GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

A. Explain in words the multiplication property of equality.
B. If an equation contains fractions, how do you use the multiplication property of equality to clear the equation of fractions?
C. Why is it okay to divide both sides of an equation by the same nonzero number?
D. Explain in words how you would solve the equation $3x = 7$ using the multiplication property of equality.

Problem Set 1.3

1. 2
2. 2
3. 4
4. 9
5. $-\frac{1}{2}$
6. $-\frac{1}{3}$
7. -2
8. -5
9. 3
10. 4
11. 4
12. 2
13. 0
14. 0
15. 0
16. 0
17. 6
18. 12
19. -50
20. -7
21. $\frac{3}{2}$
22. $-\frac{5}{2}$
23. 12
24. 8
25. -3
26. -1
27. 32
28. 4
29. -8
30. 2
31. $\frac{1}{2}$
32. 1
33. 4
34. -2
35. 8
36. 12
37. -4
38. 3
39. 4
40. -3
41. -15
42. 15
43. $-\frac{1}{2}$
44. $\frac{3}{4}$
45. 3
46. 6
47. 1
48. 2
49. $\frac{1}{4}$
50. -5
51. -3
52. -2
53. 3
54. -3
55. 2
56. 4

Solve the following equations. Be sure to show your work.

1. $5x = 10$ **2.** $6x = 12$ **3.** $7a = 28$ **4.** $4a = 36$

5. $-8x = 4$ **6.** $-6x = 2$ **7.** $8m = -16$ **8.** $5m = -25$

9. $-3x = -9$ **10.** $-9x = -36$ **11.** $-7y = -28$ **12.** $-15y = -30$

13. $2x = 0$ **14.** $7x = 0$ **15.** $-5x = 0$ **16.** $-3x = 0$

17. $\frac{x}{3} = 2$ **18.** $\frac{x}{4} = 3$ **19.** $-\frac{m}{5} = 10$ **20.** $-\frac{m}{7} = 1$

21. $-\frac{x}{2} = -\frac{3}{4}$ **22.** $-\frac{x}{3} = \frac{5}{6}$ **23.** $\frac{2}{3}a = 8$ **24.** $\frac{3}{4}a = 6$

25. $-\frac{3}{5}x = \frac{9}{5}$ **26.** $-\frac{2}{5}x = \frac{6}{15}$ **27.** $-\frac{5}{8}y = -20$ **28.** $-\frac{7}{2}y = -14$

Simplify both sides as much as possible, and then solve.

29. $-4x - 2x + 3x = 24$ **30.** $7x - 5x + 8x = 20$

31. $4x + 8x - 2x = 15 - 10$ **32.** $5x + 4x + 3x = 4 + 8$

33. $-3 - 5 = 3x + 5x - 10x$ **34.** $10 - 16 = 12x - 6x - 3x$

35. $18 - 13 = \frac{1}{2}a + \frac{3}{4}a - \frac{5}{8}a$ **36.** $20 - 14 = \frac{1}{3}a + \frac{5}{6}a - \frac{2}{3}a$

Solve the following equations by multiplying both sides by -1.

37. $-x = 4$ **38.** $-x = -3$ **39.** $-x = -4$ **40.** $-x = 3$

41. $15 = -a$ **42.** $-15 = -a$ **43.** $-y = \frac{1}{2}$ **44.** $-y = -\frac{3}{4}$

Solve each of the following equations using the method shown in Examples 5–8 in this section.

45. $3x - 2 = 7$ **46.** $2x - 3 = 9$ **47.** $2a + 1 = 3$ **48.** $5a - 3 = 7$

49. $\frac{1}{8} + \frac{1}{2}x = \frac{1}{4}$ **50.** $\frac{1}{3} + \frac{1}{7}x = -\frac{8}{21}$

51. $6x = 2x - 12$ **52.** $8x = 3x - 10$

53. $2y = -4y + 18$ **54.** $3y = -2y - 15$ **55.** $-7x = -3x - 8$

56. $-5x = -2x - 12$ **57.** $8x + 4 = 2x - 5$ **58.** $5x + 6 = 3x - 6$

59. $x + \frac{1}{2} = \frac{1}{4}x - \frac{5}{8}$ **60.** $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

61. $6m - 3 = m + 2$ **62.** $6m - 5 = m + 5$

63. $\frac{1}{2}m - \frac{1}{4} = \frac{1}{12}m + \frac{1}{6}$ **64.** $\frac{1}{2}m - \frac{5}{12} = \frac{1}{12}m + \frac{5}{12}$

65. $9y + 2 = 6y - 4$ **66.** $6y + 14 = 2y - 2$

67. Solve each equation.

a. $2x = 3$ **b.** $2 + x = 3$

c. $2x + 3 = 0$ **d.** $2x + 3 = -5$

e. $2x + 3 = 7x - 5$

57. $-\frac{3}{2}$
58. -6
59. $-\frac{3}{2}$
60. -6
61. 1
62. 2
63. 1
64. 2
65. -2
66. -4
67. a. $\frac{3}{2}$
b. 1
c. $-\frac{3}{2}$
d. -4
e. $\frac{8}{5}$
68. a. 2
b. 5
c. -2
d. $\frac{2}{5}$
e. $-\frac{2}{3}$
69. 200 tickets
70. 2 three-pointers
71. $1,390.85 per month
72. 12
73. 2
74. 8
75. 6
76. 0
77. 3,000
78. 4,000
79. $3x - 11$
80. $5x - 13$
81. $0.09x + 180$
82. $0.04x + 280$
83. $-6y + 4$
84. $-6y + 2$
85. $4x - 11$
86. $6x - 3$
87. $5x$
88. $5x$
89. $0.17x$
90. $0.10x$

68. Solve each equation.

a. $5t = 10$ **b.** $5 + t = 10$

c. $5t + 10 = 0$ **d.** $5t + 10 = 12$

e. $5t + 10 = 8t + 12$

Applying the Concepts

69. Break-Even Point Movie theaters pay a certain price for the movies that you and I see. Suppose a theater pays $1,500 for each showing of a popular movie. If they charge $7.50 for each ticket they sell, then the equation $7.5x = 1{,}500$ gives the number of tickets they must sell to equal the $1,500 cost of showing the movie. This number is called the break-even point. Solve the equation for x to find the break-even point.

70. Basketball Laura plays basketball for her community college. In one game she scored 13 points total, with a combination of free throws, field goals, and three-pointers. Each free throw is worth 1 point, each field goal is 2 points, and each three-pointer is worth 3 points. If she made 1 free throw and 3 field goals, then solving the equation

$$1 + 3(2) + 3x = 13$$

will give us the number of three-pointers she made. Solve the equation to find the number of three-point shots Laura made.

71. Taxes Suppose 21% of your monthly pay is withheld for federal income taxes and another 8% is withheld for Social Security, state income tax, and other miscellaneous items. If you are left with $987.50 a month in take-home pay, then the amount you earned before the deductions were removed from your check is given by the equation

$$G - 0.21G - 0.08G = 987.5$$

Solve this equation to find your gross income.

72. Rhind Papyrus The *Rhind Papyrus* is an ancient document that contains mathematical riddles. One problem asks the reader to find a quantity such that when it is added to one-fourth of itself the sum is 15. The equation that describes this situation is

$$x + \frac{1}{4}x = 15$$

Solve this equation.

Getting Ready for the Next Section

To understand all of the explanations and examples in the next section you must be able to work the problems below.

Solve each equation.

73. $2x = 4$ **74.** $3x = 24$ **75.** $30 = 5x$ **76.** $0 = 5x$

77. $0.17x = 510$ **78.** $0.1x = 400$

Apply the distributive property and then simplify if possible.

79. $3(x - 5) + 4$
80. $5(x - 3) + 2$
81. $0.09(x + 2{,}000)$
82. $0.04(x + 7{,}000)$
83. $7 - 3(2y + 1)$
84. $4 - 2(3y + 1)$
85. $3(2x - 5) - (2x - 4)$
86. $4(3x - 2) - (6x - 5)$

Simplify.

87. $10x + (-5x)$
88. $12x + (-7x)$
89. $0.08x + 0.09x$
90. $0.06x + 0.04x$

Solving Linear Equations

1.4

We will now use the material we have developed in the first three sections of this chapter to build a method for solving any linear equation.

DEFINITION *linear equation*

A ***linear equation*** in one variable is any equation that can be put in the form $ax + b = 0$, where a and b are real numbers and a is not zero.

Each of the equations we will solve in this section is a linear equation in one variable. The steps we use to solve a linear equation in one variable are listed here.

Note: You may have some previous experience solving equations. Even so, you should solve the equations in this section using the method developed here. Your work should look like the examples in the text. If you have learned shortcuts or a different method of solving equations somewhere else, you can always go back to them later. What is important now is that you are able to solve equations by the methods shown here.

PROPERTY *Strategy for Solving Linear Equations in One Variable*

Step 1a: Use the distributive property to separate terms, if necessary.
1b: If fractions are present, consider multiplying both sides by the LCD to eliminate the fractions. If decimals are present, consider multiplying both sides by a power of 10 to clear the equation of decimals.
1c: Combine similar terms on each side of the equation.
Step 2: Use the addition property of equality to get all variable terms on one side of the equation and all constant terms on the other side. A variable term is a term that contains the variable (for example, $5x$). A constant term is a term that does not contain the variable (the number 3, for example).
Step 3: Use the multiplication property of equality to get x (that is, $1x$) by itself on one side of the equation.
Step 4: Check your solution in the original equation to be sure that you have not made a mistake in the solution process.

As you will see as you work through the examples in this section, it is not always necessary to use all four steps when solving equations. The number of steps used depends on the equation. In Example 1 there are no fractions or decimals in the original equation, so step 1b will not be used. Likewise, after applying the distributive property to the left side of the equation in Example 1, there are no similar terms to combine on either side of the equation, making step 1c also unnecessary.

EXAMPLE 1 Solve $2(x + 3) = 10$.

SOLUTION To begin, we apply the distributive property to the left side of the equation to separate terms:

Step 1a: $2x + 6 = 10$ — Distributive property

Step 2: $2x + 6 + (-6) = 10 + (-6)$ — Addition property of equality
$2x = 4$

Step 3: $\frac{1}{2}(2x) = \frac{1}{2}(4)$ — Multiply each side by $\frac{1}{2}$
$x = 2$ — The solution is 2

The solution to our equation is 2. We check our work (to be sure we have not made either a mistake in applying the properties or an arithmetic mistake) by substituting 2 into our original equation and simplifying each side of the result separately.

Check:	When	$x = 2$	
	the equation	$2(x + 3) = 10$	
Step 4:	becomes	$2(2 + 3) \stackrel{?}{=} 10$	
		$2(5) \stackrel{?}{=} 10$	
		$10 = 10$	A true statement

Our solution checks. ■

The general method of solving linear equations is actually very simple. It is based on the properties we developed in Chapter 1 and on two very simple new properties. We can add any number to both sides of the equation and multiply both sides by any nonzero number. The equation may change in form, but the solution set will not. If we look back to Example 1, each equation looks a little different from each preceding equation. What is interesting and useful is that each equation says the same thing about x. They all say x is 2. The last equation, of course, is the easiest to read, and that is why our goal is to end up with x by itself.

The examples that follow show a variety of equations and their solutions. When you have finished this section and worked the problems in the problem set, the steps in the solution process should be a description of how you operate when solving equations. That is, you want to work enough problems so that the Strategy for Solving Linear Equations is second nature to you.

EXAMPLE 2 Solve for $x : 3(x - 5) + 4 = 13$.

SOLUTION Our first step will be to apply the distributive property to the left side of the equation:

Step 1a:		$3x - 15 + 4 = 13$	Distributive property
Step 1c:		$3x - 11 = 13$	Simplify the left side
Step 2:		$3x - 11 + 11 = 13 + 11$	Add 11 to both sides
		$3x = 24$	
Step 3:		$\frac{1}{3}(3x) = \frac{1}{3}(24)$	Multiply both sides by $\frac{1}{3}$
		$x = 8$	The solution is 8
Check:	When	$x = 8$	
	the equation	$3(x - 5) + 4 = 13$	
	becomes	$3(8 - 5) + 4 \stackrel{?}{=} 13$	
Step 4:		$3(3) + 4 \stackrel{?}{=} 13$	
		$9 + 4 \stackrel{?}{=} 13$	
		$13 = 13$	A true statement ■

EXAMPLE 3 Solve $5(x - 3) + 2 = 5(2x - 8) - 3$.

SOLUTION In this case we apply the distributive property on each side of the equation:

Step 1a:	$5x - 15 + 2 = 10x - 40 - 3$	Distributive property
Step 1c:	$5x - 13 = 10x - 43$	Simplify each side
Step 2:	$5x + (-5x) - 13 = 10x + (-5x) - 43$	Add $-5x$ to both sides
	$-13 = 5x - 43$	
	$-13 + 43 = 5x - 43 + 43$	Add 43 to both sides
	$30 = 5x$	
Step 3:	$\frac{1}{5}(30) = \frac{1}{5}(5x)$	Multiply both sides by $\frac{1}{5}$
	$6 = x$	The solution is 6

Check: Replacing x with 6 in the original equation, we have

Step 4:

$$5(6 - 3) + 2 \stackrel{?}{=} 5(2 \cdot 6 - 8) - 3$$
$$5(3) + 2 \stackrel{?}{=} 5(12 - 8) - 3$$
$$5(3) + 2 \stackrel{?}{=} 5(4) - 3$$
$$15 + 2 \stackrel{?}{=} 20 - 3$$
$$17 = 17 \quad \text{A true statement}$$

Note: It makes no difference on which side of the equal sign x ends up. Most people prefer to have x on the left side because we read from left to right, and it seems to sound better to say x is 6 rather than 6 is x. Both expressions, however, have exactly the same meaning.

EXAMPLE 4 Solve the equation $0.08x + 0.09(x + 2{,}000) = 690$.

SOLUTION We can solve the equation in its original form by working with the decimals, or we can eliminate the decimals first by using the multiplication property of equality and solving the resulting equation. Both methods follow.

Method 1

Working with the decimals.

	$0.08x + 0.09(x + 2{,}000) = 690$	Original equation
Step 1a:	$0.08x + 0.09x + 0.09(2{,}000) = 690$	Distributive property
Step 1c:	$0.17x + 180 = 690$	Simplify the left side
Step 2:	$0.17x + 180 + (-180) = 690 + (-180)$	Add -180 to each side
	$0.17x = 510$	
Step 3:	$\frac{0.17x}{0.17} = \frac{510}{0.17}$	Divide each side by 0.17
	$x = 3{,}000$	

Note that we divided each side of the equation by 0.17 to obtain the solution. This is still an application of the multiplication property of equality because dividing by 0.17 is equivalent to multiplying by $\frac{1}{0.17}$.

Method 2

Eliminating the decimals in the beginning.

	$0.08x + 0.09(x + 2{,}000) = 690$	Original equation
Step 1a:	$0.08x + 0.09x + 180 = 690$	Distributive property
Step 1b:	$100(0.08x + 0.09x + 180) = 100(690)$	Multiply both sides by 100
	$8x + 9x + 18{,}000 = 69{,}000$	
Step 1c:	$17x + 18{,}000 = 69{,}000$	Simplify the left side
Step 2:	$17x = 51{,}000$	Add $-18{,}000$ to each side
Step 3:	$\frac{17x}{17} = \frac{51{,}000}{17}$	Divide each side by 17
	$x = 3{,}000$	

Substituting 3,000 for x in the original equation, we have

Step 4:	$0.08(3{,}000) + 0.09(3{,}000 + 2{,}000) \stackrel{?}{=} 690$	
	$0.08(3{,}000) + 0.09(5{,}000) \stackrel{?}{=} 690$	
	$240 + 450 \stackrel{?}{=} 690$	
	$690 = 690$	A true statement ■

EXAMPLE 5 Solve $7 - 3(2y + 1) = 16$.

SOLUTION We begin by multiplying -3 times the sum of $2y$ and 1:

Step 1a:	$7 - 6y - 3 = 16$	Distributive property
Step 1c:	$-6y + 4 = 16$	Simplify the left side
Step 2:	$-6y + 4 + (-4) = 16 + (-4)$	Add -4 to both sides
	$-6y = 12$	
Step 3:	$-\frac{1}{6}(-6y) = -\frac{1}{6}(12)$	Multiply both sides by $-\frac{1}{6}$
	$y = -2$	■

There are two things to notice about the example that follows: first, the distributive property is used to remove parentheses that are preceded by a negative sign, and, second, the addition property and the multiplication property are not shown in as much detail as in the previous examples.

EXAMPLE 6 Solve $3(2x - 5) - (2x - 4) = 6 - (4x + 5)$.

SOLUTION When we apply the distributive property to remove the grouping symbols and separate terms, we have to be careful with the signs. Remember, we can think of $-(2x - 4)$ as $-1(2x - 4)$, so that

$$-(2x - 4) = -1(2x - 4) = -2x + 4$$

It is not uncommon for students to make a mistake with this type of simplification and write the result as $-2x - 4$, which is incorrect. Here is the complete solution to our equation:

$3(2x - 5) - (2x - 4) = 6 - (4x + 5)$	Original equation
$6x - 15 - 2x + 4 = 6 - 4x - 5$	Distributive property
$4x - 11 = -4x + 1$	Simplify each side
$8x - 11 = 1$	Add $4x$ to each side
$8x = 12$	Add 11 to each side
$x = \frac{12}{8}$	Multiply each side by $\frac{1}{8}$
$x = \frac{3}{2}$	Reduce to lowest terms

The solution, $\frac{3}{2}$, checks when replacing x in the original equation. ■

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

A. What is the first step in solving a linear equation containing parentheses?

B. What is the last step in solving a linear equation?

C. Explain in words how you would solve the equation $2x - 3 = 8$.

D. If an equation contains decimals, what can you do to eliminate the decimals?

Problem Set 1.4

Solve each of the following equations using the four steps shown in this section.

1. $2(x+3)=12$
2. $3(x-2)=6$
3. $6(x-1)=-18$
4. $4(x+5)=16$
5. $2(4a+1)=-6$
6. $3(2a-4)=12$
7. $14=2(5x-3)$
8. $-25=5(3x+4)$
9. $-2(3y+5)=14$
10. $-3(2y-4)=-6$
11. $-5(2a+4)=0$
12. $-3(3a-6)=0$
13. $1=\frac{1}{2}(4x+2)$
14. $1=\frac{1}{3}(6x+3)$
15. $3(t-4)+5=-4$
16. $5(t-1)+6=-9$

Solve each equation.

17. $4(2x+1)-7=1$
18. $6(3y+2)-8=-2$
19. $\frac{1}{2}(x-3)=\frac{1}{4}(x+1)$
20. $\frac{1}{3}(x-4)=\frac{1}{2}(x-6)$
21. $-0.7(2x-7)=0.3(11-4x)$
22. $-0.3(2x-5)=0.7(3-x)$
23. $-2(3y+1)=3(1-6y)-9$
24. $-5(4y-3)=2(1-8y)+11$
25. $\frac{3}{4}(8x-4)+3=\frac{2}{5}(5x+10)-1$
26. $\frac{5}{6}(6x+12)+1=\frac{2}{3}(9x-3)+5$
27. $0.06x+0.08(100-x)=6.5$
28. $0.05x+0.07(100-x)=6.2$
29. $6-5(2a-3)=1$
30. $-8-2(3-a)=0$
31. $0.2x-0.5=0.5-0.2(2x-13)$
32. $0.4x-0.1=0.7-0.3(6-2x)$
33. $2(t-3)+3(t-2)=28$
34. $-3(t-5)-2(2t+1)=-8$
35. $5(x-2)-(3x+4)=3(6x-8)+10$
36. $3(x-1)-(4x-5)=2(5x-1)-7$
37. $2(5x-3)-(2x-4)=5-(6x+1)$
38. $3(4x-2)-(5x-8)=8-(2x+3)$
39. $-(3x+1)-(4x-7)=4-(3x+2)$
40. $-(6x+2)-(8x-3)=8-(5x+1)$
41. $x+(2x-1)=2$
42. $x+(5x+2)=20$
43. $x-(3x+5)=-3$
44. $x-(4x-1)=7$
45. $15=3(x-1)$
46. $12=4(x-5)$
47. $4x-(-4x+1)=5$
48. $-2x-(4x-8)=-1$
49. $5x-8(2x-5)=7$
50. $3x+4(8x-15)=10$
51. $7(2y-1)-6y=-1$
52. $4(4y-3)+2y=3$
53. $0.2x+0.5(12-x)=3.6$
54. $0.3x+0.6(25-x)=12$
55. $0.5x+0.2(18-x)=5.4$
56. $0.1x+0.5(40-x)=32$
57. $x+(x+3)(-3)=x-3$
58. $x-2(x+2)=x-2$
59. $5(x+2)+3(x-1)=-9$
60. $4(x+1)+3(x-3)=2$
61. $3(x-3)+2(2x)=5$
62. $2(x-2)+3(5x)=30$
63. $5(y+2)=4(y+1)$
64. $3(y-3)=2(y-2)$

1. 3
2. 4
3. -2
4. -1
5. -1
6. 4
7. 2
8. -3
9. -4
10. 3
11. -2
12. 2
13. 0
14. 0
15. 1
16. -2
17. $\frac{1}{2}$
18. $-\frac{1}{3}$
19. 7
20. 10
21. 8
22. 6
23. $-\frac{1}{3}$
24. $\frac{1}{2}$
25. $\frac{3}{4}$
26. 8
27. 75
28. 40
29. 2
30. 7
31. 6
32. 5
33. 8
34. 3
35. 0
36. 1
37. $\frac{3}{7}$
38. $\frac{1}{3}$
39. 1
40. $-\frac{2}{3}$
41. 1
42. 3
43. -1
44. -2
45. 6
46. 8
47. $\frac{3}{4}$
48. $\frac{3}{2}$
49. 3
50. 2
51. $\frac{3}{4}$
52. $\frac{5}{6}$
53. 8
54. 10

SPOTLIGHT ON SUCCESS *Napa Valley College*

You may think that all your mathematics instructors started their college math sequence with precalculus or calculus, but that is not always the case. Diane van Deusen, a full time mathematics instructor at Napa Valley College in Napa, California, started her career in mathematics in elementary algebar. Here is part of her story from her website:

Dear Student,

Welcome to elementary algebra! Since we will be spending a significant amount of time together this semester, I thought I should introduce myself to you, and tell you how I ended up with a career in education.

I was not encouraged to attend college after high school, and in fact, had no interest in "more school". Consequently, I didn't end up taking a college class until I was 31 years old! Before returning to and while attending college, I worked locally in the restaurant business as a waitress and bartender and in catering. In fact, I sometimes wait tables a few nights a week during my summer breaks.

When I first came back to school, at Napa Valley College (NVC), I thought I might like to enter the nursing program but soon found out nursing was not for me. As I started working on general education requirements, I took elementary algebra and was surprised to learn that I really loved mathematics, even though I had failed 8th grade algebra! As I continued to appreciate and value my own education, I decided to become a teacher so that I could support other people seeking education goals. After earning my AA degree from NVC, I transferred to Sonoma State where I earned my bachelors degree in mathematics with a concentration in statistics. Finally, I attended Cal State Hayward to earn my master's degree in applied statistics. It took me ten years in all to do this.

I feel that having been a returning student while a single, working parent, also an EOPS and Financial Aid recipient, I fully understand the complexity of the life of a community college student. If at any time you have questions about the college, the class or just need someone to talk to, my door is open.

I sincerely hope that my classroom will provide a positive and satisfying learning experience for you.

Diane Van Deusen

Algebra is a great place to start you journey into college mathematics. You can start here and go as far as you want in mathematics. Who knows, you may end up teaching mathematics one day, just like Diane Van Deusen.

Formulas

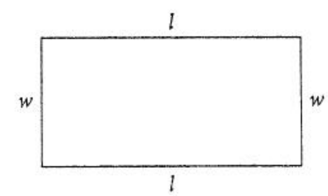
1.5

In this section we continue solving equations by working with formulas. To begin, here is the definition of a formula.

def DEFINITION *formula*

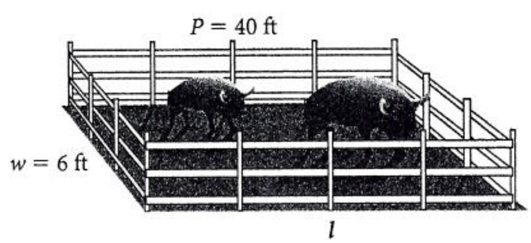
In mathematics, a ***formula*** is an equation that contains more than one variable.

The equation $P = 2l + 2w$, which tells us how to find the perimeter of a rectangle, is an example of a formula.



To begin our work with formulas, we will consider some examples in which we are given numerical replacements for all but one of the variables.

EXAMPLE 1 The perimeter P of a rectangular livestock pen is 40 feet. If the width w is 6 feet, find the length.



SOLUTION First we substitute 40 for P and 6 for w in the formula $P = 2l + 2w$. Then we solve for l:

When	$P = 40$ and $w = 6$	
the formula	$P = 2l + 2w$	
becomes	$40 = 2l + 2(6)$	
or	$40 = 2l + 12$	Multiply 2 and 6
	$28 = 2l$	Add -12 to each side
	$14 = l$	Multiply each side by $\frac{1}{2}$

To summarize our results, if a rectangular pen has a perimeter of 40 feet and a width of 6 feet, then the length must be 14 feet. ■

EXAMPLE 2 Find y when $x = 4$ in the formula $3x + 2y = 6$.

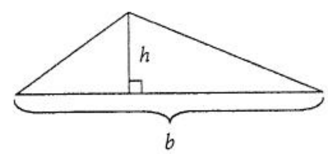
SOLUTION We substitute 4 for x in the formula and then solve for y:

When	$x = 4$	
the formula	$3x + 2y = 6$	
becomes	$3(4) + 2y = 6$	
or	$12 + 2y = 6$	Multiply 3 and 4
	$2y = -6$	Add -12 to each side
	$y = -3$	Multiply each side by $\frac{1}{2}$

In the next examples we will solve a formula for one of its variables without being given numerical replacements for the other variables.

Consider the formula for the area of a triangle:

$A = \frac{1}{2}bh$



where A = area, b = length of the base, and h = height of the triangle.

Suppose we want to solve this formula for h. What we must do is isolate the variable h on one side of the equal sign. We begin by multiplying both sides by 2, because it is the reciprocal of $\frac{1}{2}$:

$$2 \cdot A = 2 \cdot \frac{1}{2}bh$$

$$2A = bh$$

Then we divide both sides by b:

$$\frac{2A}{b} = \frac{bh}{b}$$

$$h = \frac{2A}{b}$$

The original formula $A = \frac{1}{2}bh$ and the final formula $h = \frac{2A}{b}$ both give the same relationship among A, b, and h. The first one has been solved for A and the second one has been solved for h.

[Δ≠Σ RULE

To solve a formula for one of its ***variables***, we must isolate that variable on either side of the equal sign. All other variables and constants will appear on the other side.

EXAMPLE 3 Solve $3x + 2y = 6$ for y.

SOLUTION To solve for y, we must isolate y on the left side of the equation. To begin, we use the addition property of equality to add $-3x$ to each side:

$3x + 2y = 6$	Original formula
$3x + (-3x) + 2y = (-3x) + 6$	Add $-3x$ to each side
$2y = -3x + 6$	Simplify the left side
$\frac{1}{2}(2y) = \frac{1}{2}(-3x + 6)$	Multiply each side by $\frac{1}{2}$
$y = -\frac{3}{2}x + 3$	Multiplication

■

EXAMPLE 4 Solve $h = vt - 16t^2$ for v.

SOLUTION Let's begin by interchanging the left and right sides of the equation. That way, the variable we are solving for, v, will be on the left side.

$vt - 16t^2 = h$	Exchange sides
$vt - 16t^2 + 16t^2 = h + 16t^2$	Add $16t^2$ to each side
$vt = h + 16t^2$	
$\frac{vt}{t} = \frac{h + 16t^2}{t}$	Divide each side by t
$v = \frac{h + 16t^2}{t}$	

We know we are finished because we have isolated the variable we are solving for on the left side of the equation and it does not appear on the other side. ■

EXAMPLE 5 Solve for y: $\frac{y-1}{x} = \frac{3}{2}$.

SOLUTION Although we will do more extensive work with formulas of this form later in the book, we need to know how to solve this particular formula for y in order to understand some things in the next chapter. We begin by multiplying each side of the formula by x. Doing so will simplify the left side of the equation, and make the rest of the solution process simple.

$\frac{y-1}{x} = \frac{3}{2}$	Original formula
$x \cdot \frac{y-1}{x} = \frac{3}{2} \cdot x$	Multiply each side by x
$y - 1 = \frac{3}{2}x$	Simplify each side
$y = \frac{3}{2}x + 1$	Add 1 to each side

This is our solution. If we look back to the first step, we can justify our result on the left side of the equation this way: Dividing by x is equivalent to multiplying by its reciprocal $\frac{1}{x}$. Here is what it looks like when written out completely:

$$x \cdot \frac{y-1}{x} = x\,\frac{1}{x}\,(y - 1) = 1(y - 1) = (y - 1)$$

■

FACTS FROM GEOMETRY ***More on Complementary and Supplementary Angles***

In Chapter 1 we defined complementary angles as angles that add to 90°; that is, if x and y are complementary angles, then

$$x + y = 90°$$

If we solve this formula for y, we obtain a formula equivalent to our original formula:

$$y = 90° - x$$

Because y is the complement of x, we can generalize by saying that the complement of angle x is the angle $90° - x$. By a similar reasoning process, we can say that the supplement of angle x is the angle $180° - x$. To summarize, if x is an angle, then

The complement of x is $90° - x$, and

The supplement of x is $180° - x$

If you go on to take a trigonometry class, you will see this formula again.

EXAMPLE 6 Find the complement and the supplement of 25°.

SOLUTION We can use the formulas $90° - x$ and $180° - x$.

The complement of 25° is $90° - 25° = 65°$.

The supplement of 25° is $180° - 25° = 155°$. ■

Basic Percent Problems

The next examples in this section show how basic percent problems can be translated directly into equations. To understand these examples, you must recall that *percent* means "per hundred" that is, 75% is the same as $\frac{75}{100}$, 0.75, and, in reduced fraction form, $\frac{3}{4}$. Likewise, the decimal 0.25 is equivalent to 25%. To change a decimal to a percent, we move the decimal point two places to the right and write the % symbol. To change from a percent to a decimal, we drop the % symbol and move the decimal point two places to the left. The table that follows gives some of the most commonly used fractions and decimals and their equivalent percents.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	$0.33\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.66\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%

EXAMPLE 7 What number is 25% of 60?

SOLUTION To solve a problem like this, we let $x =$ the number in question (that is, the number we are looking for). Then, we translate the sentence directly into an equation by using an equal sign for the word "is" and multiplication for the word "of." Here is how it is done:

What number is 25% of 60?

$$x = 0.25 \cdot 60$$
$$x = 15$$

Notice that we must write 25% as a decimal in order to do the arithmetic in the problem.

The number 15 is 25% of 60.

EXAMPLE 8 What percent of 24 is 6?

SOLUTION Translating this sentence into an equation, as we did in Example 7, we have:

What percent of 24 is 6?

$$x \cdot 24 = 6$$
or
$$24x = 6$$

Next, we multiply each side by $\frac{1}{24}$. (This is the same as dividing each side by 24.)

$$\frac{1}{24}(24x) = \frac{1}{24}(6)$$
$$x = \frac{6}{24}$$
$$= \frac{1}{4}$$
$$= 0.25, \text{ or } 25\%$$

25% of 24 is 6, or in other words, the number 6 is 25% of 24.

EXAMPLE 9 45 is 75% of what number?

SOLUTION Again, we translate the sentence directly:

45 is 75% of what number?

$$45 = 0.75 \cdot x$$

Next, we multiply each side by $\frac{1}{0.75}$ (which is the same as dividing each side by 0.75):

$$\frac{1}{0.75}(45) = \frac{1}{0.75}(0.75x)$$
$$\frac{45}{0.75} = x$$
$$60 = x$$

The number 45 is 75% of 60.

EXAMPLE 10 The American Dietetic Association (ADA) recommends eating foods in which the calories from fat are less than 30% of the total calories. The nutrition labels from two kinds of granola bars are shown in Figure 1. For each bar, what percent of the total calories come from fat?

BAR I

Nutrition Facts	
Serving Size 2 bars (47g)	
Servings Per Container 6	
Amount Per Serving	
Calories	210
Calories from Fat	**70**
	% Daily Value*
Total Fat 8g	**12%**
Saturated Fat 1g	**5%**
Cholesterol 0mg	**0%**
Sodium 150mg	**6%**
Total Carbohydrate 32g	**11%**
Dietary Fiber 2g	**10%**
Sugars 12g	
Protein 4g	

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.

BAR II

Nutrition Facts	
Serving Size 1 bar (21g)	
Servings Per Container 8	
Amount Per Serving	
Calories	80
Calories from Fat	**15**
	% Daily Value*
Total Fat 1.5g	**2%**
Saturated Fat 0g	**0%**
Cholesterol 0mg	**0%**
Sodium 60mg	**3%**
Total Carbohydrate 16g	**5%**
Dietary Fiber 1g	**4%**
Sugars 5g	
Protein 2g	

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.

FIGURE 1

SOLUTION The information needed to solve this problem is located towards the top of each label. Each serving of Bar I contains 210 calories, of which 70 calories come from fat. To find the percent of total calories that come from fat, we must answer this question:

70 is what percent of 210?

For Bar II, one serving contains 80 calories, of which 15 calories come from fat. To find the percent of total calories that come from fat, we must answer this question:

15 is what percent of 80?

Translating each equation into symbols, we have

70 is what percent of 210	15 is what percent of 80
$70 = x \cdot 210$	$15 = x \cdot 80$
$x = \frac{70}{210}$	$x = \frac{15}{80}$
$x = 0.33$ to the nearest hundredth	$x = 0.19$ to the nearest hundredth
$x = 33\%$	$x = 19\%$

Comparing the two bars, 33% of the calories in Bar I are fat calories, whereas 19% of the calories in Bar II are fat calories. According to the ADA, Bar II is the healthier choice. ■

Applying the Concepts

As we mentioned in Chapter 1, in the U.S. system, temperature is measured on the Fahrenheit scale. In the metric system, temperature is measured on the Celsius scale. On the Celsius scale, water boils at 100 degrees and freezes at 0 degrees. To denote a temperature of 100 degrees on the Celsius scale, we write

100°C, which is read "100 degrees Celsius"

Table 1 is intended to give you an intuitive idea of the relationship between the two temperature scales. Table 2 gives the formulas, in both symbols and words, that are used to convert between the two scales.

Table 1

	Temperature	
Situation	**Fahrenheit**	**Celsius**
Water freezes	32°F	0°C
Room temperature	68°F	20°C
Normal body temperature	98.6°F	37°C
Water boils	212°F	100°C

Table 2

To Convert from	Formula in Symbols	Formula in Words
Fahrenheit to Celsius	$C = \frac{5}{9}(F - 32)$	Subtract 32, multiply by 5, then divide by 9
Celsius to Fahrenheit	$F = \frac{9}{5}C + 32$	Multiply by $\frac{9}{5}$, then add 32

EXAMPLE 11 Mr. McKeague traveled to Buenos Aires with a group of friends. It was a hot day when they arrived. One of the bank kiosks indicated the temperature was 25°C. Someone asked what that would be on the Fahrenheit scale (the scale they were familiar with), and Budd, one of his friends said, "just multiply by 2 and add 30."

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a. What was the temperature in °F according to Budd's approximation?

b. What is the actual temperature in °F?

c. Why does Budd's estimate work?

d. Write a formula for Budd's estimate.

SOLUTION

a. According to Budd, we multiply by 2 and add 30, so

$$2 \cdot 25 + 30 = 50 + 30 = 80°F$$

b. Using the formula $F = \frac{9}{5}C + 32$, with $C = 25$, we have

$$F = \frac{9}{5}(25) + 32 = 45 + 32 = 77°F$$

c. Budd's estimate works because $\frac{9}{5}$ is approximately 2 and 30 is close to 32.

d. In symbols, Budd's estimate is $F = 2 \cdot C + 30$.

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

A. What is a formula?
B. How do you solve a formula for one of its variables?
C. What are complementary angles?
D. What does percent mean?