	$2 \cdot (-2) - 1 = -5$
-1	$2 \cdot (-1) - 1 = -3$
0	$2 \cdot 0 - 1 = -1$
1	$2 \cdot 1 - 1 = 1$
2	$2 \cdot 2 - 1 = 3$

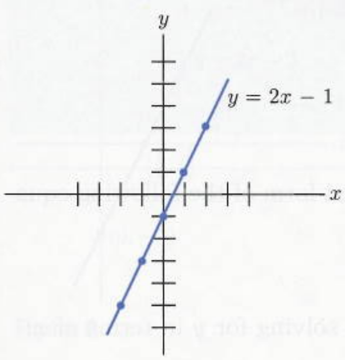


Figure 3

Thus the points $(-2, -5)$, $(-1, -3)$, $(0, -1)$, $(1, 1)$, and $(2, 3)$ are all on the graph. Plot these points (Fig. 3). It appears that the points lie on a straight line. By taking more values for x and plotting the corresponding points, it is easy to become convinced that the graph of $y = 2x - 1$ is indeed a straight line.

Now Try Exercise 27

EXAMPLE 4 **Graph of an equation** Sketch the graph of the equation $x = 3$.

Solution It is clear that the x-coordinate of any point on the graph must be 3. The y-coordinate can be anything. So some points on the graph are $(3, 0)$, $(3, 5)$, $(3, -4)$, $(3, -2)$. Again the graph is a straight line (Fig. 4 on the next page).

Now Try Exercise 29

EXAMPLE 5 **Graph of an equation** Sketch the graph of the equation $x = a$, where a is any given number.

EXAMPLE 11 **Perpendicular lines** Find the equation of the line perpendicular to the graph of $y = 2x - 5$ and passing through $(1, 2)$.

Solution The slope of the graph of $y = 2x - 5$ is 2. By the perpendicular property, the slope of a line perpendicular to it is $-\frac{1}{2}$. If a line has slope $-\frac{1}{2}$ and passes through $(1, 2)$, it has the equation

$$y - 2 = -\tfrac{1}{2}(x - 1) \quad \text{or} \quad y = -\tfrac{1}{2}x + \tfrac{5}{2}$$

Now Try Exercise 19 (by the point-slope formula). ■

The final property of slopes gives the relationship between slopes of parallel lines. A proof is outlined in Exercise 80.

Parallel Property Parallel lines have the same slope. Conversely, if two different lines have the same slope, they are parallel.

EXAMPLE 12 **Parallel lines** Find the equation of the line through $(2, 0)$ and parallel to the line whose equation is $y = \frac{1}{3}x - 11$.

Solution The slope of the line having equation $y = \frac{1}{3}x - 11$ is $\frac{1}{3}$. Therefore, any line parallel to it also has slope $\frac{1}{3}$. Thus the desired line passes through $(2, 0)$ and has slope $\frac{1}{3}$, so its equation is

Now Try Exercise 21

$$y - 0 = \tfrac{1}{3}(x - 2) \quad \text{or} \quad y = \tfrac{1}{3}x - \tfrac{2}{3}.$$ ■

Practice Problems 1.4

Suppose that the revenue y from selling x units of a certain commodity is given by the formula $y = 4x$. (Revenue is the amount of money received from the sale of the commodity.)

1. What interpretation can be given to the slope of the graph of this equation?
2. (See Example 6.) Find the coordinates of the point of intersection of $y = 4x$ and $y = 2x + 5000$.
3. What interpretation can be given to the value of the x-coordinate of the point found in Problem 2?

EXERCISES 1.4

In Exercises 1–4, find the slope of the line having the given equation.

1. $y = \frac{2}{3}x + 7$ **2.** $y = -4$

3. $y - 3 = 5(x + 4)$ **4.** $7x + 5y = 10$

In Exercises 5–8, plot each pair of points, draw the straight line between them, and find the slope.

5. $(3, 4)$, $(7, 9)$ **6.** $(-2, 1)$, $(3, -3)$

7. $(0, 0)$, $(5, 4)$ **8.** $(4, 17)$, $(-2, 17)$

9. What is the slope of any line parallel to the y-axis?

10. Why doesn't it make sense to talk about the slope of the line between the two points $(2, 3)$ and $(2, -1)$?

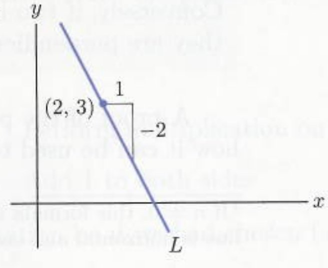
In Exercises 11–14, graph the given linear equation by beginning at the y-intercept, moving 1 unit to the right and m units in the y-direction.

11. $y = -2x + 1$ **12.** $y = 4x - 2$

13. $y = 3x$ **14.** $y = -2$

In Exercises 15–22, find the equation of line L.

15.



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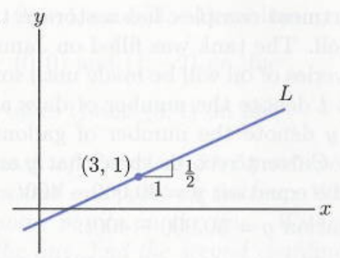

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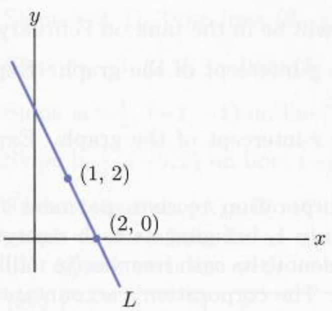

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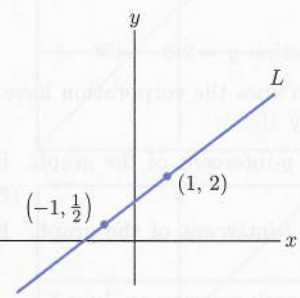

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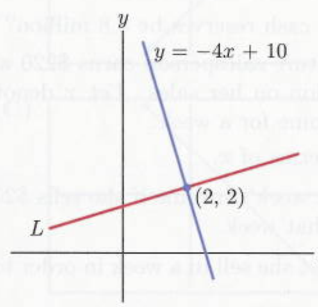
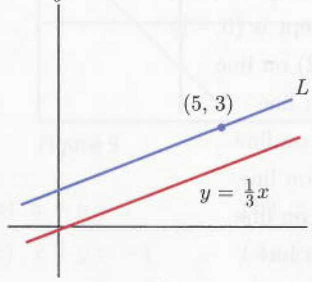


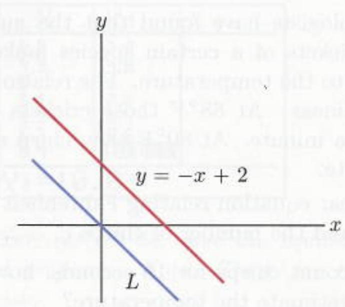
L perpendicular to $y = -4x + 10$

20.



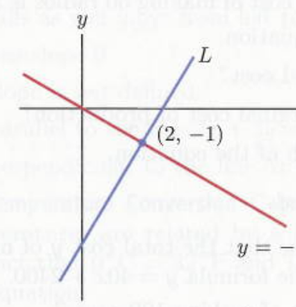
L parallel to $y = \frac{1}{3}x$

21.



L parallel to $y = -x + 2$

22.



L perpendicular to $y = -\frac{1}{2}x$

23. Find the equation of the line passing through the point $(2, 3)$ and parallel to the x-axis.

24. Find the equation of the line passing through $(0, 0)$ and having slope 1.5.

25. Find the y-intercept of the line passing through the point $(5, 6)$ and having slope $\frac{3}{5}$.

26. Find the slope of the line passing through the point $(1, 4)$ and having y-intercept $(0, 4)$.

27. **Weekly Pay** A salesperson's weekly pay depends on the volume of sales. If she sells x units of goods, then her pay is $y = 5x + 60$ dollars. Give an interpretation to the slope and the y-intercept of this straight line.

28. **Cost Curve** A manufacturer has fixed costs (such as rent and insurance) of \$2000 per month. The cost of producing each unit of goods is \$4. Give the linear equation for the cost of producing x units per month.

29. **Demand Curve** Suppose the price that must be set in order to sell q items is given by the equation $p = -3q + 1200$.

(a) Find and interpret the p-intercept of the graph of the equation.

(b) Find and interpret the q-intercept of the graph of the equation.

(c) Find and interpret the slope of the graph of the equation.

(d) What price must be set in order to sell 350 items?

(e) What quantity will be sold if the price is \$300?

(f) Sketch the graph of the equation.

30. Cricket Chirps Biologists have found that the number of chirps that crickets of a certain species make per minute is related to the temperature. The relationship is very close to linear. At 68°F those crickets chirp about 124 times a minute. At 80°F they chirp about 172 times a minute.

(a) Find the linear equation relating Fahrenheit temperature F and the number of chirps c.

(b) If you only count chirps for 15 seconds, how can you quickly estimate the temperature?

31. Cost Equation Suppose that the cost of making 20 radios is \$6800 and the cost of making 50 radios is \$9500.

(a) Find the cost equation.

(b) What is the fixed cost?

(c) What is the marginal cost of production?

(d) Sketch the graph of the equation.

Exercises 32–34 are related.

32. Cost Equation Suppose that the total cost y of making x coats is given by the formula $y = 40x + 2400$.

(a) What is the cost of making 100 coats?

(b) How many coats can be made for \$3600?

(c) Find and interpret the y-intercept of the graph of the equation.

(d) Find and interpret the slope of the graph of the equation.

33. Revenue Equation Suppose that the total revenue y from the sale of x coats is given by the formula $y = 100x$.

(a) What is the revenue if 300 coats are sold?

(b) How many coats must be sold to have a revenue of \$6000?

(c) Find and interpret the y-intercept of the graph of the equation.

(d) Find and interpret the slope of the graph of the equation.

34. Profit Equation Consider a coat factory with the cost and revenue equations given in Exercises 32 and 33.

(a) Find the equation giving the profit y resulting from making and selling x coats.

(b) Find and interpret the y-intercept of the graph of the equation.

(c) Find and interpret the x-intercept of the graph of the equation.

(d) Find and interpret the slope of the graph of the equation.

(e) How much profit will be made if 80 coats are sold?

(f) How many coats must be sold to have a profit of \$6000?

(g) Sketch the graph of the equation found in part (a).

35. Heating Oil An apartment complex has a storage tan to hold its heating oil. The tank was filled on Januar 1, but no more deliveries of oil will be made until some time in March. Let t denote the number of days afte January 1 and let y denote the number of gallons of fuel oil in the tank. Current records show that y and will be related by the equation $y = 30{,}000 - 400t$.

(a) Graph the equation $y = 30{,}000 - 400t$.

(b) How much oil will be in the tank on February 1

(c) How much oil will be in the tank on February 15

(d) Determine the y-intercept of the graph. Explai its significance.

(e) Determine the t-intercept of the graph. Explai its significance.

36. Cash Reserves A corporation receives payment for a large contract on July 1, bringing its cash reserves t \$2.3 million. Let y denote its cash reserves (in millions t days after July 1. The corporation's accountants es timate that y and t will be related by the equatio $y = 2.3 - .15t$.

(a) Graph the equation $y = 2.3 - .15t$.

(b) How much cash does the corporation have on th morning of July 16?

(c) Determine the y-intercept of the graph. Explai its significance.

(d) Determine the t-intercept of the graph. Explai its significance.

(e) Determine the cash reserves on July 4.

(f) When will the cash reserves be \$.8 million?

37. Weekly Pay A furniture salesperson earns \$220 a week plus 10% commission on her sales. Let x denote her sales and y her income for a week.

(a) Express y in terms of x.

(b) Determine her week's income if she sells \$2000 in merchandise that week.

(c) How much must she sell in a week in order to earn \$540?

Find the equations of the following lines.

38. Slope is $-\frac{1}{2}$; y-intercept is $(0, 0)$

39. Slope is 3; y-intercept is $(0, -1)$

40. Slope is $-\frac{1}{3}$; $(6, -2)$ on line

41. Slope is 1; $(1, 2)$ on line

42. Slope is $\frac{1}{2}$; $(2, -3)$ on line

43. Slope is -7; $(5, 0)$ on line

44. Slope is $-\frac{2}{5}$; $(0, 5)$ on line

45. Slope is 0; $(7, 4)$ on line

46. $(5, -3)$ and $(-1, 3)$ on line

47. $(2, 1)$ and $(4, 2)$ on line

48. (2, −1) and (3, −1) on line

49. (0, 0) and (1, −2) on line

50. (3, −1) and (3, 1) on line

In each of Exercises 51–54 we specify a line by giving the slope and one point on the line. We give the first coordinate of some points on the line. Without deriving the equation of the line, find the second coordinate of each of the points.

51. Slope is 2, (1, 3) on line; (2,); (0,); (−1,)

52. Slope is −3, (2, 2) on line; (3,); (4,); (1,)

53. Slope is $-\frac{1}{4}$, (−1, −1) on line; (0,); (1,); (−2,)

54. Slope is $\frac{1}{3}$, (−5, 2) on line; (−4,); (−3,); (−2,)

55. Each of the lines (A), (B), (C), and (D) in Fig. 9 is the graph of one of the linear equations (a), (b), (c), and (d). Match each line with its equation.

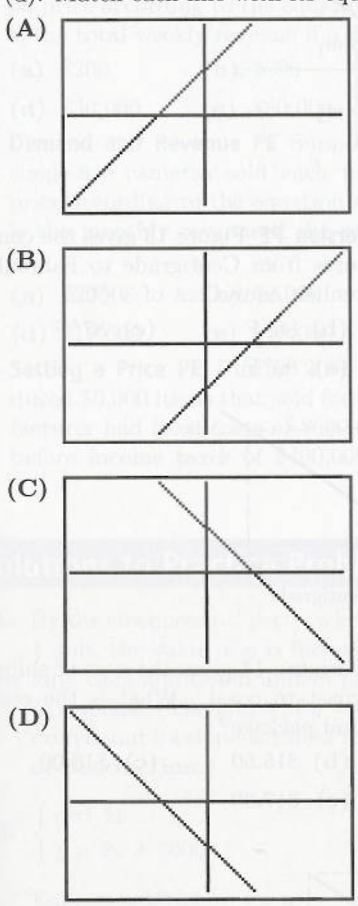


Figure 9

(a) $x + y = 1$ (b) $x - y = 1$

(c) $x + y = -1$ (d) $x - y = -1$

56. The following table gives several points on the line $Y_1 = mx + b$. Find m and b.

X	Y_1
4.8	3.6
4.9	4.8
5	6
5.1	7.2
5.2	8.4
5.3	9.6
5.4	10.8

Y_1=10.8

In Exercises 57–62, give the equation of a line with the stated property. Note: There are many answers to each exercise.

57. rises as you move from left to right

58. falls as you move from left to right

59. has slope 0

60. slope is not defined

61. parallel to the line $2x + 3y = 4$

62. perpendicular to the line $5x + 6y = 7$

63. **Temperature Conversion** Celsius and Fahrenheit temperatures are related by a linear equation. Use the fact that 0°C = 32°F and 100°C = 212°F to find an equation.

64. **Dating of Artifacts** An archaeologist dates a bone fragment discovered at a depth of 4 feet as approximately 1500 B.C. and dates a pottery shard at a depth of 8 feet as approximately 2100 B.C. Assuming there is a linear relationship between depths and dates at this archeological site, find the equation that relates depth to date. How deep should the archaeologist dig to look for relics from 3000 B.C.?

65. **College Tuition** The average college tuition and fees at four-year public colleges increased from \$2848 in 1995 to \$5685 in 2006. Assuming that average tuition and fees increased linearly with respect to time, find the equation that relates the average tuition and fees, y, to the number of years after 1995, x. What were the average tuition and fees in 2000?

66. **College Enrollments** Two-year college enrollments increased from 5.2 million in 1990 to 6.5 million in 2005. Assuming that enrollments increased linearly with respect to time, find the equation that relates the enrollment, y, to the number of years after 1990, x. When was the enrollment 6 million?

67. **Gas Mileage** A certain car gets 25 miles per gallon when the tires are properly inflated. For every pound of pressure that the tires are underinflated, the gas mileage decreases by $\frac{1}{2}$ miles per gallon. Find the equation that relates miles per gallon, y, to the amount the tires are underinflated, x. Use the equation to calculate the gas mileage when the tires are underinflated by 8 pounds of pressure.

68. **Home Health Aid Jobs** According to the U.S. Department of Labor, home health aid jobs are expected to increase from 787,000 in 2006 to 1,171,000 in 2016. Assuming that the number of home health aid jobs increases linearly during that time, find the equation that relates the number of jobs, y, to the number of years after 2006, x. Use the equation to predict the number of home health aid jobs in 2012.

69. Bachelor Degrees in Business According to the U.S. National Center of Educational Statistics, 249,165 bachelor degrees in business were awarded in 1991 and 318,042 were awarded in 2006. If the number of bachelor degrees in business continues to grow linearly, how many bachelor degrees in business will be awarded in 2011?

70. Pizza Stores According to the Pizza Marketing Quarterly, the number of U.S. Domino's Pizza stores grew from 4818 in 2001 to 5143 in 2006. If the number of stores continues to grow linearly, when will there be 6000 stores?

71. Super Bowl Commercials The average cost of a 30-second advertising slot during the Super Bowl increased linearly from \$2.4 million in 2005 to \$2.7 million in 2008. Find the equation that relates the cost (in millions of dollars) of a 30-second slot, y, to the number of years after 2000, x. What was the average cost in 2007?

72. Write an inequality whose graph consists of the points on or below the straight line passing through the two points $(2, 5)$ and $(4, 9)$.

73. Write an inequality whose graph consists of the points on or above the line with slope 4 and y-intercept $(0, 3)$.

74. Find a system of inequalities having the feasible set in Fig. 10.

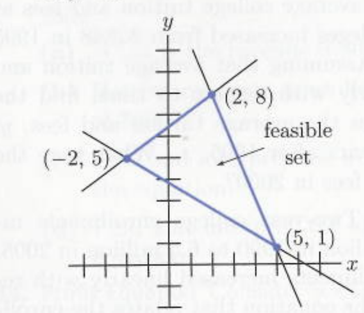


Figure 10

75. Find a system of inequalities having the feasible set in Fig. 11.

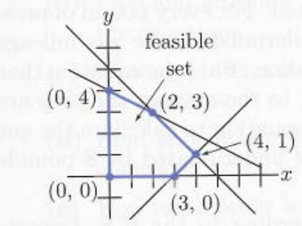


Figure 11

76. Show that the points $(1, 3)$, $(2, 4)$, and $(3, -1)$ are *not* on the same line.

77. For what value of k will the three points $(1, 5)$, $(2, 7)$ and $(3, k)$ be on the same line?

78. Find the value of a for which the line through the points $(a, 1)$ and $(2, -3.1)$ is parallel to the line through the points $(-1, 0)$ and $(3.8, 2.4)$.

79. Rework Exercise 78, where the word "parallel" is replaced by the word "perpendicular."

80. Prove the parallel property. [*Hint*: If $y = mx + b$ and $y = m'x + b'$ are the equations of two lines, then the two lines have a point in common if and only if the equation $mx + b = m'x + b'$ has a solution x.]

81. Prove the perpendicular property. [*Hint*: Without loss of generality, assume that both lines pass through the origin. Use the point-slope formula, the Pythagorean theorem, and Fig. 12.]

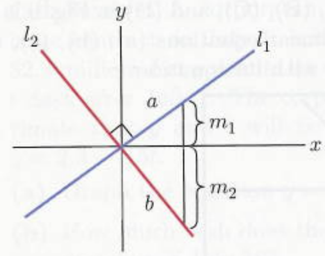


Figure 12

82. Temperature Conversion PE Figure 13 gives the conversion of temperatures from Centigrade to Fahrenheit. What is the Fahrenheit equivalent of 30°C?

(a) 85°F (b) 86°F (c) 87°F
(d) 88°F (e) 89°F

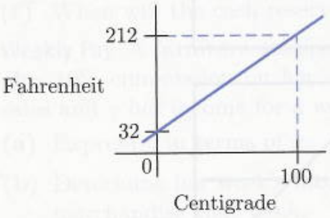


Figure 13

83. Shipping Costs PE Figure 14 gives the cost of shipping a package from coast to coast. What is the cost of shipping a 20-pound package?

(a) \$15.00 (b) \$15.50 (c) \$16.00
(d) \$16.50 (e) \$17.00

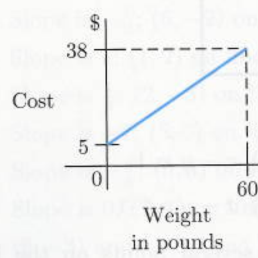


Figure 14

84. **Costs and Revenue PE** A T-shirt company has fixed costs of \$25,000 per year. Each T-shirt costs \$8.00 to produce and sells for \$12.50. How many T-shirts must the company produce and sell each year in order to make a profit of \$65,000?

(a) 9200 (b) 11,250 (c) 14,375 (d) 20,000 (e) 25,556

85. **Costs and Revenue PE** A company produces a single product for which variable costs are \$100 per unit and annual fixed costs are \$1,000,000. If the product sells for \$130 per unit, how many units must the company produce and sell in order to attain an annual profit of \$2,000,000?

(a) 10,000 (b) 15,385 (c) 20,000 (d) 66,667 (e) 100,000

86. **Demand and Revenue PE** Suppose the quantity q of a certain brand of mountain bike sold each week depends on price according to the equation $q = 800 - 4p$. What is the total weekly revenue if a bike sells for \$150?

(a) \$200 (b) \$600 (c) \$2000 (d) \$30,000 (e) \$80,000

87. **Demand and Revenue PE** Suppose the number n of single-use cameras sold each month varies with the price according to the equation $n = 2200 - 25p$. What is the monthly revenue if the price of each camera is \$8?

(a) \$200 (b) \$2,000 (c) \$16,000 (d) \$17,600 (e) \$18,000

88. **Setting a Price PE** During 2008 a manufacturer produced 50,000 items that sold for \$100 each. The manufacturer had fixed costs of \$600,000 and made a profit before income taxes of \$400,000. In 2009, rent and insurance combined increased by \$200,000. Assuming that the quantity produced and all other costs were unchanged, what should the 2009 price be if the manufacturer is to make the same \$400,000 profit before income taxes?

(a) \$96 (b) \$100 (c) \$102 (d) \$104 (e) \$106

89. **Setting a Price PE** Rework Exercise 88 with a 2008 fixed cost of \$800,000 and a profit before income taxes of \$300,000.

(a) \$96 (b) \$100 (c) \$102 (d) \$104 (e) \$106

Exercises 90–94 require the use of a graphing calculator.

90. Graph the three lines $y = 2x - 3$, $y = 2x$, and $y = 2x + 3$ together and then identify each line without using TRACE.

91. Graph the two lines $y = .5x + 1$ and $y = -2x + 9$ in the standard window $[-10, 10]$ *by* $[-10, 10]$. Do they appear perpendicular? If not, use `ZSquare` or `ZoomSqr` to obtain true aspect, and look at the graphs.

92. Graph the line $y = -.5x + 2$ with the window `ZDecimal` or `ZoomDec`. Without pressing TRACE, move the cursor to a point on the line. Then move the cursor one unit to the right and down $\frac{1}{2}$ unit to return to the line. If you start at a point on the line and move 2 units to the right, how many units down will you have to move the cursor to return to the line? Test your answer.

93. Graph the three lines $y = 2x + 1$, $y = x + 1$, and $y = .5x + 1$ together and then identify each line without using TRACE.

94. Repeat Exercise 92 for the line $y = .7x - 2$, using "up" instead of "down" and .7 instead of $\frac{1}{2}$.

Solutions to Practice Problems 1.4

1. By the steepness property, whenever x is increased by 1 unit, the value of y is increased by 4 units. Therefore, each additional unit of production brings in \$4 of revenue. (The graph of $y = 4x$ is called a **revenue curve** and its slope is called the **marginal revenue of production**.)

2. $\begin{cases} y = 4x \\ y = 2x + 5000 \end{cases}$

Equate expressions for y:

$$4x = 2x + 5000$$
$$2x = 5000$$
$$x = 2500$$
$$y = 4(2500) = 10{,}000.$$

3. When producing 2500 units, the revenue equals the cost. This value of x is called the **break-even point**. Since profit = (revenue) − (cost), the company will make a profit only if its level of production is greater than the break-even point (Fig. 15).

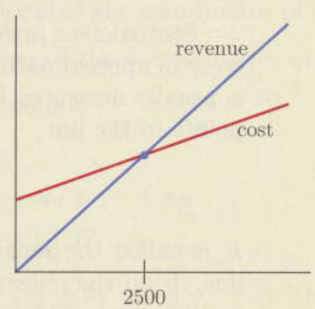


Figure 15

1.5 The Method of Least Squares

Modern people compile graphs of literally thousands of different quantities: the purchasing value of the dollar as a function of time; the pressure of a fixed volume of air as a function of temperature; the average income of people as a function of their years of formal education; or the incidence of strokes as a function of blood pressure. The observed points on such graphs tend to be irregularly distributed due to the complicated nature of the phenomena underlying them as well as to errors made in observation (for example, a given procedure for measuring average income may not count certain groups). In spite of the imperfect nature of the data, we are often faced with the problem of making assessments and predictions based on them. Roughly speaking, this problem amounts to filtering the sources of errors in the data and isolating the basic underlying trend. Frequently, on the basis of a suspicion or a working hypothesis, we may suspect that the underlying trend is linear—that is, the data should lie on a straight line. But which straight line? This is the problem that the **method of least squares** attempts to answer. To be more specific, let us consider the following problem:

Problem Given observed data points $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$ in the plane, find the straight line that "best" fits these points.

In order to completely understand the statement of the problem being considered, we must define what it means for a line to "best" fit a set of points. If (x_i, y_i) is one of our observed points, then we will measure how far it is from a given line $y = ax + b$ by the vertical distance, E_i, from the point to the line. (See Fig. 1.)

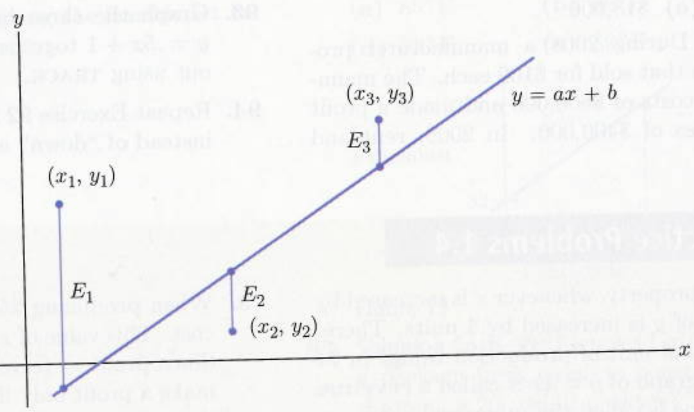


Figure 1. Fitting a line to data points.

Statisticians prefer to work with the square of the vertical distance E_i. The total error in approximating the data points $(x_1, y_1), \ldots, (x_N, y_N)$ by the line $y = ax + b$ is usually measured by the sum E of the squares of the vertical distances from the points to the line,

$$E = E_1^2 + E_2^2 + \cdots + E_N^2.$$

E is called the **least-squares error** of the observed points with respect to the line. If all the observed points lie on the line $y = ax + b$, then all the E_i are zero and the error E is zero. If a given observed point is far away from the line, the corresponding E_i^2 is large and hence makes a large contribution to the error E.

EXAMPLE 1 **Finding the least-squares error** Determine the least-squares error when the line $y = 1.5x + 3$ is used to approximate the data points $(1, 6)$, $(4, 5)$, and $(6, 14)$.

Solution Figure 2 shows the line, the points, and the vertical distances. The vertical distance of a point from the line is determined by finding the second coordinate of the point on the line having the same x-coordinate as the point. For instance, for the data point $(1, 6)$, the point on the line with x-coordinate 1 has y-coordinate $y = 1.5(1) + 3 = 4.5$ and, therefore, vertical distance $6 - 4.5 = 1.5$. Table 1 summarizes the vertical distances. The table shows that the least-squares error is $1.5^2 + 4^2 + 2^2 = 2.25 + 16 + 4 = 22.25$. ■

Now Try Exercise 1

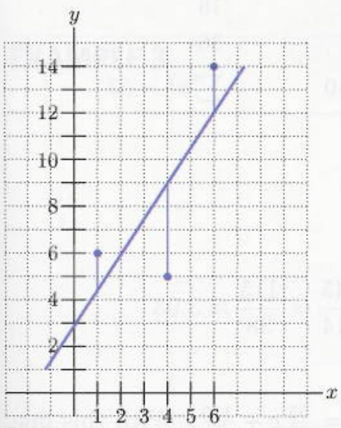


Figure 2

TABLE 1 Vertical distances from the line y =1.5x + 3

Data Point	Point on Line	Vertical Distance
$(1, 6)$	$(1, 4.5)$	1.5
$(4, 5)$	$(4, 9)$	4
$(6, 14)$	$(6, 12)$	2

In general, we cannot expect to find a line $y = mx + b$ that fits the observed points so well that the error E is zero. Actually, this situation will occur only if the observed points lie on a straight line. However, we can rephrase our original problem as follows:

Problem Given observed data points $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$ in the plane, find the straight line $y = mx + b$ for which the error E is as small as possible.

This line, called the **least-squares line** or the **regression line**, can be found from the data points with the formulas

$$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{N \cdot \sum x^2 - \left(\sum x\right)^2}$$

$$b = \frac{\sum y - m \cdot \sum x}{N},$$

where

$\sum x =$ sum of the x-coordinates of the data points

$\sum y =$ sum of the y-coordinates of the data points

$\sum xy =$ sum of the products of the coordinates of the data points

$\sum x^2 =$ sum of the squares of the x-coordinates of the data points

$N =$ number of data points.

That is,

$$\sum x = x_1 + x_2 + \cdots + x_N$$
$$\sum y = y_1 + y_2 + \cdots + y_N$$
$$\sum xy = x_1 \cdot y_1 + x_2 \cdot y_2 + \cdots + x_N \cdot y_N$$
$$\sum x^2 = x_1^2 + x_2^2 + \cdots + x_N^2.$$

EXAMPLE 2 **Finding the least-squares line** Find the least-squares line for the data points of Example 1.

Solution The sums are calculated in Table 2 and then used to determine the values of m and b.

TABLE 2

x	y	xy	x^2
1	6	6	1
4	5	20	16
6	14	84	36
$\sum x = 11$	$\sum y = 25$	$\sum xy = 110$	$\sum x^2 = 53$

$$m = \frac{3 \cdot 110 - 11 \cdot 25}{3 \cdot 53 - 11^2} = \frac{55}{38} \approx 1.45$$

$$b = \frac{25 - \frac{55}{38} \cdot 11}{3} = \frac{38 \cdot 25 - 55 \cdot 11}{38 \cdot 3} = \frac{345}{114} = \frac{115}{38} \approx 3.03$$

Therefore, the equation of the least-squares line is $y = \frac{55}{38}x + \frac{115}{38}$. With this line, the least-squares error can be shown to be about 22.13. ■

Now Try Exercise 7

INCORPORATING TECHNOLOGY

Obtaining the Least-Squares Line with Technology Least-squares lines can be obtained with graphing calculators and spreadsheets. For instance, on the TI-83/84 Plus graphing calculator screens in Fig. 3, the data points are entered into lists, the least-squares line is calculated with the item **LinReg(ax + b)** of the STAT/CALC menu, and the data points and line are plotted with [STAT PLOT] and [GRAPH]. On the TI-89, the least-squares line is calculated with the **LinReg** command found in the MATH/Statistics/Regressions menu. The end of this section and Appendix D contain the details for obtaining least-squares lines with these calculators. ■

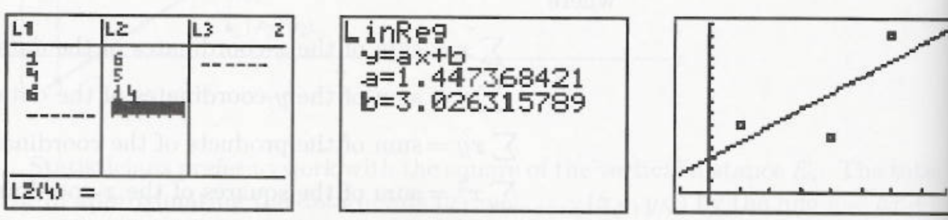


Figure 3. Obtaining a least-squares line with a TI-83/84 Plus.

Spreadsheet programs, such as Excel, have special functions that calculate the slope and y-intercept of the least-squares line for a collection of data points. In Fig. 4 the least-squares line of Example 2 is calculated and graphed in Excel. The end of this section shows how to obtain the graph in Fig. 4.

The next example obtains a least-squares line and uses the line to make projections.

	A	B
1	1	6
2	4	5
3	6	14
4	slope	1.447368421
5	y-intercept	3.026315789
6		
7		
8	Cell	Formula
9	B4	=SLOPE(B1:B3,A1:A3)
10	B5	=INTERCEPT(B1:B3,A1:A3)

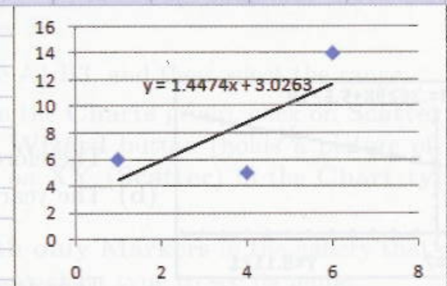


Figure 4. Obtaining a least-squares line with Excel.

EXAMPLE 3 **Finding and using the least-squares line** Table 3 gives the U.S. per capita health care expenditures[1] for several years.

TABLE 3 U.S per capita health care expenditures

Year	2002	2003	2004	2005	2006
Dollars	5560	5952	6301	6649	7026

(a) Find the least-squares line for this data.

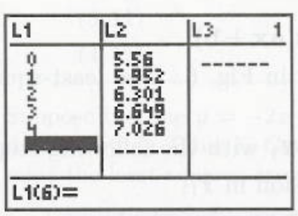
(b) Use the least-squares line to estimate the per capita health care expenditures for the year 2009.

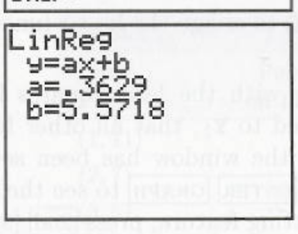
(c) Use the least-squares line to estimate the year in which the per capita health care expenditures will reach $9000.

Solution (a) The numbers can be made more manageable by counting years beginning with 2002 and measuring dollars in thousands. See Table 4.

TABLE 4 U.S per capita health care expenditures

Year	0	1	2	3	4
Dollars	5.560	5.952	6.301	6.649	7.026





The slope and y-intercept of the least-squares line can be found with the formulas involving sums or with technology. The sums are calculated in Table 5 and then used to determine the values of m and b. The screens in the margin show the results of solving Example 3 with a graphing calculator.

TABLE 5

x	y	xy	x^2
0	5.560	0	0
1	5.952	5.952	1
2	6.301	12.602	4
3	6.649	19.947	9
4	7.026	28.104	16
$\sum x = 10$	$\sum y = 31.488$	$\sum xy = 66.605$	$\sum x^2 = 30$

[1] Centers for Medicare and Medicaid Services, NHE Fact Sheet.

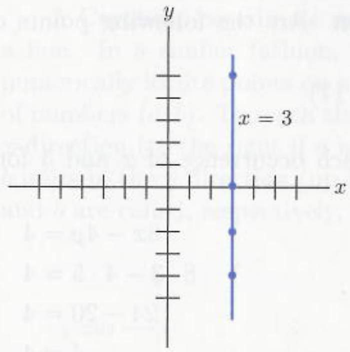


Figure 4

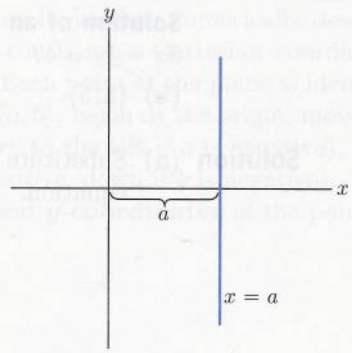


Figure 5

Solution The x-coordinate of any point on the graph must be a. Reasoning as in Example 4, the graph is a vertical line a units away from the y-axis (Fig. 5). (Of course, if a is negative, then the line lies to the left of the y-axis.) ■

Now Try Exercise 33

The equations in Examples 2 to 5 are all special cases of the general equation

$$cx + dy = e,$$

corresponding to particular choices of the constants c, d, and e. Any such equation is called a **linear equation in general form** (in the two variables x and y). A linear equation also can be put into standard form.

DEFINITION The **standard form** of a linear equation is obtained by solving for y if y appears and for x if y does not appear. In the former case the standard form looks like

$$y = mx + b \quad (m, b \text{ constants}),$$

whereas in the latter it looks like

$$x = a \quad (a \text{ constant}).$$

EXAMPLE 6

Standard form of a linear equation Find the standard form of the following equations.

(a) $8x - 4y = 4$ (b) $2x + 3y = 3$ (c) $2x = 6$

Solution (a) Since y appears, we obtain the standard form by solving for y in terms of x:

$$8x - 4y = 4$$
$$-4y = -8x + 4 \quad \text{Subtract } 8x \text{ from both sides.}$$
$$y = 2x - 1 \quad \text{Divide both sides by } -4.$$

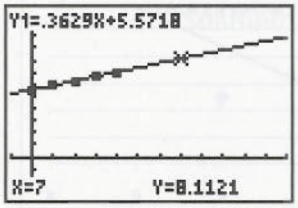
Thus the standard form of $8x - 4y = 4$ is $y = 2x - 1$—that is, $y = mx + b$ with $m = 2$, $b = -1$.

(b) Again y occurs, so we solve for y:

$$2x + 3y = 3$$
$$3y = -2x + 3 \quad \text{Subtract } 2x \text{ from both sides.}$$
$$y = -\tfrac{2}{3}x + 1 \quad \text{Divide both sides by } 3.$$

So the standard form is $y = -(2/3)x + 1$. Here $m = -2/3$ and $b = 1$.

$$m = \frac{5 \cdot 66.605 - 10 \cdot 31.488}{5 \cdot 30 - 10^2} = \frac{18.145}{50} = .3629$$



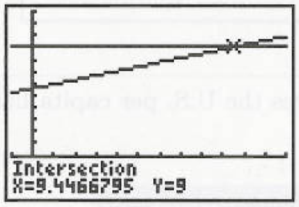
$$b = \frac{31.488 - .3629 \cdot 10}{5} = 5.5718$$

Therefore, the least-squares line is $y = .3629x + 5.5718$.

(b) The year 2009 corresponds to $x = 7$. The value of y is

$$y = .3629(7) + 5.5718 = 8.1121.$$

Therefore, an estimate of per capita health care expenditures in the year 2009 is \$8112.10.



(c) Set the value of y equal to 9 and solve for x.

$$9 = .3629x + 5.5718$$

$$x = \frac{9 - 5.5718}{.3629} \approx 9.45$$

Rounding up, expenditures are projected to reach \$9000 ten years after 2002; that is, in the year 2012.

Now Try Exercise 13

Obtaining the Least-Squares Line with a Graphing Calculator The following steps show how to use a TI-83/84 Plus calculator to find the straight line that minimizes the least-squares error for the points (1, 4), (2, 5), and (3, 8). (See Appendix D for the corresponding steps with the TI-89 calculator.)

1. Press [STAT] **1** to obtain a table used for entering the data.
2. If there are no data in columns labeled $\mathbf{L_1}$ and $\mathbf{L_2}$, proceed to step 4.
3. Move the cursor up to $\mathbf{L_1}$ and press [CLEAR] [ENTER] to delete all data in $\mathbf{L_1}$'s column. Move the cursor right and up to $\mathbf{L_2}$ and press [CLEAR] [ENTER] to delete all data in $\mathbf{L_2}$'s column.
4. If necessary, move the cursor left to the first blank row of the $\mathbf{L_1}$ column. Press **1** [ENTER] **2** [ENTER] **3** [ENTER] to place the x-coordinates of the three points into the $\mathbf{L_1}$ column.
5. Move the cursor right to the $\mathbf{L_2}$ column and press **4** [ENTER] **5** [ENTER] **8** [ENTER] to place the y-coordinates of the three points into the $\mathbf{L_2}$ column. The screen should now appear as in Fig. 5.

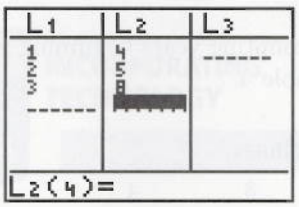


Figure 5

6. Press [STAT] [▶] and press the number for **LinReg(ax + b)**.
7. Press [ENTER]. The screen should now appear as in Fig. 6. The least-squares line is $y = 2x + \frac{5}{3}$. (*Note*: $\frac{5}{3} \approx 1.666666667$.)

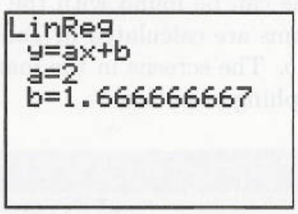


Figure 6

8. If desired, the linear function can be assigned to $\mathbf{Y_1}$ with the following steps:
   (a) Press [Y=] [CLEAR] to erase the current expression in $\mathbf{Y_1}$.
   (b) Press [VARS] **5** [▶] [▶] and the number for **RegEQ** to assign the linear function to $\mathbf{Y_1}$.
9. The original points can be easily plotted along with the least-squares line. Assume that the linear function has been assigned to $\mathbf{Y_1}$, that all other functions have been cleared or deselected, and that the window has been set to $[-4, 4]$ *by* $[-4, 9]$. Press [2nd] [STAT PLOT] [ENTER] [ENTER] [GRAPH] to see the display in Fig. 7. (*Note 1*: To turn off the point-plotting feature, press [2nd] [STAT PLOT] [ENTER] [▶] [ENTER]. *Note 2*: The point-plotting feature can be toggled from the **Y=** editor by moving the cursor to the word **"Plot1"** on the top line and pressing [ENTER].)

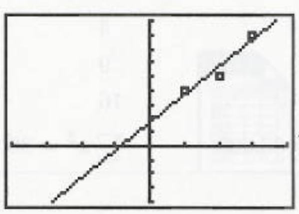


Figure 7

Obtaining the Least-Squares Line with an Excel Spreadsheet[2] The following steps obtain the graph in Fig. 4 on page 39.

1. Enter the six numbers in the range A1:B3, and then select the range.
2. (Excel 2007) On the **Insert** tab, in the **Charts** group, click on **Scatter**.
   (Excel 97-2003) Click the **Chart Wizard** button (holds a picture of a histogram) on the toolbar, and click on **XY (Scatter)** in the **Chart type** list box.
3. (Excel 2007) Click on **Scatter with only Markers** in the gallery that opens. *Note*: You can rest the mouse on any chart type to see its name.
   (Excel 97-2003) Click on the **Finish** button in the lower right corner of the window.
4. Right-click on one of the diamond-shaped points in the graph and then click on **Add Trendline**.
5. Click the box in front of **Display equation on chart.**
6. (Excel 2007) Click on the **Close** button.
   (Excel 97-2003) Click on the **OK** button.
7. Click on the legend located to the right of the graph and press the **Del** key.

Practice Problems 1.5

1. Can a vertical distance be negative?
2. Under what condition will a vertical distance be zero?

EXERCISES 1.5

1. Suppose the line $y = 3x + 1$ is used to fit the four data points in Table 6. Complete the table and determine the least-squares error E.

TABLE 6

Data Point	Point on Line	Vertical Distance
(1, 3)		
(2, 6)		
(3, 11)		
(4, 12)		

2. Suppose the line $y = -2x + 12$ is used to fit the four data points in Table 7. Complete the table and determine the least-squares error E.

TABLE 7

Data Point	Point on Line	Vertical Distance
(1, 11)		
(2, 7)		
(3, 5)		
(4, 5)		

3. Find the least-squares error E for the least-squares line fit to the four points in Fig. 8.

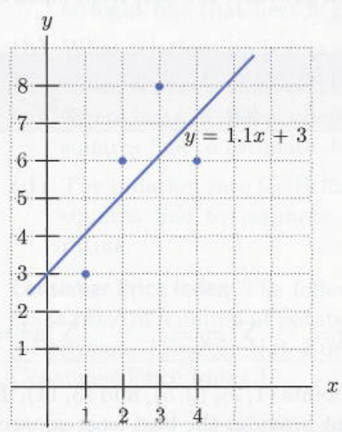


Figure 8

[2] "Excel 2007" refers to the latest version of Excel, and "Excel 97-2003" refers to the earlier versions.

4. Find the least-squares error E for the least-squares line fit to the five points in Fig. 9.

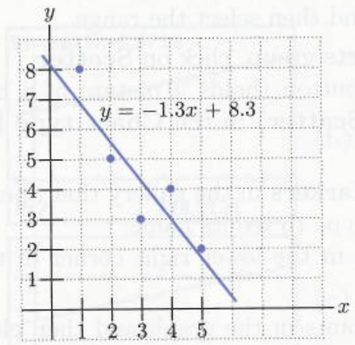


Figure 9

5. Complete Table 8 and find the values of m and b for the straight line that provides the best least-squares fit to the data.

TABLE 8

x	y	xy	x^2
1	7		
2	6		
3	4		
4	3		
$\sum x =$	$\sum y =$	$\sum xy =$	$\sum x^2 =$

6. Complete Table 9 and find the values of m and b for the straight line that provides the best least-squares fit to the data.

TABLE 9

x	y	xy	x^2
1	2		
2	4		
3	7		
4	9		
5	12		
$\sum x =$	$\sum y =$	$\sum xy =$	$\sum x^2 =$

7. Consider the data points $(1, 2)$, $(2, 5)$, and $(3, 11)$. Find the straight line that provides the best least-squares fit to these data.

8. Consider the data points $(1, 8)$, $(2, 4)$, and $(4, 3)$. Find the straight line that provides the best least-squares fit to these data.

9. Consider the data points $(1, 9)$, $(2, 8)$, $(3, 6)$, and $(4, 3)$. Find the straight line that provides the best least-squares fit to these data.

10. Consider the data points $(1, 5)$, $(2, 7)$, $(3, 6)$, and $(4, 10)$. Find the straight line that provides the best least-squares fit to these data.

Since Exercises 11–16 use real data, they are best answered with a graphing calculator or spreadsheet.

11. Lung Cancer and Smoking The following table[3] gives the crude male death rate for lung cancer in 1950 and the per capita consumption of cigarettes in 1930 in various countries.

Country	Cigarette Consumption (per Capita)	Lung Cancer Deaths (per Million Males)
Norway	250	95
Sweden	300	120
Denmark	350	165
Australia	470	170

(a) Use the method of least-squares to obtain the straight line that best fits these data.

(b) In 1930 the per capita cigarette consumption in Finland was 1100. Use the straight line found in part (a) to estimate the male lung cancer death rate in Finland in 1950.

12. Cost and Usage of Fuel The accompanying table shows the 1999 price of a gallon (in U.S. dollars) of fuel and the average miles driven per automobile for several countries.[4]

Country	Price per Gallon	Average Miles per Automobile
France	\$4.01	9090
Germany	\$3.78	7752
Japan	\$3.65	6009
Sweden	\$4.22	9339
United Kingdom	\$5.13	10,680

(a) Find the straight line that provides the best least squares fit to these data.

(b) In 1999, the price of gas in Canada was \$2.04 per gallon. Use the straight line of part (a) to estimate the average number of miles automobiles were driven in Canada.

(c) In 1999 the average miles driven in the United States was 11,868. Use the straight line of part (a) to estimate the 1999 price of a gallon of gasoline in the United States.

[3]These data were obtained from *Smoking and Health*, Report of the Advisory Committee to the Surgeon General of the Public Heal[th] Service, U.S. Department of Health, Education, and Welfare, Washington, D.C., Public Health Service Publication No. 1103, p. 17[...]
[4]U.S. Department of Transportation, Federal Highway Administration, *Highway Statistics*, 2000.

13. College Gradu[...] of persons 25 [...] or more years [...]
(a) Use the [...] straight [...]
(b) Estimate [...]
(c) If the tre[...] (a) conti[...]

14. College Enroll[...] ment (in mill[...] States[6] for cer[...]

(a) Use the [...] straight li[...]
(b) Estimate t[...]
(c) If the tren[...] (a) contin[...] million?

15. Life Expectancy [...] life expectancy [...]

Curr[...]

(a) Find the st[...] squares fit [...]

[5]U.S. Bureau of the C[...]
[6]U.S. Dept. of Educat[...]
[7]J. Bradford DeLong ([...]

13. **College Graduates** The following table gives the percent of persons 25 years and over who have completed four or more years of college.[5]

(a) Use the method of least squares to obtain the straight line that best fits these data.

(b) Estimate the percent for the year 2003.

(c) If the trend determined by the straight line in part (a) continues, when will the percent reach 30?

Year	Percent
1980	17.0
1985	19.4
1990	21.3
1995	23.0
2000	25.6
2005	27.6

14. **College Enrollments** The following table gives enrollment (in millions) in public colleges in the United States[6] for certain years.

Year	Enrollment
1995	11.1
2000	11.8
2002	12.8
2003	12.9
2004	13.0
2005	13.0
2006	13.3

(a) Use the method of least squares to obtain the straight line that best fits these data.

(b) Estimate the enrollment in 1998.

(c) If the trend determined by the straight line in part (a) continues, when will the enrollment reach 15 million?

15. **Life Expectancy** The following table is an abbreviated life expectancy table for U.S. males.

Current Age	Life Expectancy
0	75.2
20	76.2
40	77.7
60	80.8
80	88.2

(a) Find the straight line that provides the best least-squares fit to these data.

(b) Use the straight line of part (a) to estimate the life expectancy of a 30-year-old U.S. male. (*Note*: The actual life expectancy is 77.0 years.)

(c) Use the straight line of part (a) to estimate the life expectancy of a 50-year-old U.S. male. (*Note*: The actual life expectancy is 78.9 years.)

(d) Use the straight line of part (a) to estimate the life expectancy of a 90-year-old U.S. male. (*Note*: The actual life expectancy is 94.4 years.)

16. **Banking** Two Harvard economists studied countries' relationships between the independence of banks and inflation rates from 1955 to 1990.[7] The independence of banks was rated on a scale of -1.5 to 2.5, with -1.5, 0, and 2.5 corresponding to least, average, and most independence, respectively. The following table gives the values for various countries.

Country	Independence Rating	Inflation Rate (%)
New Zealand	−1.4	7.6
Italy	−.75	7.2
Belgium	.3	4.0
France	.4	6.0
Canada	.9	4.5
United States	1.6	4.0
Switzerland	2.2	3.1

(a) Use the method of least squares to obtain the straight line that best fits these data.

(b) What relationship between independence of banks and inflation is indicated by the least-squares line?

(c) Japan has a .6 independence rating. Use the least-squares line to estimate Japan's inflation rate.

(d) The inflation rate for Britain is 6.8. Use the least-squares line to estimate Britain's independence rating.

17. **Consumer Price Index** The following table gives the average price of a pound of potato chips in January of the given years. (*Source*: U.S. Bureau of Labor Statistics, Consumer Price Index.)

Year	Price
1993	\$2.83
1996	\$3.00
2000	\$3.39
2004	\$3.43
2008	\$3.53

[5] U.S. Bureau of the Census.

[6] U.S. Dept. of Education, National Center for Education Statistics, *Digest of Education Statistics, 2007.*

[7] J. Bradford DeLong (Harvard) and L. H. Summers (World Bank).

(a) Use the method of least squares to obtain the straight line that best fits these data.

(b) Estimate the average price of a pound of potato chips in January 1999.

(c) If this trend continues, when will the average price of a pound of potato chips be \$3.85?

18. **Greenhouse Gases** Although greenhouse gases are essential to maintaining the temperature of the Earth, an excess of greenhouse gases can raise the temperature to dangerous levels. One of the most threatening greenhouse gases is carbon dioxide (CO_2). The Mauna Loa atmospheric CO_2 measurements constitute the longest continuous record of atmospheric CO_2 concentrations available in the world. The following table shows the concentration of CO_2 (in parts per million) at Mauna Loa, Hawaii for three years.

Year	CO_2 (ppm)
1958	315
1983	342
2008	384

(a) Use the method of least squares to obtain th straight line that best fits these data. (Let $x =$ correspond to 1958.)

(b) Estimate the concentration of CO_2 in the ye 2000.

(c) If the trend continues, in what year will the co centration of CO_2 reach 398 ppm.

19. Find the best least-squares fit to the points $(5, 4)$ a $(7, 3)$. Show that it is the straight line through the tw points.

Solutions to Practice Problems 1.5

1. No. The word "distance" implies a nonnegative number. It is the absolute value of the difference between the y-coordinate of the data point and the y-coordinate of the point on the line.

2. The vertical distance will be zero when the point ac tually lies on the least-squares line.

CHAPTER SUMMARY

1. Cartesian coordinate systems associate a number with each point of a line and associate a pair of numbers with each point of a plane.

2. The collection of points in the plane that satisfy the linear equation $ax + by = c$ (where a and b are not both zero) lies on a straight line. After this equation is put into one of the standard forms $y = mx + b$ or $x = a$, the graph can be easily drawn.

3. The direction of the inequality sign in an inequality is unchanged when a number is added to or subtracted from both sides of the inequality, or when both sides of the inequality are multiplied by the same positive number. The direction of the inequality sign is reversed when both sides of the inequality are multiplied by the same negative number.

4. The collection of points in the plane that satisfy the linear inequality $ax + by \leq c$ or $ax + by \geq c$ consists of all points on and to one side of the graph of the corresponding linear equation. After this inequality is put into standard form, the graph can be easily pictured by crossing out the half-plane consisting of the points that do not satisfy the inequality.

5. The feasible set of a system of linear inequalities (that is, the collection of points that satisfy all the inequa ities) is best obtained by crossing out the points n satisfied by each inequality.

6. The point of intersection of a pair of lines can be o tained by first converting the equations to standa form and then either equating the two expressions f y or substituting the value of x from the form $x =$ into the other equation.

7. The slope of the line $y = mx + b$ is the numb m. It is also the ratio of the difference betwe the y-coordinates and the difference between the coordinates of any pair of points on the line.

8. The steepness property states that if we start at a point on a line of slope m and move 1 unit to the rig then we must move m units vertically to return to t line.

9. The point-slope formula states that the line of slo m passing through the point (x_1, y_1) has the equati $y - y_1 = m(x - x_1)$.

10. Two nonvertical lines are parallel if, and only if, th have the same slope. Two nonvertical lines are perpe dicular if, and only if, the product of their slopes is -

11. The method of least squares finds the straight line that gives the best fit to a collection of points in the sense that the sum of the squares of the vertical distances from the points to the line is as small as possible. The slope and y-intercept of the least-squares line are usually found with formulas involving sums of coordinates or by using technology.

REVIEW OF FUNDAMENTAL CONCEPTS

1. How do you determine the coordinates of a point in the plane?
2. What is meant by the graph of an equation in x and y?
3. What is the general form of a linear equation in x and y?
4. What is the standard form of a linear equation in x and y?
5. What is the y-intercept of a line? How do you find the y-intercept from an equation of a line?
6. What is the x-intercept of a line? How do you find the x-intercept from an equation of a line?
7. Give a method for graphing the equation $y = mx + b$.
8. State the inequality properties for addition, subtraction, and multiplication.
9. What are the general and standard forms of a linear inequality in x and y?
10. Explain how to obtain the graph of a linear inequality.
11. What is meant by the feasible set of a system of linear inequalities?
12. Describe how to obtain the point of intersection of two lines.
13. Define the slope of a line, and give a physical description.
14. Suppose you know the slope and the coordinates of a point on a line. How could you draw the line without first finding its equation?
15. What is the point-slope form of the equation of a line?
16. Describe how to find the equation for a line when you know the coordinates of two points on the line.
17. What can you say about the slopes of perpendicular lines?
18. What can you say about the slopes of parallel lines?
19. What is the least-squares line approximation to a set of data points?

KEY FORMULAS

Equation of line with slope m and y-intercept $(0, b)$:

$$y = mx + b$$

Equation of line with slope m and passing through (x_1, y_1):

$$y - y_1 = m(x - x_1)$$

Slope of line passing through (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let m and n be slopes of parallel lines:

$$m = n$$

Let m and n be slopes of perpendicular lines:

$$m = -\frac{1}{n}$$

Let $y = mx + b$ be the least-squares line for the points $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$:

$$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{N \cdot \sum x^2 - \left(\sum x\right)^2}$$

$$b = \frac{\sum y - m \cdot \sum x}{N},$$

where

$$\sum x = x_1 + x_2 + \ldots + x_N$$

$$\sum y = y_1 + y_2 + \ldots + y_N$$

$$\sum xy = x_1 \cdot y_1 + x_2 \cdot y_2 + \cdots + x_N y_N$$

$$\sum x^2 = x_1^2 + x_2^2 + \cdots + x_N^2.$$

SUPPLEMENTARY EXERCISES

1. What is the equation of the y-axis?

2. Graph the linear equation $y = -\frac{1}{2}x$.

3. Find the point of intersection of the pair of straight lines $x - 5y = 6$ and $3x = 6$.

4. Find the slope of the line having equation $3x - 4y = 8$.

5. Find the equation of the line having y-intercept $(0, 5)$ and x-intercept $(10, 0)$.

6. Graph the linear inequality $x - 3y \geq 12$.

7. Does the point $(1, 2)$ satisfy the linear inequality $3x + 4y \geq 11$?

8. Find the point of intersection of the pair of straight lines $2x - y = 1$ and $x + 2y = 13$.

9. Find the equation of the straight line passing through the point $(15, 16)$ and parallel to the line $2x - 10y = 7$.

10. Find the y-coordinate of the point having x-coordinate 1 and lying on the line $y = 3x + 7$.

11. Find the x-intercept of the straight line with equation $x = 5$.

12. Graph the linear inequality $y \leq 6$.

13. Solve the following system of linear equations:

$$\begin{cases} 3x - 2y = 1 \\ 2x + y = 24. \end{cases}$$

14. Graph the feasible set for the following system of inequalities:

$$\begin{cases} 2y + 7x \geq 28 \\ 2y - x \geq 0 \\ y \leq 8. \end{cases}$$

15. Find the y-intercept of the line passing through the point $(4, 9)$ and having slope $\frac{1}{2}$.

16. **Cost of Moving** The fee charged by a local moving company depends on the amount of time required for the move. If t hours are required, then the fee is $y = 35t + 20$ dollars. Give an interpretation of the slope and y-intercept of this line.

17. Are the points $(1, 2)$, $(2, 0)$, and $(3, 1)$ on the same line?

18. Write an equation of the line with x-intercept $(3, 0)$ and y-intercept $(0, -2)$.

19. **PE** If $x + 7y = 30$ and $x = -2y$, then y equals
(a) -12 (b) 12 (c) 30/9 (d) 6 (e) -6

20. Write the inequality whose graph is the half-plane below the line with slope $\frac{2}{3}$ and y-intercept $\left(0, \frac{3}{2}\right)$.

21. Write the inequality whose graph is the half-plane above the line through $(2, -1)$ and $(6, 8.6)$.

22. Solve the system of linear equations

$$\begin{cases} 1.2x + 2.4y = .6 \\ 4.8y - 1.6x = 2.4. \end{cases}$$

23. Find the equation of the line through $(1, 1)$ and the tersection point of the lines $y = -x + 1$ and $y = 2x$

24. Find all numbers x such that $2x + 3(x - 2) \geq 0$.

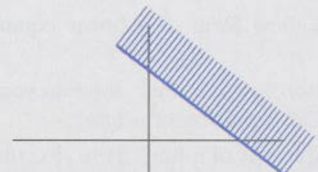
25. Graph the equation $x + \frac{1}{2}y = 4$, and give the slope both intercepts.

26. Do the three graphs of the linear equations $2x - 3y$ $5x + 2y = 0$, and $x + y = 1$ contain a common poi

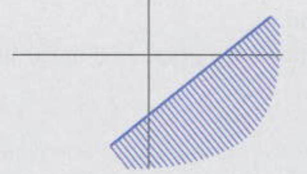
27. Show that the lines with equations $2x - 3y = 1$ $3x + 2y = 4$ are perpendicular.

28. Each of the half-planes (A), (B), (C), and (D) is graph of one of the linear inequalities (a), (b), (c), (d). Match each half-plane with its inequality.

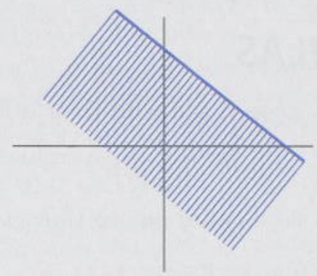
(A)



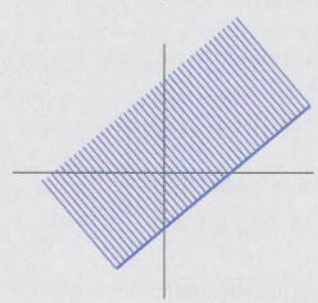
(B)



(C)



(D)



(a) $x + y \geq 1$ (b) $x + y \leq 1$

(c) $x - y \leq 1$ (d) $y - x \leq -1$

29. Each of the l of one of the of the followi

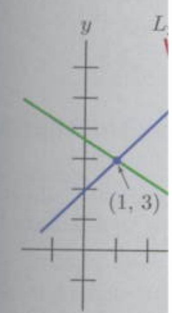


Figure 1

(a) $4x + y =$

(c) $2x + 3y$

30. Find a systen Fig. 2 and fin tex. [*Hint*: T

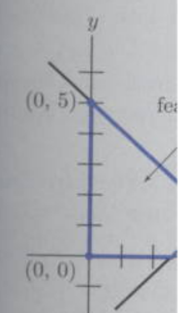


Figure 2

31. Consider the f

One is the equ the equation o tions and then curves.

32. Find the vertic

The American Fresh

29. Each of the lines L_1, L_2, and L_3 in Fig. 1 is the graph of one of the equations (a), (b), and (c). Match each of the following equations with its corresponding line.

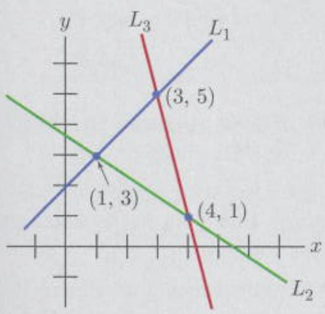


Figure 1

(a) $4x + y = 17$ (b) $y = x + 2$

(c) $2x + 3y = 11$

30. Find a system of inequalities having the feasible set in Fig. 2 and find the coordinates of the unspecified vertex. [*Hint*: There is a right angle at the vertex $\left(4, \frac{3}{2}\right)$.]

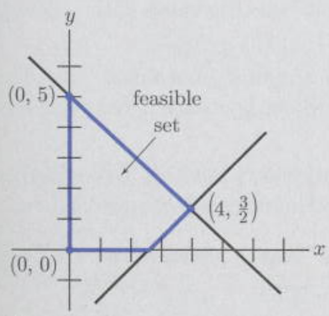


Figure 2

31. Consider the following four equations:

$$p = .01q - 5$$
$$p = .005q + .5$$
$$p = -.01q - 10$$
$$p = -.01q + 5.$$

One is the equation of a supply curve and another is the equation of a demand curve. Identify the two equations and then find the intersection point of those two curves.

32. Find the vertices of the following feasible set.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 5x + y \leq 50 \\ 2x + 3y \leq 33 \\ x - 2y \geq -8 \end{cases}$$

33. College Freshmen The percent of college freshmen who enter college intending to eventually obtain a degree in a medical field (M.D., D.O., D.D.S., D.V.M.) has increased steadily in recent years.[1] The percent, y, who entered college x years after 2000 intending to obtain a degree in a medical field is approximated by the linear equation $y = .1x + 8.8$.

(a) What interpretation can be given to the y-intercept of the graph of the equation?

(b) Approximately what percent of college freshmen entering in 2008 intended to eventually obtain a degree in a medical field?

(c) In what year did approximately 9.2% of entering college freshmen intend to eventually obtain a degree in a medical field?

34. Profit Equation For a certain manufacturer, the production and sale of each additional unit yields an additional profit of \$10. The sale of 1000 units yields a profit of \$4000.

(a) Find the profit equation.

(b) Find the y- and x-intercepts of the graph of the profit equation.

(c) Sketch the graph of the profit equation.

35. Car Rentals One-day car rentals cost \$50 plus 10 cents per mile from company A and \$40 plus 20 cents per mile from company B.

(a) For each company, give the linear equations for the cost, y, when x miles are driven.

(b) Which company offers the best value when the car is driven for 80 miles?

(c) Which company offers the best value when the car is driven for 160 miles?

(d) For what mileage do the two companies offer the same value?

36. Inflation In 1990, white bread cost an average of \$0.69 per pound. In 2007, white bread cost an average of \$1.21 per pound. (*Source*: U.S. Bureau of Labor Statistics, Consumer Price Index)

(a) Assuming a linear increase in the price per pound of bread, find the equation that relates the cost, y, to the number of years after 1990, x.

(b) When did white bread cost an average of \$1.00 per pound?

37. Graph the linear inequality $x \leq 3y + 2$.

38. Sales Commission A furniture store offers its new employees a weekly salary of \$200 plus a 3% commission on sales. After one year, employees receive \$100 per week plus a 5% sales commission. For what weekly sales level will the two scales produce the same salary?

[1] *The American Freshman: National Norms*; American Council on Education, University of California-Los Angeles.

39. Find a system of linear inequalities having the feasible set of Fig. 3.

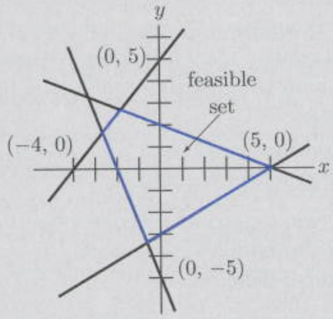


Figure 3

40. Find a system of linear inequalities having the feasible set of Fig. 4.

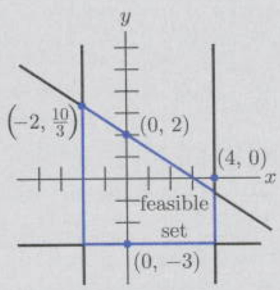


Figure 4

41. Medical Assistant Jobs According to the U.S. Department of Labor, medical assistant jobs are expected to increase from 417,000 in 2006 to 565,000 in 2016. Assuming that the number of medical assistant jobs increases linearly during that time, find the equation that relates the number of jobs, y, to the number of years after 2006, x. Use the equation to predict the number of medical assistant jobs in 2012.

42. Bachelor Degrees in Education According to the U.S. National Center of Educational Statistics, 110,807 bachelor degrees in education were awarded in 1991 and 107,238 were awarded in 2006. If the number of bachelor degrees in education continues to decline linearly, how many bachelor degrees in education will be awarded in 2010?

43. Soft Drinks According to *Beverage Digest*, Coke Classic's percentage of the soft drink market declined from 20.4 in 2000 to 17.2 in 2007. If the percentage declined linearly during that time, estimate Coke Classic's percentage of the soft drink market in 2005.

44. Life Expectancy The following table gives the 2008 life expectancy at birth for several countries.[2]

	Male	Female
Portugal	74.8	81.5
United States	75.3	81.1
Australia	77.9	83.8
Japan	78.7	85.6

(a) Use the method of least squares to obtain the straight line that best fits these data.

(b) In Norway the life expectancy of men is 77.2 years. Use the least-squares line from (a) to estimate the life expectancy for women.

(c) In France the life expectancy for women is 84.2 years. Use the least-squares line from (a) to estimate the life expectancy for men.

45. Prospective Nurses Students entering college in 2007 expressed the greatest interest in nursing careers in many years. The following table gives the percent of college freshmen in 2003 through 2007 who said that their probable career choice would be in nursing.[3]

(a) Use the method of least squares to obtain the straight line that best fits these data. (Let $x = 0$ represent 2000.)

(b) Estimate the percentage of students who entered college in 2009 whose probable career was in nursing.

(c) If the trend continues, when will 5.4% of freshmen have nursing as their probable career choice?

Year	Percent
2003	3.5
2004	3.9
2005	4.0
2006	4.1
2007	4.3

46. Cancer and Diet The following table[4] gives the (age-adjusted) death rate per 100,000 women from breast cancer and the daily dietary fat intake (in grams per day) for various countries.

Country	Fat Intake	Death Rate
Japan	41	4
Poland	90	10
Finland	118	13
United States	148	21

(a) Use the method of least squares to obtain the straight line that best fits these data.

[2] U.S. Bureau of the Census, International Database.

[3] *The American Freshman: National Norms*; American Council on Education, University of California–Los Angeles.

[4] B. S. Reddy et al., "Nutrition and Its Relationship to Cancer," *Advances in Cancer Research* 32:237, 1980.

(b) In Denmark women consume an average of 160 grams of fat per day. Use the least-squares line to estimate the breast cancer death rate.

(c) In New Zealand the breast cancer death rate is 22 women per 100,000. Use the least-squares line to estimate the daily fat intake in New Zealand.

Conceptual Exercises

47. Consider an equation of the form $y = mx + b$. When the value of m remains fixed and the value of b changes, the graph is translated vertically. As the value of b increases, does the graph move up or down?

48. Consider a linear equation of the form $y = mx + b$. When the value of b remains fixed and the value of m changes, the graph rotates about the point $(0, b)$. As the value of m increases, will the graph rotate in a clockwise or counterclockwise direction?

49. When is the x-intercept of a line the same as the y-intercept?

50. What is the difference between a line having undefined slope and zero slope?

51. Does every line have an x-intercept? y-intercept?

52. Suppose you have found the line of best least-squares fit to a collection of points and that you edit the data by adding a point on the line to the data. Will the expanded data have the same least-squares line? Explain the rationale for your conclusion, and then experiment to test whether your conclusion is correct.

CHAPTER TEST

1. Graph the linear equation $y = 3x$.

2. Find the y-coordinate of the point having x-coordinate $\frac{1}{2}$ and lying on the line $y = -2x + 6$.

3. Find the equation of the line through the point $(-1, 3)$ and parallel to the line $y = -2x + 6$.

4. Solve the system of linear equations

$$\begin{cases} 2x - 3y = 9 \\ -3x + 7y = -11. \end{cases}$$

5. Show that the lines $3x - y = 1$ and $-\frac{1}{3}x - 4 = y$ are perpendicular.

6. Graph the inequality $x + y \geq 8$ in the plane.

7. Find the equation of the line of slope 2 that passes through the intersection point of the lines $4x + 5y = 11$ and $2x - 3y = 7$.

8. Find the coordinates of the vertices of the feasible set for the following system of inequalities.

$$\begin{cases} x + y \geq 16 \\ -2x + y \leq 10 \\ 3x + y \leq 75 \\ x \geq 0,\ y \geq 0 \end{cases}$$

9. **Sales Commissions** Fred sells carpeting. He earns \$200 each week plus 5% commission on sales. His sister thinks she has a better job selling carpet because she earns \$250 a week plus 3% commission on her sales. For what volume of sales is Fred's sister correct? When is she incorrect?

10. **Inflation** The following table gives the average price of a pound of Red Delicious apples in January of the given years. (*Source*: U.S. Bureau of Labor Statistics, Consumer Price Index.)

Year	Price
1999	\$0.86
2002	\$0.88
2005	\$0.97
2008	\$1.16

(a) Use the method of least squares to obtain the straight line that best fits these data.

(b) Estimate the average price of a pound of Red Delicious apples in January 2004.

(c) If this trend continues, when will the average price of a pound of Red Delicious apples be \$1.25?

(c) Here y does not occur, so we solve for x:

$$2x = 6$$
$$x = 3 \quad \text{Divide both sides by 2.}$$

Now Try Exercise 19

Thus the standard form of $2x = 6$ is $x = 3$—that is, $x = a$, where a is 3. ■

We have seen that any linear equation has one of the two standard forms $y = mx + b$ and $x = a$. From Example 5, the graph of $x = a$ is a vertical line, a units from the y-axis. What can be said about the graph of $y = mx + b$? In Example 3, we saw that the graph is a straight line in the special case $y = 2x - 1$. Actually, the graph of $y = mx + b$ is always a straight line. To sketch the graph, we need only locate two points. Two convenient points to locate are the **intercepts**, the points where the line crosses the x- and y-axes. When x is 0, $y = m \cdot 0 + b = b$. Thus $(0, b)$ is on the graph of $y = mx + b$ and is the y-intercept of the line. The x-intercept is found as follows: A point on the x-axis has y-coordinate 0. So the x-coordinate of the x-intercept can be found by setting $y = 0$—that is, $mx + b = 0$—and solving this equation for x.

EXAMPLE 7 **Graph of a linear equation** Sketch the graph of the equation $y = 2x - 1$.

Solution Here $m = 2$, $b = -1$. The y-intercept is $(0, b) = (0, -1)$. To find the x-intercept, we must solve $2x - 1 = 0$. But then $2x = 1$ and $x = 1/2$. So the x-intercept is $(1/2, 0)$. Plot the two points $(0, -1)$ and $(1/2, 0)$, and draw the straight line through them (Fig. 6). ■

Now Try Exercise 23

NOTE The line in Example 7 had different x- and y-intercepts. Two other circumstances can occur, however. First, the two intercepts may be the same, as in $y = 3x$. Second, there may be no x-intercept, as in $y = 1$. In both of these circumstances, we must plot some point other than an intercept in order to graph the line. ■

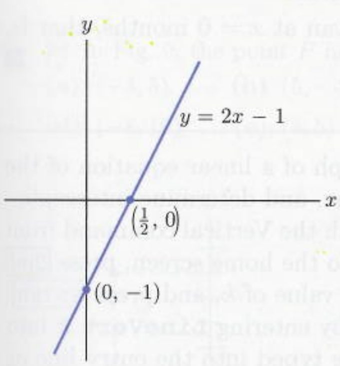


Figure 6

To summarize,

To graph the equation $y = mx + b$,
1. Plot the y-intercept $(0, b)$.
2. Plot some other point. [The most convenient choice is often the x-intercept $(x, 0)$, where x is determined by setting $y = 0$ and solving for x.]
3. Draw a line through the two points.

The next example gives an application of linear equations.

EXAMPLE 8 **Linear depreciation** For tax purposes, businesses must keep track of the current values of each of their assets. A common mathematical model is to assume that the current value y is related to the age x of the asset by a linear equation. A moving company buys a 40-foot van with a useful lifetime of 5 years. After x months of use, the value y of the van is estimated by the linear equation

$$y = 25{,}000 - 400x.$$

(a) Sketch the graph of this linear equation.

(b) What is the value of the van after 5 years?

(c) When will the value of the van be \$15,000?

(d) What economic interpretation can be given to the y-intercept of the graph?

Solution **(a)** The y-intercept is $(0, 25{,}000)$. To find the x-intercept, set $y = 0$ and solve for x.

$$0 = 25{,}000 - 400x \qquad \text{Set } y = 0.$$
$$400x = 25{,}000 \qquad \text{Add } 400x \text{ to both sides.}$$
$$x = 62.5 \qquad \text{Divide both sides by 400.}$$

The x-intercept is $(62.5, 0)$. The graph of the linear equation is sketched in Fig. 7. Note how the value decreases as the age of the van increases. The value of the van reaches 0 after 62.5 months. Note also that we have sketched only the portion of the graph that has physical meaning, namely the portion for x between 0 and 62.5.

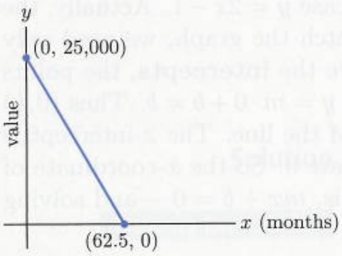


Figure 7

(b) After 5 years (or 60 months), the value of the van is

$$y = 25{,}000 - 400(60) = 25{,}000 - 24{,}000 = 1000.$$

Since the useful life of the van is 5 years, this value represents the *salvage value* of the van.

(c) Set the value of y to 15,000 and solve for x.

$$15{,}000 = 25{,}000 - 400x \qquad \text{Set } y = 15{,}000.$$
$$400x + 15{,}000 = 25{,}000 \qquad \text{Add } 400x \text{ to both sides.}$$
$$400x = 10{,}000 \qquad \text{Subtract 15,000 from both sides.}$$
$$x = 25 \qquad \text{Divide both sides by 400.}$$

The value of the van will be \$15,000 after 25 months.

(d) The y-intercept corresponds to the value of the van at $x = 0$ months, that is, the initial value of the van, \$25,000.

Now Try Exercise 41

INCORPORATING TECHNOLOGY

GC[1] Appendices B and D show how to obtain a graph of a linear equation of the form $y = mx + b$, find coordinates of points on the line, and determine intercepts.

Vertical lines can be drawn on a TI-83/84 Plus with the Vertical command from the DRAW menu. To draw the vertical line $x = k$, go to the home screen, press [2nd] [DRAW] 4 to display the word **`Vertical`**, type in the value of k, and press [ENTER]. With the TI-89, the vertical line $x = k$ can be drawn by entering **`LineVert k`** into the home screen. (The word **`LineVert`** either can be typed into the entry line or selected from the CATALOG menu.)

Practice Problems 1.1

1. Plot the point $(500, 200)$.
2. Is the point $(4, -7)$ on the graph of the linear equation $2x - 3y = 1$? Is the point $(5, 3)$?
3. Graph the linear equation $5x + y = 10$.
4. Graph the straight line $y = 3x$.

[1]GC is an abbreviation for "Graphing Calculator." Throughout this text we discuss specifics of using TI-83/84 Plus and TI-89 calculators. Additional details are provided in Appendices B and D. For the specifics of using other calculators, consult the guidebook that comes with the calculator.

EXERCISES 1.1

In Exercises 1–8, plot the given point.

1. $(2, 3)$ **2.** $(-1, 4)$
3. $(0, -2)$ **4.** $(2, 0)$
5. $(-2, 1)$ **6.** $(-1, -\frac{5}{2})$
7. $(-20, 40)$ **8.** $(25, 30)$

9. PE[2] In Fig. 8, the point P has coordinates
(a) $(5, 3)$ **(b)** $(-3, 5)$ **(c)** $(-5, 3)$
(d) $(3, -5)$ **(e)** $(5, -3)$

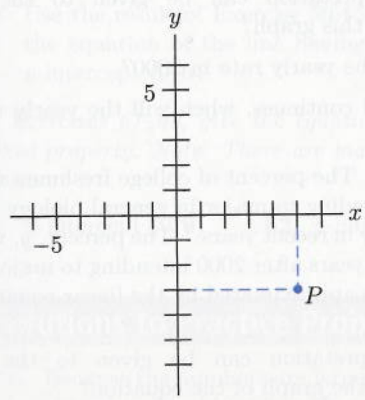


Figure 8

10. PE In Fig. 9, the point P has coordinates
(a) $(-4, 5)$ **(b)** $(5, -4)$ **(c)** $(10, -8)$
(d) $(-8, 10)$ **(e)** $(4, 5)$

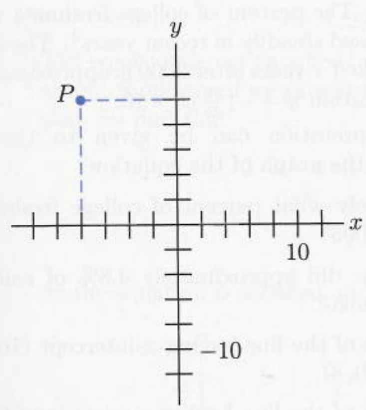


Figure 9

In Exercises 11–14, determine if the point is on the graph of the equation $-2x + \frac{1}{3}y = -1$.

11. $(1, 3)$ **12.** $(2, 6)$ **13.** $(\frac{1}{2}, 3)$ **14.** $(\frac{1}{3}, -1)$

In Exercises 15–18, each linear equation is in the standard form $y = mx + b$. Identify m and b.

15. $y = 5x + 8$ **16.** $y = -2x - 6$
17. $y = 3$ **18.** $y = \frac{2}{3}x$

In Exercises 19–22, put the linear equations into standard form.

19. $14x + 7y = 21$ **20.** $x - y = 3$
21. $3x = 5$ **22.** $-\frac{1}{2}x + \frac{2}{3}y = 10$

In Exercises 23–26, find the x-intercept and the y-intercept of each line.

23. $y = -4x + 8$ **24.** $y = 5$
25. $x = 7$ **26.** $y = -8x$

In Exercises 27–34, graph the given linear equation.

27. $y = \frac{1}{3}x - 1$ **28.** $y = 2x$ **29.** $y = \frac{5}{2}$
30. $x = 0$ **31.** $3x + 4y = 24$ **32.** $x + y = 3$
33. $x = -\frac{5}{2}$ **34.** $\frac{1}{2}x - \frac{1}{3}y = -1$

35. Which of the following equations describe the same line as the equation $2x + 3y = 6$?
(a) $4x + 6y = 12$ **(b)** $y = -\frac{2}{3}x + 2$
(c) $x = 3 - \frac{3}{2}y$ **(d)** $6 - 2x - y = 0$
(e) $y = 2 - \frac{2}{3}x$ **(f)** $x + y = 1$

36. Which of the following equations describe the same line as the equation $\frac{1}{2}x - 5y = 1$?
(a) $2x - \frac{1}{5}y = 1$ **(b)** $x = 5y + 2$
(c) $2 - 5x + 10y = 0$ **(d)** $y = .1(x - 2)$
(e) $10y - x = -2$ **(f)** $1 + .5x = 2 + 5y$

37. Each of the lines L_1, L_2, and L_3 in Fig. 10 is the graph of one of the equations (a), (b), and (c). Match each of the equations with its corresponding line.
(a) $x + y = 3$ **(b)** $2x - y = -2$ **(c)** $x = 3y + 3$

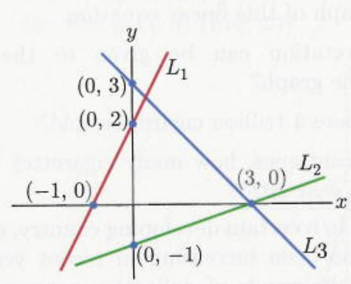


Figure 10

[2]Exercises denoted PE are similar to questions appearing in professional exams such as GMAT and CPA exams.

38. Which of the following equations is graphed in Fig. 11?

(a) $x + y = 3$ (b) $y = x - 1$ (c) $2y = x + 3$

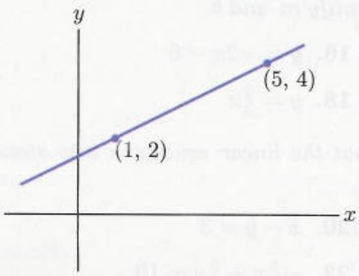


Figure 11

39. Heating Water The temperature of water in a heating tea kettle rises according to the equation $y = 30x + 72$, where y is the temperature (in degrees Fahrenheit) x minutes after the kettle was put on the burner.

(a) What physical interpretation can be given to the y-intercept of the graph?

(b) What will the temperature of the water be after 3 minutes?

(c) After how many minutes will the water be at its boiling point of 212°?

40. Deforestation The amount of tropical rainforest area in Central America has been decreasing steadily in recent years. The amount y (in thousands of square miles) x years after 1969 is estimated by the linear equation $y = \left(-\frac{25}{8}\right)x + 130$.

(a) Sketch the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of the graph?

(c) When were there 80,000 square miles of tropical rain forests?

(d) Approximately how large was the tropical rainforest in the year 2007?

41. Cigarette Consumption The worldwide consumption of cigarettes has been increasing steadily in recent years. The number of trillions of cigarettes, y, purchased x years after 1960, is estimated by the linear equation $y = .075x + 2.5$.

(a) Sketch the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of the graph?

(c) When were there 4 trillion cigarettes sold?

(d) If this trend continues, how many cigarettes will be sold in the year 2020?

42. Ecotourism Income In a certain developing country, ecotourism income has been increasing in recent years. The income y (in thousands of dollars) x years after 2000 can be modeled by $y = 1.15x + 14$.

(a) Sketch the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of this graph?

(c) When was there \$20,000 in ecotourism income?

(d) If this trend continues, how much ecotourism income will there be in 2016?

43. Insurance Rates Yearly car insurance rates have been increasing steadily in the last few years. The rate y (in dollars) for a small car x years after 1997 can be modeled by $y = 60x + 678$.

(a) Sketch the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of this graph?

(c) What was the yearly rate in 2000?

(d) If this trend continues, when will the yearly rate be \$1578?

44. College Freshmen The percent of college freshmen who enter college intending to major in general biology has increased steadily in recent years.[3] The percent, y, who entered college x years after 2000 intending to major in general biology is approximated by the linear equation $y = .15x + 3.85$.

(a) What interpretation can be given to the y-intercept of the graph of the equation?

(b) Approximately what percent of college freshmen entering in 2005 intended to major in general biology?

(c) In what year did approximately 4.9% of entering college freshmen intend to major in general biology?

45. College Freshmen The percent of college freshmen who smoke has decreased steadily in recent years[3]. The percent, y, who smoked x years after 2000 is approximated by the linear equation $y = -(\frac{26}{35})x + 10$.

(a) What interpretation can be given to the y-intercept of the graph of the equation?

(b) Approximately what percent of college freshmen smoked in 2005?

(c) In what year did approximately 4.8% of college freshmen smoke?

46. Find an equation of the line having x-intercept $(16, 0)$ and y-intercept $(0, 8)$.

47. Find an equation of the line having x-intercept $(.6, 0)$ and y-intercept $(0, .9)$.

48. Find an equation of the line having y-intercept $(0, 5)$ and x-intercept $(4, 0)$.

49. What is the equation of the x-axis?

50. Can a line have more than one x-intercept?

[3] *The American Freshman: National Norms*; American Council on Education, University of California-Los Angeles.

51. What is the general form of the equation of a line that is parallel to the x-axis?

52. Give the x- and y-intercepts of the line $\frac{x}{a}+\frac{y}{b}=1$.

*Every straight line has an equation of the form $cx+dy=e$, where not both c and d are 0. Such an equation is said to be a **general form** of the equation of the line. In Exercises 53–58, find a general form of the given equation.*

53. $y=2x+3$ **54.** $x=5$ **55.** $x=-3$

56. $y=3x-4$ **57.** $y=-\frac{2}{3}x-5$ **58.** $y=4x-\frac{5}{6}$

59. Show that the straight line with x-intercept $(a,0)$ and y-intercept $(0,b)$, where a and b are not both zero, has $bx+ay=ab$ as a general form of its equation.

60. Use the result of Exercise 59 to find a general form of the equation of the line having x-intercept $(5,0)$ and y-intercept $(0,6)$.

In Exercises 61–68, give the equation of a line having the stated property. Note: There are many answers to each exercise.

61. x-intercept $(9,0)$ **62.** y-intercept $(0,10)$

63. passes through the point $(-2,5)$

64. passes through the point $(3,-3)$

65. crosses the positive part of the y-axis

66. passes through the origin

67. crosses the negative part of the x-axis

68. crosses the positive part of the x-axis

Exercises 69–74 require the use of a graphing calculator.

In Exercises 69–72, (a) graph the line in the standard window $[-10,10]$ by $[-10,10]$, (b) use the calculator to find the y-coordinate of the point on the line with x-coordinate 2, (c) use the calculator to determine the two intercepts.

69. $y=-3x+6$ **70.** $y=.25x-2$

71. $3y-2x=9$ **72.** $2y+5x=8$

73. Determine an appropriate window and graph the line $2y+x=100$.

74. Determine an appropriate window and graph the line $x-3y=60$.

Solutions to Practice Problems 1.1

1. Because the numbers are large, make each hatchmark correspond to 100. Then the point $(500,200)$ is found by starting at the origin, moving 500 units to the right and 200 units up (Fig. 12).

2. Replace x by 4 and y by -7 in the equation $2x-3y=1$.

$$2(4)-3(-7)=1$$
$$8+21=1$$
$$29=1$$

The equation is not satisfied, so $(4,-7)$ is not on the graph. Similarly, if we substitute $(5,3)$ into the equation, we find that

$$2\cdot 5-3\cdot 3=1$$
$$10-9=1$$
$$1=1.$$

So the equation is satisfied and $(5,3)$ is on the graph.

3. The standard form is obtained by solving for y:

$$y=-5x+10.$$

Thus $m=-5$ and $b=10$. The y-intercept is $(0,10)$. To find the x-intercept, set $y=0$:

$$0=-5x+10$$
$$5x=10$$
$$x=2.$$

So the x-intercept is $(2,0)$ (Fig. 13).

4. To find the y-intercept, set $x=0$; then $y=3\cdot 0=0$. To find the x-intercept, set $y=0$; then $3x=0$ or $x=0$. The two intercepts are the same point $(0,0)$. We must therefore plot some other point. Setting $x=1$, we obtain $y=3\cdot 1=3$, so another point on the line is $(1,3)$ (Fig. 14).

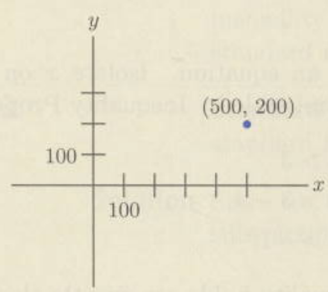


Figure 12

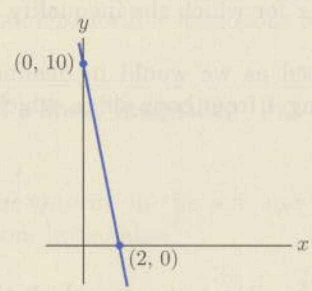


Figure 13

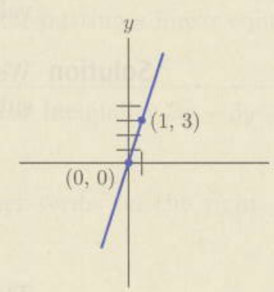


Figure 14