Activity 6 Write a term paper on one of the following topics in mathematics according to the guidelines of your instructor.

- Babylonian mathematics
- Egyptian mathematics
- The origin of zero
- Plimpton 322
- The Rhind papyrus
- Origins of the Pythagorean theorem
- The regular (Platonic) solids
- The Pythagorean brotherhood
- The Golden Ratio (Golden Section)
- The three famous construction problems of the Greeks
- The history of the approximations of π
- Euclid and his "Elements"
- Early Chinese mathematics
- Early Hindu mathematics
- Origin of the word *algebra*
- Magic squares
- Figurate numbers
- The Fibonacci sequence
- The Cardano/Tartaglia controversy
- Historical methods of computation (logarithms, the abacus, Napier's rods, the slide rule, etc.)
- Pascal's triangle
- The origins of probability theory
- Women in mathematics
- Mathematical paradoxes
- Unsolved problems in mathematics
- The four color theorem
- The proof of Fermat's Last Theorem
- The search for large primes
- Fractal geometry
- The co-inventors of calculus
- The role of the computer in the study of mathematics
- Mathematics and music
- Police mathematics
- The origins of complex numbers
- Goldbach's conjecture
- The use of the Internet in mathematics education
- The development of graphing calculators
- Mathematics education reform movement
- Multicultural mathematics
- The Riemann Hypothesis

Activity 7 Investigate a computer program that focuses on teaching children elementary mathematics and write a critical review of it as if you were writing for a journal that contains software reviews of educational material. Be sure to address the higher-level thinking skills in addition to drill and practice.

Activity 8 The following Web sites provide a fascinating list of mathematics-related topics. Go to one of them, choose a topic that interests you, and report on it, according to the guidelines of your instructor.

www.mathworld.wolfram.com

www.world.std.com/~reinhold/mathmovies.html

www.mcs.surrey.ac.uk/Personal/R.Knott/

www.dir.yahoo.com/Science/Mathematics/

www.cut-the-knot.com/

www.ics.uci.edu/~eppstein/recmath.html

Activity 9 A theme of mathematics-related scenes in movies and television is found throughout this book. Prepare a report on one or more such scenes, and determine whether the mathematics involved is correct or incorrect. If correct, show why; if incorrect, find the correct answer. The Website

www.world.std.com/~reinhold/mathmovies.html

provides a wealth of information on mathematics in the movies.

Activity 10 The longest running animated television show is *The Simpsons*, having begun in 1989. The Website

www.simpsonsmath.com

explores the occurrence of mathematics in the episodes on a season-by-season basis. Watch several episodes and elaborate on the mathematics found in them.

COLLABORATIVE INVESTIGATION

Discovering Patterns in Pascal's Triangle

One fascinating array of numbers, *Pascal's triangle,* consists of rows of numbers, each of which contains one more entry than the one before. The first five rows are shown here.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

To discover some of its patterns, divide the class into groups of four students each. Within each group designate one student as A, one as B, one as C, and one as D. Then perform the following activities in order.

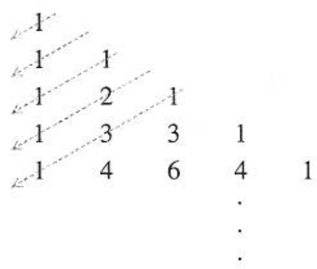
1. Discuss among group members some of the properties of the triangle that are obvious from observing the first five rows shown.
2. It is fairly obvious that each row begins and ends with 1. Discover a method whereby the other entries in a row can be determined from the entries in the row immediately above it. (*Hint*: In the fifth row, $6 = 3 + 3$.) Then, as a group, find the next four rows of the triangle, and have each member prepare his or her own copy of the entire first nine rows for later reference.
3. Now each student in the group will investigate a particular property of the triangle. In some cases, a calculator will be helpful. All students should begin working at the same time. (A discussion follows.)

   Student A: Find the sum of the entries in each row. Notice the pattern that emerges. Now write the tenth row of the triangle.

   Student B: Investigate the successive differences in the diagonals from upper left to lower right. For example, in the diagonal that begins 1, 2, 3, 4, . . . , the successive differences are all 1; in the diagonal that begins 1, 3, 6, . . . , the successive differences are 2, 3, 4, and so on. Do this up through the diagonal that begins 1, 6, 21, ...

   Student C: Find the values of the first five powers of the number 11, starting with 11^0 (recall $11^0 = 1$).

Student D: Arrange the nine rows of the triangle with all rows "flush left," and then draw lightly dashed arrows as shown:



and so on. Then add along the diagonals. Write these sums in order from left to right.

4. After all students have concluded their individual investigations in Item 3, return to a group discussion.

(a) Have student A report the result found in Item 3, and then make a prediction concerning the sum of the entries in the tenth row.

(b) Have student B report the successive differences discovered in the diagonals. Then have all students in the group investigate the successive differences in the diagonal that begins 1, 7, 28. . . . (It may be necessary to write a few more rows of the triangle.)

(c) Have student C report the relationship between the powers of 11 found, and then determine the value of 11^5. Why does the pattern not continue here?

(d) Have student D report the sequence of numbers found. Then, as a group, predict what the next sum will be by observing the pattern in the sequence. Confirm your prediction by actual computation.

5. Choose a representative from each group to report to the entire class the observations made throughout this investigation.

6. Find a reference to Pascal's triangle using a search engine of the Internet and prepare a report on the reference.

CHAPTER 1 TEST

In Exercises 1 and 2, decide whether the reasoning involved is an example of inductive or deductive reasoning.

1. Jane Fleming is a sales representative for a publishing company. For the past 14 years, she has exceeded her annual sales goal, primarily by selling mathematics textbooks. Therefore, she will also exceed her annual sales goal this year. inductive

2. For all natural numbers n, n^2 is also a natural number. 101 is a natural number. Therefore, 101^2 is a natural number. deductive

3. What are the fourth and fifth numbers in this sequence?

$$1, 4, 27, \underline{256}, \underline{3125}, 46656, \ldots$$

(From *Mathematics Teacher* monthly calendar, April 25, 1994) (The n^{th} term of the sequence is n^n.)

4. Use the list of equations and inductive reasoning to predict the next equation, and then verify your conjecture.

$$65{,}359{,}477{,}124{,}183 \times 17 = 1{,}111{,}111{,}111{,}111{,}111$$
$$65{,}359{,}477{,}124{,}183 \times 34 = 2{,}222{,}222{,}222{,}222{,}222$$
$$65{,}359{,}477{,}124{,}183 \times 51 = 3{,}333{,}333{,}333{,}333{,}333$$

5. Use the method of successive differences to find the next term in the sequence

$$3, 11, 31, 69, 131, 223, \ldots. \quad 351$$

6. Find the sum $1 + 2 + 3 + \cdots + 250$. 31,375

7. Consider the following equations, where the left side of each is an octagonal number.

$$1 = 1$$
$$8 = 1 + 7$$
$$21 = 1 + 7 + 13$$
$$40 = 1 + 7 + 13 + 19$$

Use the pattern established on the right sides to predict the next octagonal number. What is the next equation in the list? 65; $65 = 1 + 7 + 13 + 19 + 25$

8. Use the result of Exercise 7 and the method of successive differences to find the first eight octagonal numbers. Then divide each by 4 and record the remainder. What is the pattern obtained?

9. Describe the pattern used to obtain the terms of the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, ...

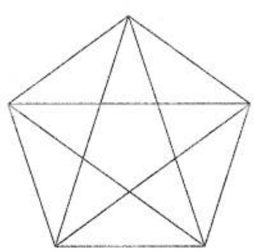
Use problem-solving strategies to solve each problem, taken from the date indicated in the monthly calendar of Mathematics Teacher.

10. ***Building a Fraction*** Each of the four digits 2, 4, 6, and 9 is placed in one of the boxes to form a fraction. The numerator and the denominator are both two-digit whole numbers. What is the smallest value of all the common fractions that can be formed? Express your answer as a common fraction. (November 17, 2004) $\frac{1}{4}$

$$\frac{\square\square}{\square\square}$$

11. ***Units Digit of a Power of 9*** What is the units digit (ones digit) in the decimal representation of 9^{1997}? (January 27, 1997) 9

12. ***Counting Puzzle (Triangles)*** How many triangles are in this figure? (January 6, 2000) 35



13. ***Devising a Correct Addition Problem*** Can you put the digits 1 through 9, each used once, in the boxes of the problem below to make an addition problem that has carrying and that is correct? If so, find a solution. If not, explain why no solution exists. (April 10, 2002)

$$\square\square\square + \square\square\square = \square\square\square$$

629 + 154 = 783 is one of several solutions.

14. ***Missing Pages in a Newspaper*** A sixty-page newspaper, which consists of only one section, has the sheet with page 7 missing. What other pages are missing? (February 6, 1998) 8, 53, and 54

15. ***Units Digit of a Sum*** Find the units digit (ones digit) of the decimal numeral representing the number $11^{11} + 14^{14} + 16^{16}$. (February 14, 1994) 3

16. Based on your knowledge of elementary arithmetic, describe the pattern that can be observed when the following operations are performed: 9×1, 9×2, $9 \times 3, \ldots, 9 \times 9$. (*Hint:* Add the digits in the answers. What do you notice?) The sum of the digits is always 9.

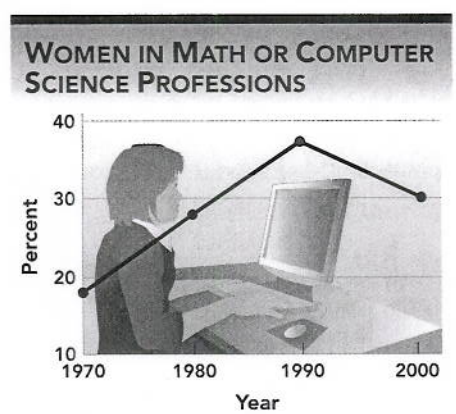
Use your calculator to evaluate each of the following. Give as many decimal places as the calculator displays.

17. $\sqrt{98.16}$ 9.907572861 (Answers may vary due to the model of calculator used.)

18. 3.25^3 34.328125

19. ***Basketball Scoring Results*** During her NCAA women's basketball career, Seimone Augustus of LSU made 800 of her 1488 field goal attempts. This means that for every 15 attempts, she made approximately __B__ of them.
A. 4 **B.** 8 **C.** 6 **D.** 2

20. ***Women in Mathematics*** The accompanying graph shows the number of women in mathematics or computer science professions during the past three decades.



Source: U.S. Bureau of the Census and Bureau of Labor Statistics.

(a) In what decade (10-year period) did the percent of women in math or computer science professions decrease? 1990–2000

(b) When did the percent of women in math or computer science professions reach a maximum? 1990

(c) In what year was the percent of women in math or computer science professions about 27%? 1980

ANSWERS TO SELECTED EXERCISES

CHAPTER 1 The Art of Problem Solving

1.1 Exercises (pages 7–10)

39. $5(6) + 5(36) + 5(216) + 5(1296) + 5(7776) = 6(7776 - 1)$ **40.** $3 + 9 + 27 + 81 + 243 = \frac{3(243 - 1)}{2}$

41. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = 1 - \frac{1}{32}$ **42.** $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} = \frac{5}{6}$

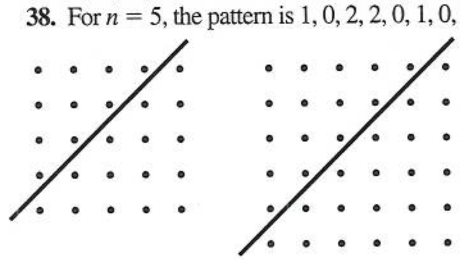
51.

52. 0 2 10 30 60 90 102 90 60 (To find any term, choose the term directly above it and add to it the two preceding terms. If there are fewer than two terms, add as many as there are.)

53. 1 (These are the numbers of chimes a clock rings, starting with 12 o'clock, if it rings the number of hours on the hour, and 1 chime on the half-hour.) **54.** E (One, Two, Three, and so on) **55. (a)** The middle digit is always 9, and the sum of the first and third digits is always 9 (considering 0 as the first digit if the difference has only two digits). **(b)** Answers will vary. **56. (f)** The final result will always be half of the number added in Step (b). **57.** 142,857; 285,714; 428,571; 571,428; 714,285; 857,142. Each result consists of the same six digits, but in a different order. $142{,}857 \times 7 = 999{,}999$ **60.** For example, suppose the age is 50 and the amount is 35 cents. Then the numbers obtained are 50, 200, 210, 5250, 4885, 4920, 5035. The final number here, 5035, provides the information.

1.2 Exercises (pages 17–19)

10. (a) 5, 11, 19, 29 **(b)** Add 12 to 29 to obtain 41. **(c)** $5^2 + 3(5) + 1 = 41$; The results agree. **35.** $8(1) + 1 = 9 = 3^2$; $8(3) + 1 = 25 = 5^2$; $8(6) + 1 = 49 = 7^2$; $8(10) + 1 = 81 = 9^2$ **36.** The pattern is 1, 0, 0, 1, 0, 0, ... **37.** The pattern is 1, 0, 1, 0, 1, 0, ... **38.** For $n = 5$, the pattern is 1, 0, 2, 2, 0, 1, 0, 2, 2, 0, ... For $n = 6$, the pattern is 1, 0, 3, 4, 3, 0, 1, 0, 3, 4, 3, 0, ... **39.**



40. (a) $\frac{4}{9}, \frac{9}{16}, \frac{16}{25}, \frac{25}{36}, \frac{36}{49}, \frac{49}{64}, \frac{64}{81}$ **(b)** $\frac{3}{6}, \frac{6}{10}, \frac{10}{15}, \frac{15}{21}, \frac{21}{28}, \frac{28}{36}, \frac{36}{45}$ **(c)** The fraction formed by two consecutive square numbers is always in lowest terms, while the fraction formed by two consecutive triangular numbers is never in lowest terms.

42. $N = 6$; They are the third triangular number and the second hexagonal number.

1.3 Exercises (pages 27–33)

5. If you multiply the two digits in the numbers in the first row, you will get the second row of numbers. The second row of numbers is a pattern of two numbers (8 and 24) repeating. **7.** I put the ring in the box and put my lock on the box. I send you the box. You put your lock on, as well, and send it back to me. I then remove my lock with my key and send the box (with your lock still on) back to you, so you can remove your lock with your key and get the ring. **11.** You should choose a sock from the box labeled *red and green socks*. Because it is mislabeled, it contains only red socks or only green socks, determined by the sock you choose. If the sock is green, relabel this box