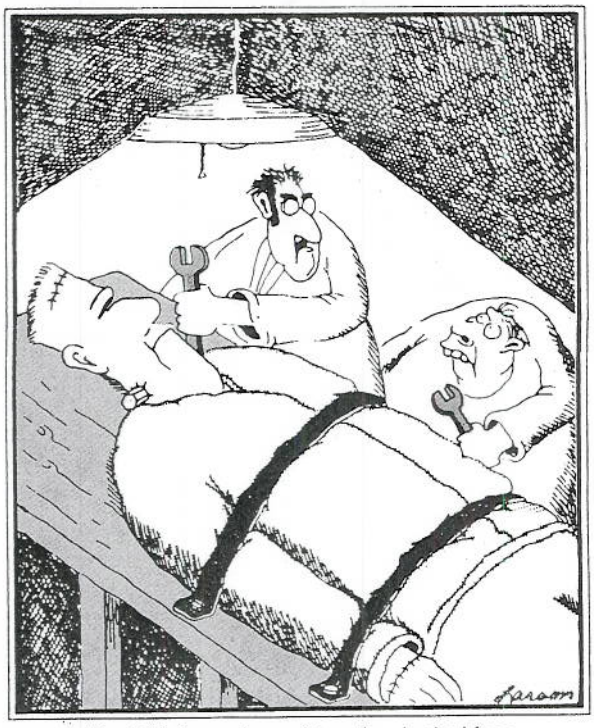
2 Fractions

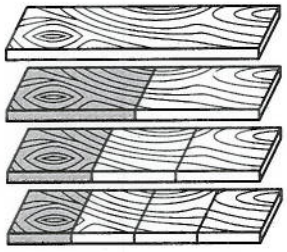


"Fool! This is an eleven-sixteenths...I asked for a five-eighths!"

2-1 WORKING WITH FRACTIONS

The word *fraction* comes from a Latin word meaning "to break," and fraction numbers are used when we need to break down standard measuring units into smaller parts. Anyone doing practical mathematics problems will find that fractions are used in many different trade and technical areas. Unfortunately, fraction arithmetic usually *must* be done with pencil and paper, since not all calculators handle fractions directly.

Look at the following piece of lumber. What happens when we break it into equal parts?



2 parts, each one-half of the whole

3 parts, each one-third of the whole

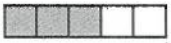
4 parts, each one-fourth of the whole

Suppose we divide this area ☐ into fifths by drawing vertical lines.

Notice that the five parts or "fifths" are equal in area. ☐☐☐☐☐

A fraction is normally written as the division of two whole numbers: $\frac{2}{3}$, $\frac{3}{4}$, or $\frac{9}{16}$. One of the five equal areas below would be "one-fifth" or $\frac{1}{5}$ of the entire area.

EXAMPLE

How would you label this portion of the area?  = ?

$$\frac{3}{5} = \frac{3 \text{ shaded parts}}{5 \text{ total parts}}$$

The fraction $\frac{3}{5}$ implies an area equal to three of the original parts.

$$\frac{3}{5} = 3 \times \left(\frac{1}{5}\right)$$

There are three equal parts, and the name of each part is $\frac{1}{5}$ or one-fifth.

YOUR TURN

In this collection of letters, HHHHPPT, what fraction are Hs?

SOLUTION

$$\text{Fraction of Hs} = \frac{\text{number of Hs}}{\text{total number of letters}} = \frac{4}{7} \text{ (read it "four sevenths")}$$

The fraction of Ps is $\frac{2}{7}$, and the fraction of Ts is $\frac{1}{7}$.

Numerator The two numbers that form a fraction are given special names to simplify talking about them. In the fraction $\frac{3}{5}$ the upper number 3 is called the *numerator* from the Latin *numero* meaning "number." It is a count of the number of parts.

Denominator The lower number 5 is called the *denominator* from the Latin *nomen* or "name." It tells us the name of the part being counted. The numerator and denominator are called the *terms* of the fraction.

$\frac{3}{5}$ ← Numerator, the number of parts

$\frac{3}{5}$ ← Denominator, the name of part, "fifths"

Learning Help ▶ A handy memory aid is to remember that the denominator is the "down part"—D for down. ◀

YOUR TURN

A paperback book costs \$6 and I have \$5. What fraction of its cost do I have? Write the answer as a fraction.

numerator = _____ , denominator = _____

ANSWER

$\underbrace{\$\ \$\ \$\ \$\ \$}_{5}$ \$5 is $\frac{5}{6}$ of the total cost.

numerator = 5, denominator = 6

PROBLEMS

Complete the following sentences by writing in the correct fraction.

(a) If we divide a length into eight equal parts, each part will be _____ of the total length.

(b) Then three of these parts will represent _____ of the total length.

(c) Eight of these parts will be _____ of the total length.

(d) Ten of these parts will be _____ of the total length.

ANSWERS

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{8}{8}$ (d) $\frac{10}{8}$

Proper Fraction

The original length is used as a standard for comparison, and any other length—smaller or larger—can be expressed as a fraction of it. A *proper fraction* is a number less than 1, as you would suppose a fraction should be. It represents a quantity less than the standard. For example, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{17}{20}$ are all proper fractions. Notice that for a proper fraction, the numerator is less than the denominator—the top number is less than the bottom number in the fraction.

Improper Fraction

An *improper fraction* is a number greater than 1 and represents a quantity greater than the standard. If a standard length is 8 in., a length of 11 in. will be $\frac{11}{8}$ of the standard. Notice that for an improper fraction the numerator is greater than the denominator—top number greater than the bottom number in the fraction.

YOUR TURN

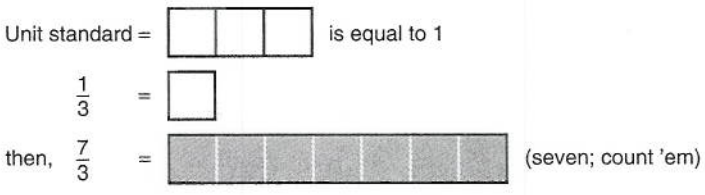
Circle the proper fractions in the following list.

$\frac{3}{2}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{5}{4}$ $\frac{15}{12}$ $\frac{1}{16}$ $\frac{35}{32}$ $\frac{7}{50}$ $\frac{65}{64}$ $\frac{105}{100}$

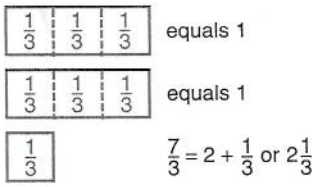
ANSWERS

You should have circled the following proper fractions: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{7}{50}$. In each fraction the numerator (top number) is less than the denominator (bottom number). Each of these fractions represents a number less than 1.

The improper fraction $\frac{7}{3}$ can be shown graphically as follows:



We can rename this number by regrouping.



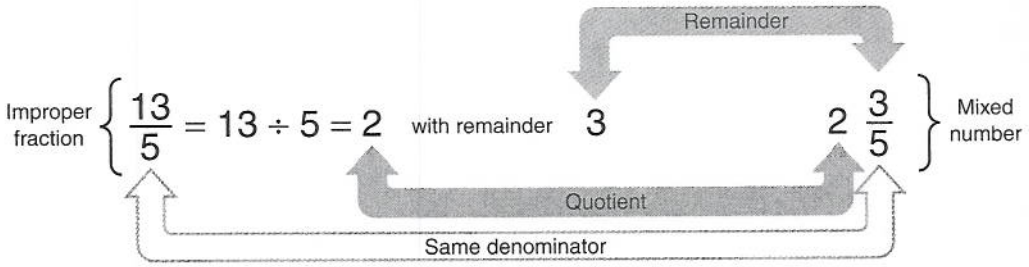
Mixed Numbers

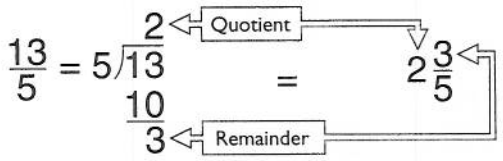
A *mixed number* is an improper fraction written as the sum of a whole number and a proper fraction.

$$\frac{7}{3} = 2 + \frac{1}{3} \text{ or } 2\frac{1}{3}$$

We usually omit the + sign and write $2 + \frac{1}{3}$ as $2\frac{1}{3}$, and read it as "two and one-third." The numbers $1\frac{1}{2}$, $2\frac{2}{5}$, and $16\frac{2}{3}$ are all written as mixed numbers.

To write an improper fraction as a mixed number, divide numerator by denominator and form a new fraction as shown:





YOUR TURN

Now you try it. Rename $\frac{23}{4}$ as a mixed number. $\frac{23}{4}$ = _____

SOLUTION

$\frac{23}{4} = 23 \div 4 = 5$ with remainder $3 \longrightarrow 5\frac{3}{4}$

If in doubt, check your work with a diagram like this:

PROBLEMS

Now try these for practice. Write each improper fraction as a mixed number.

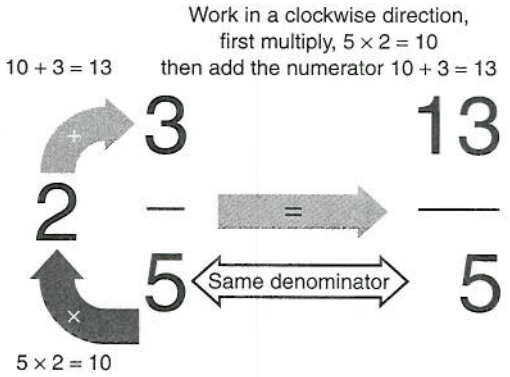
(a) $\frac{9}{5}$ (b) $\frac{13}{4}$ (c) $\frac{27}{8}$ (d) $\frac{31}{4}$ (e) $\frac{41}{12}$ (f) $\frac{17}{2}$

ANSWERS

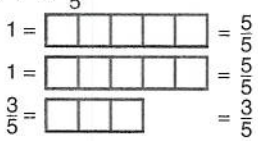
(a) $\frac{9}{5} = 1\frac{4}{5}$ (b) $\frac{13}{4} = 3\frac{1}{4}$ (c) $\frac{27}{8} = 3\frac{3}{8}$

(d) $\frac{31}{4} = 7\frac{3}{4}$ (e) $\frac{41}{12} = 3\frac{5}{12}$ (f) $\frac{17}{2} = 8\frac{1}{2}$

The reverse process, rewriting a mixed number as an improper fraction, is equally simple.



Graphically, $2\frac{3}{5} = 1 + 1 + \frac{3}{5}$



or $\frac{13}{5}$; (count them)

YOUR TURN

Now you try it. Rewrite these mixed numbers as improper fractions.

(a) $3\frac{1}{5}$ (b) $4\frac{3}{8}$ (c) $1\frac{1}{16}$ (d) $5\frac{1}{2}$ (e) $15\frac{3}{8}$ (f) $9\frac{3}{4}$

ANSWERS

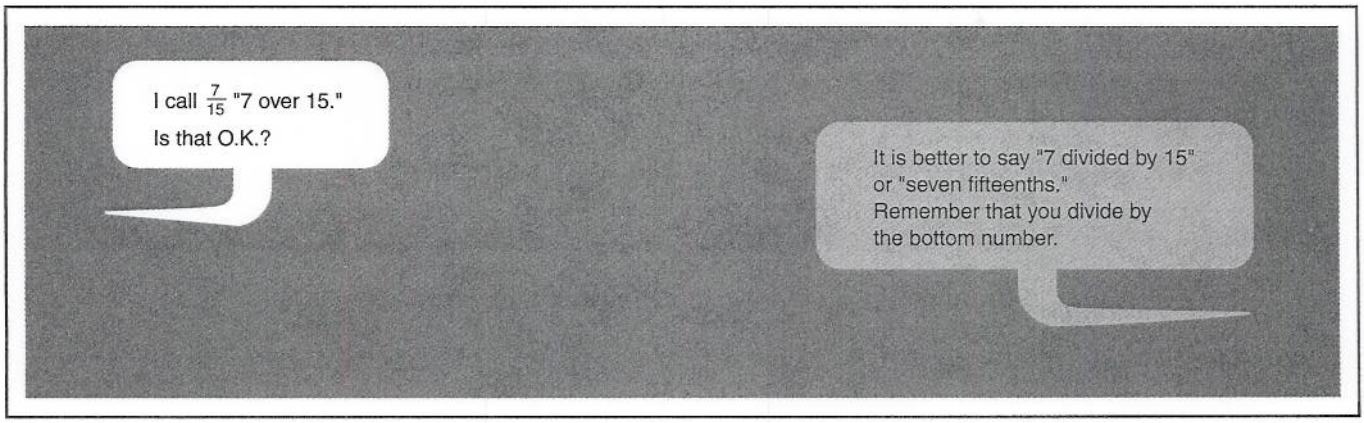
(a) **Step 1** $3 \times 5 = 15$

Step 2 $15 + 1 = 16 \leftarrow$ The new numerator

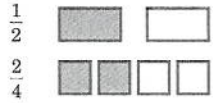
Step 3 $3\frac{1}{5} = \frac{16}{5} \swarrow$ The original denominator

(b) $4\frac{3}{8} = \frac{35}{8}$ (c) $1\frac{1}{16} = \frac{17}{16}$ (d) $5\frac{1}{2} = \frac{11}{2}$

(e) $15\frac{3}{8} = \frac{123}{8}$ (f) $9\frac{3}{4} = \frac{39}{4}$



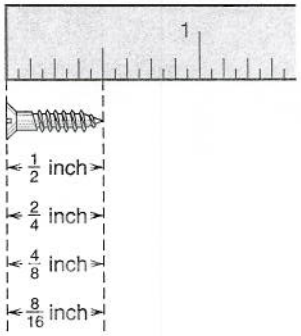
Equivalent Fractions



Two fractions are said to be *equivalent* if they represent the same number. For example, $\frac{1}{2} = \frac{2}{4}$ since both fractions represent the same portion of some standard amount.

There is a very large set of fractions equivalent to $\frac{1}{2}$.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \cdots = \frac{46}{92} = \frac{61}{122} = \frac{1437}{2874} \quad \text{and so on}$$



Each fraction is the same number, and we can use these fractions interchangeably in any mathematics problem.

To obtain a fraction equivalent to any given fraction, multiply the original numerator and denominator by the same nonzero number.

EXAMPLE

Rewrite the fraction $\frac{2}{3}$ as an equivalent fraction with denominator 15.

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

Multiply top and bottom by 5

YOUR TURN

Rewrite the fraction $\frac{3}{4}$ as an equivalent fraction with denominator equal to 20.

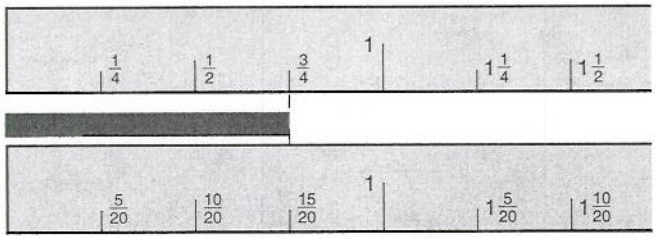
$$\frac{3}{4} = \frac{?}{20}$$

SOLUTION

$$\frac{3}{4} = \frac{3 \times \square}{4 \times \square} = \frac{?}{20} \qquad 4 \times \square = 20, \text{ so } \square \text{ must be } 5.$$

$$= \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

The number value of the fraction has not changed; we have simply renamed it.



PROBLEMS

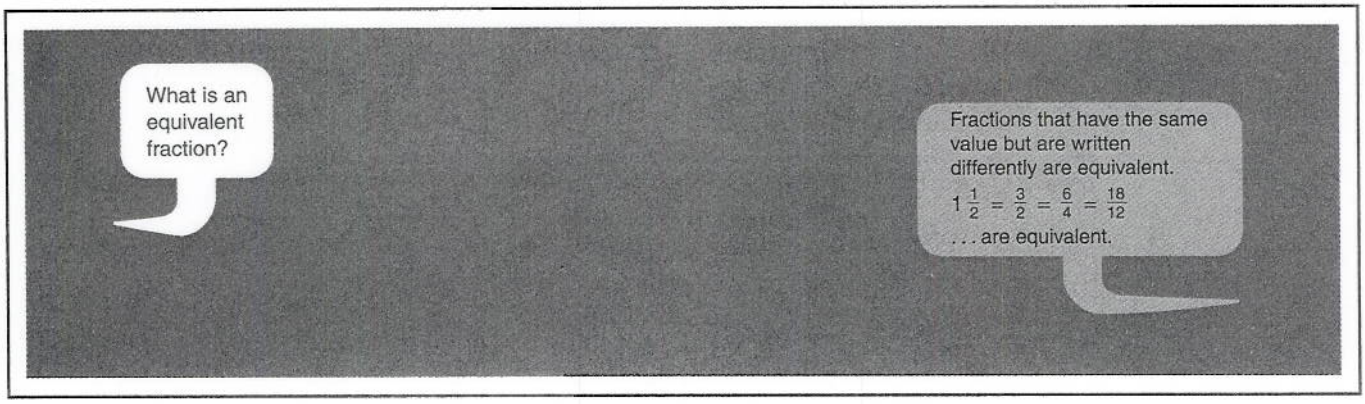
Practice with these.

(a) $\frac{5}{8} = \frac{?}{32}$ (b) $\frac{7}{16} = \frac{?}{48}$ (c) $1\frac{2}{3} = \frac{?}{12}$

SOLUTIONS

(a) $\frac{5}{8} = \frac{5 \times 4}{8 \times 4} = \frac{20}{32}$ (b) $\frac{7}{16} = \frac{7 \times 3}{16 \times 3} = \frac{21}{48}$

(c) $1\frac{2}{3} = \frac{5}{3} = \frac{5 \times 4}{3 \times 4} = \frac{20}{12}$



Writing in Lowest Terms

Very often in working with fractions you will be asked to *write a fraction in lowest terms.* This means to replace it with the most simple fraction in its set of equivalent fractions. To write $\frac{15}{30}$ in its lowest terms means to replace it by $\frac{1}{2}$. They are equivalent.

$$\frac{15}{30} = \frac{15 \div 15}{30 \div 15} = \frac{1}{2}$$

We have divided both the top and bottom of the fraction by 15.

In general, you would write a fraction in lowest terms as follows:

EXAMPLE

Write $\frac{30}{48}$ in lowest terms.

First, find the largest number that divides both top and bottom of the fraction exactly.

$30 = 5 \times 6$

$48 = 8 \times 6$

In this case, the factor 6 is the largest number that divides both parts of the fraction exactly.

Second, eliminate this common factor by dividing.

$$\frac{30}{48} = \frac{30 \div 6}{48 \div 6} = \frac{5}{8}$$

The fraction $\frac{5}{8}$ is the simplest fraction equivalent to $\frac{30}{48}$. No whole number greater than 1 divides both 5 and 8 exactly.

EXAMPLE

Write $\frac{90}{105}$ in lowest terms.

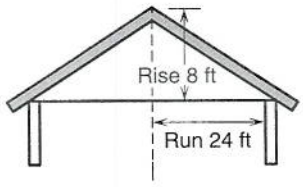
$$\frac{90}{105} = \frac{90 \div 15}{105 \div 15} = \frac{6}{7}$$

This process of eliminating a common factor is usually called *canceling*. When you cancel a factor, you divide both top and bottom of the fraction by that number. We would write $\frac{90}{105}$ as $\frac{6 \times \cancel{15}}{7 \times \cancel{15}} = \frac{6}{7}$. When you "cancel," you must divide by a *pair* of common factors, one factor in the numerator and the same factor in the denominator.

EXAMPLE

In roof construction, the *pitch* or steepness of a roof is defined as the fraction

$$\text{Pitch} = \frac{\text{rise}}{\text{run}}$$



The **rise** is the increase in height and the **run** is the corresponding horizontal distance. Typically, roofers express the pitch as the amount of rise per foot, or per 12 inches, of run. Therefore, rather than write the pitch as a fraction in lowest terms, they express it as an equivalent fraction with a denominator of 12. For the roof shown,

$$\text{Pitch} = \frac{\text{Rise}}{\text{Run}} = \frac{8\text{ ft}}{24\text{ ft}} = \frac{8 \div 2}{24 \div 2} = \frac{4}{12}$$

Divide by 2 ... to get a denominator of 12

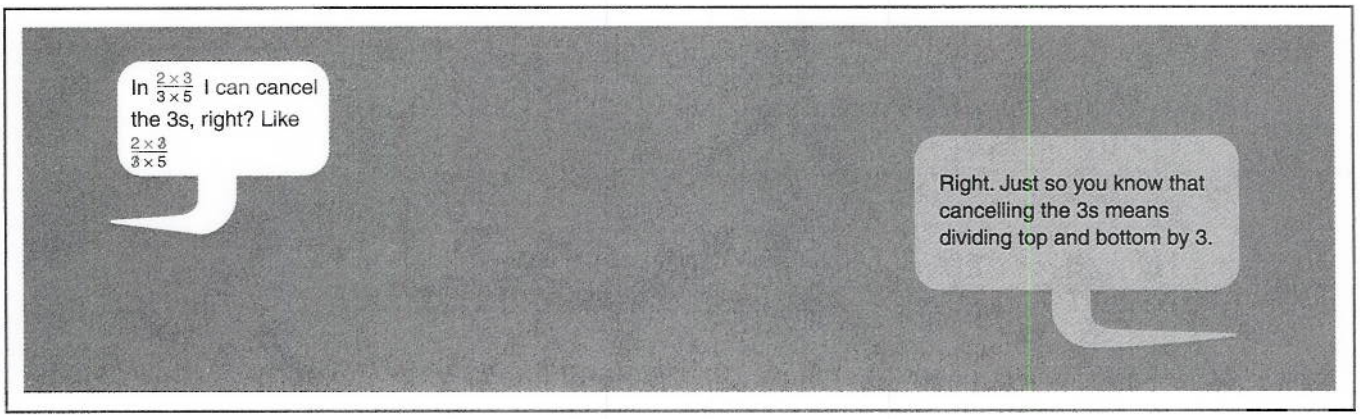
YOUR TURN

Write the following fractions in lowest terms.

(a) $\frac{6}{8}$ (b) $\frac{12}{16}$ (c) $\frac{2}{4}$ (d) $\frac{4}{12}$

(e) $\frac{15}{84}$ (f) $\frac{21}{35}$ (g) $\frac{12}{32}$

(h) **Roofing** Calculate the pitch of a storage shed roof having a rise of 6 ft over a run of 36 ft.



SOLUTIONS

(a) $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ (b) $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

(c) $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$ (d) $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$

(e) $\frac{15}{84} = \frac{15 \div 3}{84 \div 3} = \frac{5}{28}$ (f) $\frac{21}{35} = \frac{21 \div 7}{35 \div 7} = \frac{3}{5}$

(g) $\frac{12}{32} = \frac{12 \div 4}{32 \div 4} = \frac{3}{8}$ (h) Pitch $= \frac{6\text{ ft}}{36\text{ ft}} = \frac{6 \div 3}{36 \div 3} = \frac{2}{12}$

EXAMPLE

This one is a little tricky. Write $\frac{6}{3}$ in lowest terms.

$\frac{6}{3} = \frac{6 \div 3}{3 \div 3} = \frac{2}{1}$ or simply 2

Any whole number may be written as a fraction by using a denominator equal to 1.

$3 = \frac{3}{1}$ $4 = \frac{4}{1}$ and so on.

Writing numbers in this way will be helpful when you learn to do arithmetic with fractions.

Comparing Fractions

If you were offered your choice between $\frac{2}{3}$ of a certain amount of money and $\frac{5}{8}$ of it, which would you choose?

EXAMPLE

Which is the larger fraction, $\frac{2}{3}$ or $\frac{5}{8}$?

Can you decide? Rewriting the fractions as equivalent fractions will help.

To compare two fractions, rename each by changing them to equivalent fractions with the same denominator.

$$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24} \quad \text{and} \quad \frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

Now compare the new fractions: $\frac{16}{24}$ is greater than $\frac{15}{24}$ and therefore $\frac{2}{3}$ is larger than $\frac{5}{8}$.

Learning Help ▶

1. The new denominator is the product of the original ones ($24 = 8 \times 3$).
2. Once both fractions are written with the same denominator, the one with the larger numerator is the larger fraction. (16 of the fractional parts is more than 15 of them.) ◀

YOUR TURN

Which of the following quantities is the larger?

(a) $\frac{3}{4}$ in. or $\frac{5}{7}$ in.

(b) $\frac{7}{8}$ or $\frac{19}{21}$

(c) 3 or $\frac{40}{13}$

(d) $1\frac{7}{8}$ lb or $\frac{5}{3}$ lb

(e) $2\frac{1}{4}$ ft or $\frac{11}{6}$ ft

(f) $\frac{5}{16}$ or $\frac{11}{36}$

SOLUTIONS

(a) $\frac{3}{4} = \frac{21}{28}$, $\frac{5}{7} = \frac{20}{28}$; $\frac{21}{28}$ is larger than $\frac{20}{28}$, so $\frac{3}{4}$ in. is larger than $\frac{5}{7}$ in.

(b) $\frac{7}{8} = \frac{147}{168}$, $\frac{19}{21} = \frac{152}{168}$; $\frac{152}{168}$ is larger than $\frac{147}{168}$, so $\frac{19}{21}$ is larger than $\frac{7}{8}$.

(c) $3 = \frac{39}{13}$; $\frac{40}{13}$ is larger than $\frac{39}{13}$, so $\frac{40}{13}$ is larger than 3.

(d) $1\frac{7}{8} = \frac{15}{8} = \frac{45}{24}$, $\frac{5}{3} = \frac{40}{24}$; $\frac{45}{24}$ is larger than $\frac{40}{24}$, so $1\frac{7}{8}$ lb is larger than $\frac{5}{3}$ lb.

(e) $2\frac{1}{4} = \frac{9}{4} = \frac{54}{24}$, $\frac{11}{6} = \frac{44}{24}$; $\frac{54}{24}$ is larger than $\frac{44}{24}$, so $2\frac{1}{4}$ ft is larger than $\frac{11}{6}$ ft.

(f) $\frac{5}{16} = \frac{180}{576}$, $\frac{11}{36} = \frac{176}{576}$; $\frac{180}{576}$ is larger than $\frac{176}{576}$, so $\frac{5}{16}$ is larger than $\frac{11}{36}$.

Now turn to Exercises 2-1 for some practice in working with fractions.

Exercises 2-1 Working with Fractions

A. Write as an improper fraction.

1. $2\frac{1}{3}$ 2. $7\frac{1}{2}$ 3. $8\frac{3}{8}$ 4. $1\frac{1}{16}$ 5. $2\frac{7}{8}$
6. 2 7. $2\frac{2}{3}$ 8. $4\frac{3}{64}$ 9. $4\frac{5}{6}$ 10. $1\frac{13}{16}$

B. Write as a mixed number.

1. $\frac{17}{2}$ 2. $\frac{8}{5}$ 3. $\frac{11}{8}$ 4. $\frac{40}{16}$ 5. $\frac{3}{2}$
6. $\frac{11}{3}$ 7. $\frac{100}{6}$ 8. $\frac{4}{3}$ 9. $\frac{80}{32}$ 10. $\frac{5}{2}$

C. Write in lowest terms.

1. $\frac{12}{16}$ 2. $\frac{4}{6}$ 3. $\frac{6}{16}$ 4. $\frac{18}{4}$ 5. $\frac{4}{10}$
6. $\frac{35}{30}$ 7. $\frac{24}{30}$ 8. $\frac{10}{4}$ 9. $4\frac{3}{12}$ 10. $\frac{34}{32}$
11. $\frac{42}{64}$ 12. $\frac{10}{35}$ 13. $\frac{15}{36}$ 14. $\frac{45}{18}$ 15. $\frac{38}{24}$

D. Complete.

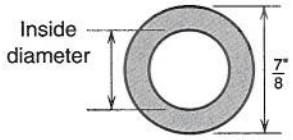
1. $\frac{7}{8} = \frac{?}{16}$ 2. $\frac{3}{4} = \frac{?}{16}$ 3. $\frac{1}{8} = \frac{?}{64}$
4. $\frac{3}{8} = \frac{?}{64}$ 5. $1\frac{1}{4} = \frac{?}{16}$ 6. $2\frac{7}{8} = \frac{?}{32}$
7. $3\frac{3}{5} = \frac{?}{10}$ 8. $1\frac{1}{16} = \frac{?}{32}$ 9. $1\frac{40}{60} = \frac{?}{3}$
10. $4 = \frac{?}{6}$ 11. $2\frac{5}{8} = \frac{?}{16}$ 12. $2\frac{5}{6} = \frac{?}{12}$

E. Which is larger?

1. $\frac{3}{5}$ or $\frac{4}{7}$ 2. $\frac{3}{2}$ or $\frac{13}{8}$ 3. $1\frac{1}{2}$ or $1\frac{3}{7}$
4. $\frac{3}{4}$ or $\frac{13}{16}$ 5. $\frac{7}{8}$ or $\frac{5}{6}$ 6. $2\frac{1}{2}$ or $1\frac{11}{8}$
7. $1\frac{2}{5}$ or $\frac{6}{4}$ 8. $\frac{3}{16}$ or $\frac{25}{60}$ 9. $\frac{13}{5}$ or $\frac{5}{2}$
10. $3\frac{1}{2}$ or $2\frac{7}{4}$ 11. $\frac{3}{8}$ or $\frac{5}{12}$ 12. $1\frac{1}{5}$ or $\frac{8}{7}$

F. Practical Problems

1. **Carpentry** Maria, an apprentice carpenter, measured the length of a 2 by 4 as $15\frac{6}{8}$ in. Express this measurement in lowest terms.
2. **Electrical Technology** An electrical light circuit in John's welding shop had a load of 2800 watts. He changed the circuit to ten 150-watt bulbs and six 100-watt bulbs. What fraction represents a comparison of the new load with the old load?



Problem 5

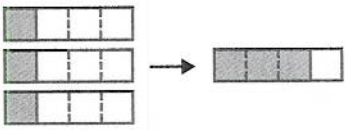
3. The numbers $\frac{22}{7}$, $\frac{19}{6}$, $\frac{47}{15}$, $\frac{25}{8}$, and $\frac{41}{13}$ are all reasonable approximations to the number π. Which is the largest approximation? Which is the smallest approximation?
4. **Sheet Metal Technology** Which is thicker, a $\frac{3}{16}$-in. sheet of metal or a $\frac{13}{64}$-in. fastener?
5. **Plumbing** Is it possible to have a $\frac{7}{8}$-in. pipe with an inside diameter of $\frac{29}{32}$-in.?
6. **Sheet Metal Technology** Fasteners are equally spaced on a metal vent cover, with nine spaces between fasteners covering 24 in. Write the distance between spaces as a mixed number.

   $\frac{24}{9} = ?$
7. **Printing** A printer has 15 rolls of newsprint in the warehouse. What fraction of this total will remain if six rolls are used?
8. **Machine Technology** A machinist who had been producing 40 parts per day increased the output to 60 parts per day by going to a faster machine. How much faster is the new machine? Express your answer as a mixed number.
9. **Landscaping** Before it can be used, a 12-ounce container of liquid fertilizer must be mixed with 48 ounces of water. What fraction of fertilizer is in the final mixture?
10. **Roofing** A ridge beam rises 18 inches over a horizontal run of 72 inches. Calculate the pitch of the roof and express it as a fraction with a denominator of 12.

When you have had the practice you need, check your answers in the Appendix.

2-2 MULTIPLICATION OF FRACTIONS

The simplest arithmetic operation with fractions is multiplication and, happily, it is easy to show graphically. The multiplication of a whole number and a fraction may be illustrated this way.

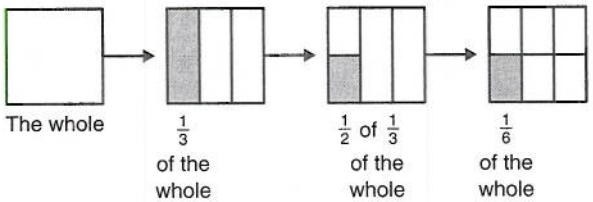


$3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ three segments each $\frac{1}{4}$ unit long.

Any fraction such as $\frac{3}{4}$ can be thought of as a product: $3 \times \frac{1}{4}$

The product of two fractions can also be shown graphically.

$\frac{1}{2} \times \frac{1}{3}$ means $\frac{1}{2}$ of $\frac{1}{3}$



The product $\frac{1}{2} \times \frac{1}{3}$ is

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \frac{\text{1 shaded area}}{\text{6 equal areas in the square}}$$

In general, we calculate this product as

$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3}$$

Multiply the numerators (top)

Multiply the denominators (bottom)

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

The product of two fractions is a fraction whose numerator is the product of their numerators and whose denominator is the product of their denominators.

EXAMPLE

Multiply $\frac{5}{6} \times \frac{2}{3}$.

$$\frac{5}{6} \times \frac{2}{3} = \frac{5 \times 2}{6 \times 3} = \frac{10}{18} = \frac{5 \times \overset{1}{\cancel{2}}}{9 \times \underset{1}{\cancel{2}}} = \frac{5}{9}$$

Note ▶ Always write your answer in lowest terms. In this problem you probably recognized that both 10 and 18 were evenly divisible by 2, so you canceled out that common factor. It will save you time and effort if you cancel common factors, such as the 2 above, *before* you multiply this way:

$$\frac{5 \times \overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}} \times 3} = \frac{5}{9} \quad \blacktriangleleft$$

EXAMPLE

Whole Number Times a Fraction

When multiplying a whole number by a fraction, first write the whole number as a fraction.

Step 1 Rewrite 6 as a fraction with denominator 1.

$$6 \times \frac{1}{8} = \frac{6}{1} \times \frac{1}{8}$$

Step 2 Divide by common factors and multiply.

$$= \frac{\overset{3}{\cancel{6}} \times 1}{1 \times \underset{4}{\cancel{8}}} = \frac{3}{4}$$

EXAMPLE

Mixed Number Times a Fraction

Step 1 Write the mixed number $2\frac{1}{2}$ as the improper fraction $\frac{5}{2}$.

$$2\frac{1}{2} \times \frac{3}{4} = \frac{5}{2} \times \frac{3}{4}$$

Step 2 Multiply.

$$= \frac{5 \times 3}{2 \times 4} = \frac{15}{8}$$

Step 3 Write the product as a mixed number.

$$= 1\frac{7}{8}$$

EXAMPLE

Mixed Number Times a Mixed Number

Step 1 Write as improper fractions. $3\frac{1}{3} \times 2\frac{1}{4} = \frac{10}{3} \times \frac{9}{4}$

Step 2 Multiply. $= \frac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}} \times \underset{2}{\cancel{4}}}$

Step 3 Write the answer as a mixed number. $= \frac{15}{2} = 7\frac{1}{2}$

Note ▶ In many word problems the words "of" or "product of" appear as signals that you are to multiply. For example, the phrase "one-half of 16" means $\frac{1}{2} \times 16 = \frac{1}{2} \times \frac{16}{1} = \frac{16}{2} = 8$, and the phrase "the product of $\frac{2}{3}$ and $\frac{1}{4}$" should be translated as $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$. ◀

YOUR TURN

Now test your understanding with these problems. Multiply as shown. Change any mixed numbers to improper fractions *before* you multiply.

(a) $\frac{7}{8}$ of $\frac{2}{3}$ = _____ (b) $\frac{8}{12} \times \frac{3}{16}$ = _____ (c) $\frac{3}{32}$ of $\frac{4}{15}$ = _____

(d) $\frac{15}{4} \times \frac{9}{10}$ = _____ (e) $\frac{3}{2}$ of $\frac{2}{3}$ = _____ (f) $1\frac{1}{2} \times \frac{2}{5}$ = _____

(g) $4 \times \frac{7}{8}$ = _____ (h) $3\frac{5}{6} \times \frac{3}{10}$ = _____ (i) $1\frac{4}{5} \times 1\frac{3}{4}$ = _____

(j) **Plumbing** Polly the Plumber needs six lengths of PVC pipe each $26\frac{3}{4}$ in. long. What total length of pipe will she need?

Remember to write your answer in lowest terms.

SOLUTIONS

(a) (*Hint:* "Of" means multiply.)

$$\frac{7}{8} \times \frac{2}{3} = \frac{7 \times 2}{(4 \times 2) \times 3} = \frac{7}{4 \times 3} = \frac{7}{12}$$

Eliminate common factors before you multiply.

Your work will look like this when you learn to do these operations mentally:

$$\frac{7}{\underset{4}{8}} \times \frac{\overset{1}{2}}{3} = \frac{7}{12}$$

(b) $$\frac{8}{12} \times \frac{3}{16} = \frac{\overset{1}{8} \times \overset{1}{3}}{(4 \times \underset{1}{3}) \times (\underset{1}{8} \times 2)} = \frac{1}{4 \times 2} = \frac{1}{8}$$

or $\overset{1}{\cancel{8}}{}^{2} \times \overset{1}{\cancel{3}}{}_{2} = \frac{1}{8}$

(c) $\overset{1}{\cancel{3}}{}_{8} \times \overset{1}{\cancel{4}}{}_{5} = \frac{1}{40}$

(d) $\overset{3}{\cancel{15}}{}_{4} \times \frac{9}{\cancel{10}_{2}} = \frac{27}{8} = 3\frac{3}{8}$

(e) $\overset{1}{\cancel{3}}{}_{1} \times \overset{1}{\cancel{2}}{}_{1} = 1$

(f) $1\frac{1}{2} \times \frac{2}{5} = \frac{3}{\cancel{2}_{1}} \times \overset{1}{\cancel{2}}{}_{5} = \frac{3}{5}$

If you don't remember how to change a mixed number to an improper fraction see page 63.

(g) $4 \times \frac{7}{8} = \frac{\overset{1}{\cancel{4}}{}_{1}}{1} \times \frac{7}{\cancel{8}_{2}} = \frac{7}{2} = 3\frac{1}{2}$

(h) $3\frac{5}{6} \times \frac{3}{10} = \frac{23}{\cancel{6}_{2}} \times \frac{\overset{1}{\cancel{3}}{}_{10}}{} = \frac{23}{20} = 1\frac{3}{20}$

(i) $1\frac{4}{5} \times 1\frac{3}{4} = \frac{9}{5} \times \frac{7}{4} = \frac{63}{20} = 3\frac{3}{20}$

(j) Total length of pipe = number of pieces × length of each piece

$$= 6 \times 26\frac{3}{4}$$

$$= 6 \times \frac{107}{4}$$

$$= \frac{\overset{3}{\cancel{6}}{}_{1}}{1} \times \frac{107}{\cancel{4}_{2}}$$

$$= \frac{321}{2} = 160\frac{1}{2} \text{ in.}$$

Exercises 2-2

Multiplication of Fractions

A. Multiply and write the answer in lowest terms.

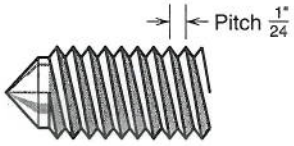
1. $\frac{1}{2} \times \frac{1}{4}$
2. $\frac{2}{5} \times \frac{2}{3}$
3. $\frac{4}{5} \times \frac{1}{6}$
4. $6 \times \frac{1}{2}$
5. $\frac{8}{9} \times 3$
6. $\frac{11}{12} \times \frac{4}{15}$
7. $\frac{8}{3} \times \frac{5}{12}$
8. $\frac{7}{8} \times \frac{13}{14}$
9. $\frac{12}{8} \times \frac{15}{9}$
10. $\frac{4}{7} \times \frac{49}{2}$
11. $4\frac{1}{2} \times \frac{2}{3}$
12. $6 \times 1\frac{1}{3}$
13. $2\frac{1}{6} \times 1\frac{1}{2}$
14. $\frac{5}{7} \times 1\frac{7}{15}$
15. $4\frac{3}{5} \times 15$
16. $10\frac{5}{6} \times 3\frac{3}{10}$
17. $34 \times 2\frac{3}{17}$
18. $7\frac{9}{10} \times 1\frac{1}{4}$
19. $11\frac{6}{7} \times \frac{7}{8}$
20. $18 \times 1\frac{5}{27}$
21. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
22. $1\frac{4}{5} \times \frac{2}{3} \times \frac{1}{4}$
23. $\frac{1}{4} \times \frac{2}{3} \times \frac{2}{5}$
24. $2\frac{1}{2} \times \frac{3}{5} \times \frac{8}{9}$
25. $\frac{2}{3} \times \frac{3}{2} \times 2$

B. Find.

1. $\frac{1}{2}$ of $\frac{1}{3}$
2. $\frac{1}{4}$ of $\frac{3}{8}$
3. $\frac{2}{3}$ of $\frac{3}{4}$
4. $\frac{7}{8}$ of $\frac{1}{2}$
5. $\frac{1}{2}$ of $1\frac{1}{2}$
6. $\frac{3}{4}$ of $1\frac{1}{4}$
7. $\frac{5}{8}$ of $2\frac{1}{10}$
8. $\frac{5}{3}$ of $1\frac{2}{3}$
9. $\frac{4}{3}$ of $\frac{3}{4}$
10. $\frac{3}{5}$ of $1\frac{1}{6}$
11. $\frac{7}{8}$ of $1\frac{1}{5}$
12. $\frac{3}{5}$ of 4
13. $\frac{7}{16}$ of 6
14. $\frac{5}{16}$ of $1\frac{1}{7}$
15. $\frac{3}{8}$ of $2\frac{2}{3}$
16. $\frac{15}{16}$ of $1\frac{3}{5}$

C. Practical Problems

1. **Building Construction** Find the width of floor space covered by 38 boards with $3\frac{5}{8}$-in. exposed surface each.
2. **Building Construction** There are 14 risers in the stairs from the basement to the first floor of a house. Find the total height of the stairs if the risers are $7\frac{1}{8}$ in. high.
3. **Roofing** Shingles are laid so that 5 in. or $\frac{5}{12}$ ft is exposed in each layer. How many feet of roof will be covered by 28 courses?
4. **Carpentry** A board $5\frac{3}{4}$ in. wide is cut to three-fourths of its original width. Find the new width.
5. **Carpentry** What length of 2-in. by 4-in. material will be required to make six bench legs each $28\frac{1}{4}$ in. long?
6. **Electrical Technology** Find the total length of 12 pieces of wire each $9\frac{3}{16}$ in. long.
7. **Auto Services** If a car averages $22\frac{3}{4}$ miles to a gallon of gas, how many miles can it travel on 14 gallons of gas?
8. **Machine Technology** What is the shortest bar that can be used for making six chisels each $6\frac{1}{8}$ in. in length?
9. **Manufacturing** How many pounds of grease are contained in a barrel if a barrel holds $46\frac{1}{2}$ gallons, and a gallon of grease weighs $7\frac{2}{3}$ lb?
10. **Plumbing** A plumber cut eight pieces of copper tubing from a coil. Each piece is $14\frac{3}{4}$ in. long. What is the total length used?



Problem 11

11. **Machine Technology** How far will a nut advance if it is given 18 turns on a $\frac{1}{4}$-in. 20-NF (National Fine thread) bolt? (*Hint:* The designation 20-NF means that the nut advances $\frac{1}{20}$ in. for each complete turn.)
12. **Building Construction** What width of floor space can be covered by 48 boards each with $4\frac{3}{8}$ in. of exposed surface?
13. **Drafting** If $\frac{3}{8}$ in. on a drawing represents 1 ft, how many inches on the drawing will represent 26 ft?
14. **Manufacturing** What is the volume of a rectangular box with interior dimensions $12\frac{1}{2}$ in. long, $8\frac{1}{8}$ in. wide, and $4\frac{1}{4}$ in. deep? (*Hint:* Volume = length $\times$ width $\times$ height.)
15. **Machine Technology** How long will it take to machine 45 pins if each pin requires $6\frac{3}{4}$ minutes? Allow 1 minute per pin for placing stock in the lathe.
16. **Manufacturing** There are 231 cu in. in a gallon. How many cubic inches are needed to fill a container with a rated capacity of $4\frac{1}{3}$ gallons?

17. **Printing** In a print shop, 1 unit of labor is equal to $\frac{1}{6}$ hour. How many hours are involved in 64 units of work?

18. **Carpentry** Find the total width of 36 2-by-4s if the finished width of each board is actually $3\frac{1}{2}$ in.

19. **Photography** A photograph must be reduced to four-fifths of its original size to fit the space available in a newspaper. Find the length of the reduced photograph if the original was $6\frac{3}{4}$ in. long.

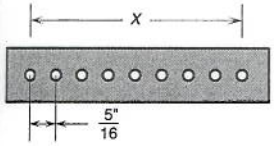
20. **Printing** A bound book weighs $1\frac{5}{8}$ lb. How many pounds will 20 cartons of 12 books each weigh?

21. **Printing** There are 6 picas in 1 in. If a line of type is $3\frac{3}{4}$ in. long, what is this length in picas?

22. **Wastewater Technology** The normal daily flow of raw sewage into a treatment plant is 32 MGD (million gallons per day). Because of technical problems one day, the plant had to cut back to three-fourths of its normal intake. What was this reduced flow?

23. **Masonry** What is the height of 12 courses of $2\frac{1}{4}$-in. bricks with $\frac{3}{8}$-in. mortar joints? (12 rows of brick and 11 mortar joints)

24. **Plumbing** To find the degree measure of the bend of a pipe fitting, multiply the fraction of bend by 360 degrees. What is the degree measure of a $\frac{1}{8}$ bend? A $\frac{1}{5}$ bend? A $\frac{1}{6}$ bend?



Problem 26

25. **Plumbing** A drain must be installed with a grade of $\frac{1}{8}$ in. of vertical drop per foot of horizontal run. How much drop will there be for 26 ft of run?

26. **Machine Technology** The center-to-center distance between consecutive holes in a strip of metal is $\frac{5}{16}$ in. What is the total distance x between the first and last centers as shown in the figure?

27. **Sheet Metal Technology** The allowance for a wired edge on fabricated metal is $2\frac{1}{2}$ times the diameter of the wire. Calculate the allowance for a wired edge if the diameter of the wire is $\frac{3}{16}$ in.

Check your answers in the Appendix, then continue in Section 2-3.

2-3 DIVISION OF FRACTIONS

Addition and multiplication are both reversible arithmetic operations. For example,

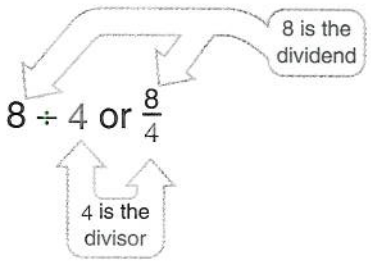
2×3 and 3×2 both equal 6

$4 + 5$ and $5 + 4$ both equal 9

The order in which you add or multiply is not important. This reversibility is called the *commutative* property of addition and multiplication.

In division this type of exchange is not allowed, and because it is not allowed many people find division very troublesome. In the division of fractions it is very important that you set up the problem correctly.

The phrase "8 divided by 4" can be written as



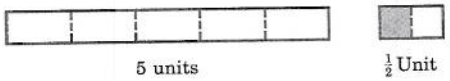
In the previous problem you were being asked to divide a set of eight objects into sets of four objects.

EXAMPLE

In the division $5 \div \frac{1}{2}$, which number is the divisor?

The divisor is $\frac{1}{2}$.

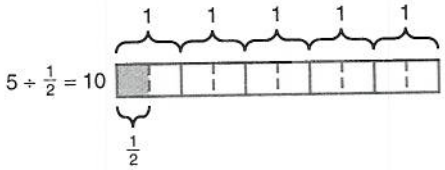
The division $5 \div \frac{1}{2}$, read "5 divided by $\frac{1}{2}$," asks how many $\frac{1}{2}$-unit lengths are included in a length of 5 units.



Division answers the question: How many of the divisor are in the dividend?

$8 \div 4 = \square$ asks you to find how many 4s are in 8.
It is easy to see that $\square = 2$.

$5 \div \frac{1}{2} = \square$ asks you to find how many $\frac{1}{2}$s are in 5. Do you see that $\square = 10$?



There are ten $\frac{1}{2}$-unit lengths contained in the 5-unit length.

Using a drawing like this to solve a division problem is difficult and clumsy. We need a simple rule.

Reciprocal

We can simplify the division of fractions with the help of a new concept called the **reciprocal**. The reciprocal of a fraction is obtained by switching its numerator and denominator. (This is often called *inverting* the fraction.)

EXAMPLE

The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4.

The reciprocal of 8 or $\frac{8}{1}$ is $\frac{1}{8}$.

To find the reciprocal of a mixed number, first convert it to an improper fraction.

The reciprocal of $2\frac{3}{5}$ or $\frac{13}{5}$ is $\frac{5}{13}$.

YOUR TURN

Find the reciprocal of each of the following numbers.

(a) $\frac{2}{5}$ (b) $\frac{7}{8}$ (c) $\frac{1}{6}$ (d) 7 (e) $2\frac{1}{5}$

ANSWERS

(a) $\frac{5}{2}$ (b) $\frac{8}{7}$ (c) $\frac{6}{1}$ or 6 (d) $7 = \frac{7}{1}$ so its reciprocal is $\frac{1}{7}$.

(e) $2\frac{1}{5}$ is $\frac{11}{5}$ and its reciprocal is $\frac{5}{11}$.

We can now use the following simple rule to divide fractions.

> To divide by a fraction, multiply by its reciprocal.

EXAMPLE

$5 \div \frac{1}{2} = ?$

$$5 \div \frac{1}{2} = 5 \times \frac{2}{1} = \frac{10}{1} = 10$$

(reciprocal; Multiply) as shown graphically on the previous page

EXAMPLE

$\frac{2}{5} \div \frac{4}{3} = ?$

$$\frac{2}{5} \div \frac{4}{3} = \frac{2}{5} \times \frac{3}{4} = \frac{\overset{1}{\cancel{2}} \times 3}{5 \times \underset{2}{\cancel{4}}} = \frac{3}{10}$$

(Multiply; The reciprocal)

Careful ▶ When dividing fractions, never attempt to cancel common factors until *after* converting the division to multiplication by the reciprocal. ◀

EXAMPLE

$\frac{5}{8} \div 3\frac{3}{4} = ?$

$$\frac{5}{8} \div 3\frac{3}{4} = \frac{5}{8} \div \frac{15}{4} = \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{1}{6}$$

We have converted division problems that are difficult to picture into simple multiplication.

The final, and very important, step in every division is checking the answer. To check, multiply the divisor by the quotient and compare this answer with the original fraction or dividend.

If $\frac{2}{5} \div \frac{4}{3} = \frac{3}{10}$ then $\frac{4}{3} \times \frac{3}{10}$ should equal $\frac{2}{5}$.

$$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}} = \frac{2}{5}$$

YOUR TURN

Divide:

$\frac{7}{8} \div \frac{3}{2} =$ _____

SOLUTION

Reciprocal

$$\frac{7}{8} \div \frac{3}{2} = \frac{7}{8} \times \frac{2}{3} = \frac{7 \times \overset{1}{\cancel{2}}}{\underset{4}{\cancel{8}} \times 3} = \frac{7}{12}$$

Multiply

$$\frac{\overset{1}{\cancel{3}}}{2} \times \frac{7}{\underset{4}{\cancel{12}}} = \frac{7}{8}$$

Learning Help ▶ The chief source of confusion for many people in dividing fractions is deciding which fraction to invert. It will help if you:

1. Put every division problem in the form

   (dividend) ÷ (divisor)

   then find the reciprocal of the divisor, and finally, multiply to obtain the quotient.

2. Check your answer by multiplying. The product

   (divisor) × (quotient or answer)

   should equal the dividend. ◀

PROBLEMS

Here are a few problems to test your understanding.

(a) $\frac{2}{5} \div \frac{3}{8}$

(b) $\frac{7}{40} \div \frac{21}{25}$

(c) $3\frac{3}{4} \div \frac{5}{2}$

(d) $4\frac{1}{5} \div 1\frac{4}{10}$

(e) $3\frac{2}{3} \div 3$

(f) Divide $\frac{3}{4}$ by $2\frac{5}{8}$.

(g) Divide $1\frac{1}{4}$ by $1\frac{7}{8}$.

(h) **Carpentry** How many sheets of plywood, each $\frac{3}{4}$ in. thick, are in a stack 18 in. high?

Work carefully and check each answer.

SOLUTIONS

(a) $\frac{2}{5} \div \frac{3}{8} = \frac{2}{5} \times \frac{8}{3} = \frac{16}{15} = 1\frac{1}{15}$

The answer is $1\frac{1}{15}$. ☑ $\overset{1}{\cancel{8}} \times \overset{2}{\cancel{16}} = \frac{2}{5}$

(b) $\frac{7}{40} \div \frac{21}{25} = \overset{1}{\cancel{7}} \times \overset{5}{\cancel{25}} = \frac{5}{24}$ ☑ $\overset{7}{\cancel{21}} \times \overset{1}{\cancel{8}} = \frac{7}{40}$

(c) $3\frac{3}{4} \div \frac{5}{2} = \frac{15}{4} \div \frac{5}{2} = \overset{3}{\cancel{15}} \times \overset{1}{\cancel{2}} = \frac{3}{2} = 1\frac{1}{2}$ ☑ $\frac{5}{2} \times \frac{3}{2} = \frac{15}{4} = 3\frac{3}{4}$

(d) $4\frac{1}{5} \div 1\frac{4}{10} = \frac{21}{5} \div \frac{14}{10} = \overset{3}{\cancel{21}} \times \overset{2}{\cancel{10}} = \frac{6}{2} = 3$

☑ $1\frac{4}{10} \times 3 = \overset{7}{\cancel{14}} \times \frac{3}{1} = \frac{21}{5} = 4\frac{1}{5}$

(e) $3\frac{2}{3} \div 3 = \frac{11}{3} \div \frac{3}{1} = \frac{11}{3} \times \frac{1}{3} = \frac{11}{9} = 1\frac{2}{9}$

☑ $3 \times \frac{11}{9} = \overset{1}{\cancel{3}} \times \frac{11}{\cancel{9}} = \frac{11}{3} = 3\frac{2}{3}$

(f) $\frac{3}{4} \div 2\frac{5}{8} = \frac{3}{4} \div \frac{21}{8} = \overset{1}{\cancel{3}} \times \overset{2}{\cancel{8}} = \frac{2}{7}$ ☑ $2\frac{5}{8} \times \frac{2}{7} = \overset{3}{\cancel{21}} \times \overset{1}{\cancel{2}} = \frac{3}{4}$

(g) $1\frac{1}{4} \div 1\frac{7}{8} = \frac{5}{4} \div \frac{15}{8} = \overset{1}{\cancel{5}} \times \overset{2}{\cancel{8}} = \frac{2}{3}$

☑ $1\frac{7}{8} \times \frac{2}{3} = \overset{5}{\cancel{15}} \times \overset{1}{\cancel{2}} = \frac{5}{4} = 1\frac{1}{4}$

(h) A question of the form "How many X are in Y?" tells us we must divide Y by X. In this problem, divide 18 in., the total height, by $\frac{3}{4}$ in., the thickness of each sheet.

$18 \div \frac{3}{4} = \overset{6}{\cancel{18}} \times \frac{4}{\cancel{3}} = 24$ There are 24 sheets of plywood in the stack.

WHY DO WE USE THE RECIPROCAL WHEN WE DIVIDE FRACTIONS?

Notice that $8 \div 4 = 2$ and $8 \times \frac{1}{4} = \frac{\overset{2}{\cancel{8}}}{1} \times \frac{1}{\underset{1}{\cancel{4}}} = 2$.

Dividing by a number gives the same result as multiplying by its reciprocal. The following shows why this is so.

The division $8 \div 4$ can be written $\frac{8}{4}$. Similarly, $\frac{1}{2} \div \frac{2}{3}$ can be written $\frac{\frac{1}{2}}{\frac{2}{3}}$.

To simplify this fraction, multiply by $\frac{\frac{3}{2}}{\frac{3}{2}}$ (which is equal to 1).

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{\frac{1}{2} \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}} = \frac{\frac{1}{2} \times \frac{3}{2}}{1} = \frac{1}{2} \times \frac{3}{2}$$

$$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$$

Therefore, $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2}$, the reciprocal of the divisor $\frac{2}{3}$.

Turn to Exercises 2-3 for a set of practice problems on dividing fractions.

Exercises 2-3

Division of Fractions

A. Divide and write the answer in lowest terms.

1. $\frac{5}{6} \div \frac{1}{2}$	2. $6 \div \frac{2}{3}$	3. $\frac{5}{12} \div \frac{4}{3}$	4. $8 \div \frac{1}{4}$
5. $\frac{6}{16} \div \frac{3}{4}$	6. $\frac{1}{2} \div \frac{1}{2}$	7. $\frac{3}{16} \div \frac{6}{8}$	8. $\frac{3}{4} \div \frac{5}{16}$
9. $1\frac{1}{2} \div \frac{1}{6}$	10. $6 \div 1\frac{1}{2}$	11. $3\frac{1}{7} \div 2\frac{5}{14}$	12. $3\frac{1}{2} \div 2$
13. $6\frac{2}{5} \div 5\frac{1}{3}$	14. $10 \div 1\frac{1}{5}$	15. $8 \div \frac{1}{2}$	16. $\frac{2}{3} \div 6$
17. $\frac{\frac{12}{2}}{3}$	18. $\frac{\frac{3}{4}}{\frac{7}{8}}$	19. $\frac{5}{\frac{2}{3}}$	20. $\frac{1\frac{1}{2}}{2\frac{1}{2}}$
21. $\frac{5}{16} \div \frac{3}{8}$	22. $\frac{7}{12} \div \frac{2}{3}$	23. $\frac{7}{32} \div 1\frac{3}{4}$	24. $1\frac{2}{3} \div 1\frac{1}{4}$

B. Practical Problems

1. **Drafting** How many feet are represented by a 4-in. line if it is drawn to a scale of $\frac{1}{2}$ in. = 1 ft?
2. **Drafting** If $\frac{1}{4}$ in. on a drawing represents 1 ft 0 in., then $3\frac{1}{2}$ in. on the drawing will represent how many feet?