76. Perform the multiplications and simplify.

a. $(x + yi)(x - yi)$

b. $(1 - i)^5$

c. $(1 - \sqrt{3})^4$

d. $[x - (2 + 5i)][x - (2 - 5i)]$

For Exercises 77–85, redo Exercises 49–57 using the i key on your graphing calculator. Remember to use parentheses appropriately! (*Note:* The values of a and b in each answer will be in decimal form.)

86. *Mathematics in Writing:* Consider the addition and the multiplication of complex numbers. How does i differ from a variable like x? If you always treat i as though it is a variable, at what step in the procedures of addition or multiplication would you run into trouble?

■ ■ ■

Chapter Summary

Terms and Symbols

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Key Ideas for Review

Topic	Page	Key Idea
Set	3	A set is a collection of objects or numbers.
The Set of Real Numbers	3	The set of real numbers is composed of the rational and irrational numbers. The rational numbers are those that can be written as the ratio of two integers, $\frac{p}{q}$, with $q \neq 0$; the irrational numbers cannot be written as the ratio of two integers.
Properties	6	The real number system satisfies a number of important properties, including: closure, commutativity, associativity, identities, inverses, distributivity
Equality	6	If two numbers are identical, we say that they are equal.
Properties	7	Equality satisfies these basic properties: reflexive property, symmetric property, transitive property, substitution property
Real Number Line	12	There is a one-to-one correspondence between the set of all real numbers and the set of all points on the real number line. That is, for every point on the line there is a real number, and for every real number there is a point on the line.
Inequalities	13	Algebraic statements using inequality symbols have geometric interpretations using the real number line. For example, $a < b$ says that a lies to the left of b on the real number line.
Operations	13	Inequalities can be operated on in the same manner as statements involving an equal sign with one important exception: when an inequality is multiplied or divided by a negative number, the direction of the inequality is reversed.
Absolute Value	15	Absolute value specifies distance independent of direction. Four important properties of absolute value are: • $\lvert a \rvert \geq 0$ • $\lvert a \rvert = \lvert -a \rvert$ • $\lvert a - b \rvert = \lvert b - a \rvert$ • $\lvert ab \rvert = \lvert a \rvert \, \lvert b \rvert$
Distance	17	The distance between points A and B whose coordinates are a and b, respectively, is given by $\overline{AB} = \lvert b - a \rvert$
Polynomials	22	Algebraic expressions of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ are called polynomials.

Key Ideas for Review

Topic	Page	Key Idea
Operations	23	To add (subtract) polynomials, just add (subtract) like terms. To multiply polynomials, form all possible products, using the rule for exponents: $$a^m a^n = a^{m+n}$$
Factoring	31	A polynomial is said to be factored when it is written as a product of polynomials of lower degree.
Rational Expressions	40	Most of the rules of arithmetic for handling fractions carry over to rational expressions. For example, the LCD has the same meaning except that we deal with polynomials in factored form rather than with integers.
Exponents	49	The rules for positive integer exponents also apply to zero, negative integer exponents, and, in fact, to all rational exponents.
Scientific Notation	52	A number in scientific notation is of the form $$\pm\, a \times 10^m$$ where $1 \leq a < 10$ and m is some integer.
Radicals	60	Radical notation is another way of writing a rational exponent. That is, $$\sqrt[n]{b} = b^{1/n}$$
Principal nth Root	57	If n is even and b is positive, there are two real numbers a such that $b^{1/n} = a$. Under these circumstances, we insist that the nth root be positive. That is, $\sqrt[n]{b}$ is a positive number if n is even and b is positive. Thus $\sqrt{16} = 4$. Similarly, we must write $$\sqrt{x^2} = \lvert x \rvert$$ to ensure that the result is a positive number.
Simplifying	62	To be in simplified form, a radical must satisfy the following conditions: • $\sqrt[n]{x^m}$ has $m < n$. • $\sqrt[n]{x^m}$ has no common factors between m and n. • The denominator has been rationalized.
Complex Numbers	68	Complex numbers were created because there were no real numbers that satisfy a polynomial equation such as $$x^2 + 5 = 0$$

Topic	Page	Key Idea
Imaginary Unit i	69	Using the imaginary unit $i = \sqrt{-1}$, a complex number is of the form $a + bi$, where a and b are real numbers; the real part of $a + bi$ is a and the imaginary part of $a + bi$ is b.
Real Number System	70	The real number system is a subset of the complex number system.

Review Exercises

Solutions to exercises whose numbers are in **blue** are in the Solutions section in the back of the book.

In Exercises 1–3, write each set by listing its elements within braces.

1. The set of natural numbers from -5 to 4, inclusive
2. The set of integers from -3 to -1, inclusive
3. The subset of $x \in S$, $S = \{0.5, 1, 1.5, 2\}$ such that x is an even integer

For Exercises 4–7, determine whether the statement is true (T) or false (F).

4. $\sqrt{7}$ is a real number.
5. -35 is a natural number.
6. -14 is not an integer.
7. 0 is an irrational number.

In Exercises 8–11, identify the property of the real number system that justifies the statement. All variables represent real numbers.

8. $3a + (-3a) = 0$
9. $(3 + 4)x = 3x + 4x$
10. $2x + 2y + z = 2x + z + 2y$
11. $9x \cdot 1 = 9x$

In Exercises 12–14, sketch the given set of numbers on a real number line.

12. The negative real numbers
13. The real numbers x such that $x > 4$
14. The real numbers x such that $-1 \le x < 1$
15. Find the value of $|-3| - |1 - 5|$.
16. Find $\overline{PQ}$ if the coordinates of P and Q are $\frac{9}{2}$ and 6, respectively.
17. A salesperson receives $3.25x + 0.15y$ dollars, where x is the number of hours worked and y is the number of miles driven. Find the amount due the salesperson if $x = 12$ hours and $y = 80$ miles.
18. Which of the following expressions are not polynomials?

    a. $-2xy^2 + x^2y$ b. $3b^2 + 2b - 6$

    c. $x^{-1/2} + 5x^2 - x$ d. $7.5x^2 + 3x - \frac{1}{2}x^0$

In Exercises 19 and 20, indicate the leading coefficient and the degree of each polynomial.

19. $-0.5x^7 + 6x^3 - 5$
20. $2x^2 + 3x^4 - 7x^5$

Review Exercises

In Exercises 21–23, perform the indicated operations.

21. $(3a^2b^2 - a^2b + 2b - a) - (2a^2b^2 + 2a^2b - 2b - a)$

22. $x(2x - 1)(x + 2)$

23. $3x(2x + 1)^2$

In Exercises 24–29, factor each expression.

24. $2x^2 - 2$

25. $x^2 - 25y^2$

26. $2a^2 + 3ab + 6a + 9b$

27. $4x^2 + 19x - 5$

28. $x^8 - 1$

29. $27r^6 + 8s^6$

In Exercises 30–33, perform the indicated operations and simplify.

30. $\dfrac{14(y-1)}{3(x^2-y^2)} \cdot \dfrac{9(x+y)}{-7xy^2}$

31. $\dfrac{4-x^2}{2y^2} \div \dfrac{x-2}{3y}$

32. $\dfrac{x^2-2x-3}{2x^2-x} \div \dfrac{x^2-4x+3}{3x^3-3x^2}$

33. $\dfrac{a+b}{a+2b} \cdot \dfrac{a^2-4b^2}{a^2-b^2}$

In Exercises 34–37, find the LCD.

34. $\dfrac{-1}{2x^2}, \quad \dfrac{2}{x^2-4}, \quad \dfrac{3}{x-2}$

35. $\dfrac{4}{x}, \quad \dfrac{5}{x^2-x}, \quad \dfrac{-3}{(x-1)^2}$

36. $\dfrac{2}{(x-1)y}, \quad \dfrac{-4}{y^2}, \quad \dfrac{x+2}{5(x-1)^2}$

37. $\dfrac{y-1}{x^2(y+1)}, \quad \dfrac{x-2}{2xy-2x}, \quad \dfrac{3x}{4y^2+8y+4}$

In Exercises 38–41, perform the indicated operations and simplify.

38. $2 + \dfrac{4}{a^2-4}$

39. $\dfrac{3}{x^2-16} - \dfrac{2}{x-4}$

40. $\dfrac{\dfrac{3}{x+2} - \dfrac{2}{x-1}}{x-1}$

41. $x^2 + \dfrac{\dfrac{1}{x} + 1}{x - \dfrac{1}{x}}$

In Exercises 42–50, simplify and express the answers using only positive exponents. All variables are positive numbers.

42. $(2a^2b^{-3})^{-3}$

43. $2(a^2 - 1)^0$

44. $\left(\dfrac{x^3}{y^{-6}}\right)^{-4/3}$

45. $\dfrac{x^{3+n}}{x^n}$

46. $\sqrt{80}$

47. $\dfrac{2}{\sqrt{12}}$

48. $\sqrt{x^7y^5}$

49. $\sqrt[4]{32x^8y^6}$

50. $\dfrac{\sqrt{x}}{\sqrt{x} + \sqrt{y}}$

51. Compute

$$\frac{(5.10 \times 10^7)(3.45 \times 10^{-2})}{7.10 \times 10^4}$$

to two decimal places and express the answer in scientific notation.

52. Rationalize the numerator for

$$\frac{\sqrt{x} - \sqrt{y}}{x - y}$$

In Exercises 53 and 54, perform the indicated operations. Simplify the answer.

53. $\sqrt[4]{x^2y^2} + 2\sqrt[4]{x^2y^2}$

54. $(\sqrt{3} + \sqrt{5})^2$

55. Evaluate the given expressions using your calculator.

a. $\dfrac{12}{5} - \dfrac{3}{7}$

b. $|(-4)^3 - 5^6|$

c. $\sqrt{8}$

d. π^8

e. $\sqrt[5]{-27}$

f. $\dfrac{|2 + \sqrt{3}|}{-6}$

g. $\sqrt[3]{4} + \sqrt{\frac{1}{8}}$ h. $\sqrt[10]{0.5}$

i. $\left(\frac{2}{3}\right)^4$ j. $9^{5/8}$

56. Solve for x and y:

$$(x - 2) + (2y - 1)i = -4 + 7i$$

57. Simplify i^{47}.

In Exercises 58–61, perform the indicated operations and write all answers in the form $a + bi$.

58. $2 + (6 - i)$ **59.** $(2 + i)^2$

60. $(4 - 3i)(2 + 3i)$ **61.** $\dfrac{4 - 3i}{2 + 3i}$

62. Perform the indicated operations.

a. Combine into one term with a common denominator

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

b. Simplify the quotient

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{c} + \frac{1}{d}}$$

63. Dan, at 200 pounds, wishes to reduce his weight to 180 pounds in time to attend his college reunion in 8 weeks. He learns that it takes 2400 calories per day to maintain his weight. A reduction of his caloric intake to 1900 calories per day will result in his losing weight at the rate of 1 pound per week. What should his daily caloric intake be to achieve this goal?

64. The executive committee of the student government association consists of a president, vice-president, secretary, and treasurer.

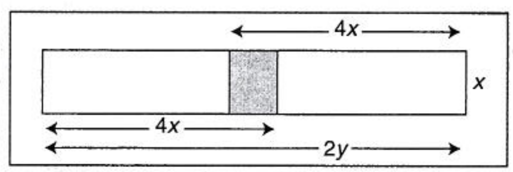
a. In how many ways can a committee of three persons be formed from among the executive committee members?

b. According to the by-laws, there must be at least three affirmative votes to carry a motion. If the president automatically has two votes, list all the minimal winning coalitions.

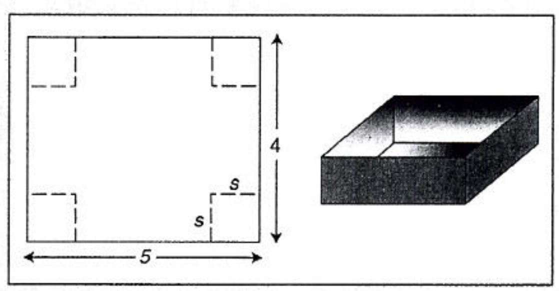
65. If 6 children can devour 6 hot dogs in $\frac{1}{10}$ of an hour, how many children would it take to devour 100 hot dogs in 6000 seconds?

66. A CD player costs a dealer \$80. If he wishes to make a profit of at least 25% of his cost, what must be the lowest selling price for the player?

67. Find the area of the shaded rectangle.



68. An open box is to be made from a 4 feet × 5 feet piece of tin by cutting out squares of equal size from the four corners and bending up the flaps to form sides. Find a formula for the volume in terms of s, the side of the square. Write the inequality that describes the restriction on s.



Review Exercises

69. Compute the following products:

a. $(x - y)(x^2 + xy + y^2)$

b. $(x - y)(x^3 + x^2y + xy^2 + y^3)$

c. $(x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

70. Using Exercise 69, find a general formula that allows you to factor $x^n - y^n$, where n is a positive integer.

71. In ancient Alexandria, numbers were multiplied by using an abacus as follows:

$$\begin{aligned} 19 \times 28 &= (20 - 1)(30 - 2) \\ &= (20)(30) - (20)(2) - 30 + 2 \\ &= 600 - 40 - 30 + 2 \\ &= 532 \end{aligned}$$

Set up a comparable sequence of steps for 13 × 17.

72. Find two ways of grouping and then factoring $ac + ad - bc - bd$.

73. The following calculation represents a sum. If each letter represents a different digit, find the appropriate correspondence between letters and digits so that the sum is correct.

```
 FORTY
   TEN
   TEN
 -----
 SIXTY
```

74. A natural number is said to be perfect if it is the sum of its divisors other than itself. For example, 6 is the first perfect number since $6 = 1 + 2 + 3$. Show that 28 is the second perfect number.

Every number of the form $2^{p-1}(2^p - 1)$, where $2^p - 1$ is prime, is an even perfect number. (Check your answer when $p = 2$.) Find the third and fourth even perfect numbers. The ancient Greeks could not find the fifth even perfect number. See if you can.

75. The speed of light is 3×10^8 meters per second. Write all answers using scientific notation.

a. How many seconds does it take an object traveling at the speed of light to go 1×10^{26} meters?

b. How many seconds are there in 1 year of 365 days?

c. Write the answer to part (a) in years. (This answer is the approximate age of the universe.)

76. Write $\sqrt{x + \sqrt{x + \sqrt{x}}}$ using exponents.

77. Determine if $(\sqrt{5} - \sqrt{24})^2$ and $(\sqrt{2} - \sqrt{3})^2$ have the same value.

78. The irrational number called the golden ratio

$$T = \frac{\sqrt{5} + 1}{2}$$

has properties that have intrigued artists, philosophers, and mathematicians through the ages. Show that T satisfies the identity

$$T = 1 + \frac{1}{T}$$

79. Rationalize the numerator in the following:

a. $\dfrac{\sqrt{x + h + 1} - \sqrt{x - 1}}{h}$

b. $\dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$

80. In alternating-current theory, the current I (amps), voltage V (volts), and impedance Z (ohms) are treated as complex numbers. The formula relating these quantities is $V = IZ$. If $I = 2 - 3i$ amps and $Z = 6 + 2i$ ohms, find the voltage across this part of the circuit.

Review Test

In Problems 1 and 2, write each set by listing its elements within braces.

1. The set of positive, even integers less than 13
2. The subset of $x \in S$, $S = \{-1, 2, 3, 5, 7\}$, such that x is a multiple of 3

In Problems 3 and 4, determine whether the statement is true (T) or false (F).

3. -1.36 is an irrational number.
4. π is equal to $\frac{22}{7}$.

In Problems 5 and 6, identify the property of the real number system that justifies the statement. All variables represent real numbers.

5. $xy(z + 1) = (z + 1)xy$
6. $(-6)\left(-\frac{1}{6}\right) = 1$

In Problems 7 and 8, sketch the given set of numbers on a real number line.

7. The integers that are greater than -3 and less than or equal to 3
8. The real numbers x such that $-2 \le x < \frac{1}{2}$
9. Find the value of $|2 - 3| - |4 - 2|$.
10. Find $\overline{AB}$ if the coordinates of A and B are -6 and -4, respectively.
11. The area of a region is given by the expression $3x^2 - xy$. Find the area if $x = 5$ meters and $y = 10$ meters.
12. Evaluate the expression

$$\frac{-|y-2x|}{|xy|}$$

if $x = 3$ and $y = -1$.

13. Which of the following expressions are not polynomials?

a. x^5 b. $5x^{-4}y + 3x^2 - y$

c. $4x^3 + x$ d. $2x^2 + 3x^0$

In Problems 14 and 15, indicate the leading coefficient and the degree of each polynomial.

14. $-2.2x^5 + 3x^3 - 2x$
15. $14x^6 - 2x + 1$

In Problems 16 and 17, perform the indicated operations.

16. $3xy + 2x + 3y + 2 - (1 - y - x + xy)$
17. $(a + 2)(3a^2 - a + 5)$

In Problems 18 and 19, factor each expression.

18. $8a^3b^5 - 12a^5b^2 + 16a^2b$
19. $4 - 9x^2$

In Problems 20 and 21, perform the indicated operations and simplify.

20. $\dfrac{m^4}{3n^2} \div \left(\dfrac{m^2}{9n} \cdot \dfrac{n}{2m^3}\right)$
21. $\dfrac{16 - x^2}{x^2 - 3x - 4} \cdot \dfrac{x - 1}{x + 4}$
22. Find the LCD of

$$\frac{-1}{2x^2} \qquad \frac{2}{4x^2 - 4} \qquad \frac{3}{x - 2}$$

In Problems 23 and 24, perform the indicated operations and simplify.

23. $\dfrac{2x}{x^2 - 9} + \dfrac{5}{3x + 9}$
24. $\dfrac{2 - \dfrac{4}{x + 1}}{x - 1}$

Review Test

In Problems 25–28, simplify and express the answers using only positive exponents.

25. $\left(\frac{x^{7/2}}{x^{2/3}}\right)^{-6}$

26. $\frac{y^{2n}}{y^{n-1}}$

27. $\frac{-1}{(x-1)^0}$

28. $(2a^2b^{-1})^2$

In Problems 29–31, perform the indicated operations.

29. $3\sqrt[3]{24} - 2\sqrt[3]{81}$

30. $(\sqrt{7} - 5)^2$

31. $\frac{1}{2}\sqrt{\frac{xy}{4}} - \sqrt{9xy}$

32. For what values of x is $\sqrt{2-x}$ a real number?

In Problems 33–35, perform the indicated operations and write all answers in the form $a + bi$.

33. $(2 - i) + (-3 + i)$

34. $(5 + 2i)(2 - 3i)$

35. $\frac{5 + 2i}{2 - i}$

Writing Exercises

1. Evaluate (8)(1.4142) and $(8)(\sqrt{2})$. Are these results close to one another? Why?
2. Discuss the need for the complex number system.
3. Compare and contrast the properties of the complex numbers with those of the real numbers.
4. Discuss why division by zero is not permitted.

Chapter 1 Project

Polynomial expressions are used by physicists to study the motion of objects in free fall. Free fall means that the attraction of gravity is the only force operating on the object. In reality, other forces like air resistance play a role.

Take a look at Exercises 86 and 87 in Section 1.3 and Exercises 84–86 in Section 1.4. Set up a table for various planets or moons in our solar system, and use the Internet or other resources to find the data you need to write free-fall equations for objects on those worlds. (*Hint:* The value of a is all you need.) Here are some values to start you off:

Mars: $a = 3.72$

Earth: $a = 4.9$

The Moon: $a = 1.6$

All these values are in SI units, so the accelerations given above are in meters per second squared.

Try to redo the Exercises listed above for various planets. Write a paragraph explaining the problem described in the chapter opener.

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