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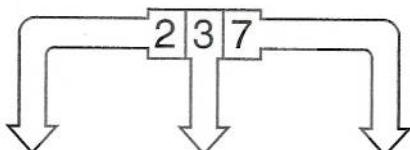
Decimal Numbers



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3-1 ADDITION AND SUBTRACTION OF DECIMALS

Place Value of Decimal Numbers By now you know that whole numbers are written in a form based on powers of ten. A number such as



means $(2 \times 100) + (3 \times 10) + (7 \times 1)$ or $200 + 30 + 7$

This way of writing numbers can be extended to fractions. A *decimal* number is a fraction whose denominator is 10 or some multiple of 10.

A decimal number may have both a whole-number part and a fraction part. For example, the number 324.576 means

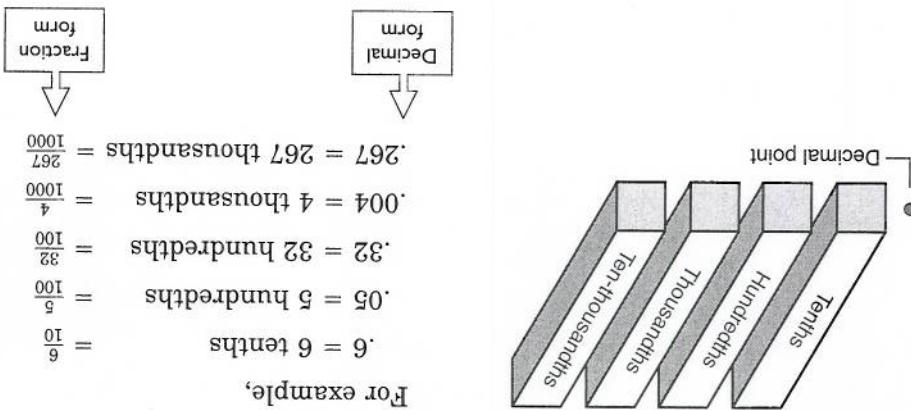
- (c) Six and four tenths
 (b) Nineteen and two hundred seventy-eight thousandths
 (a) Fifty-six hundredths

ANSWERS

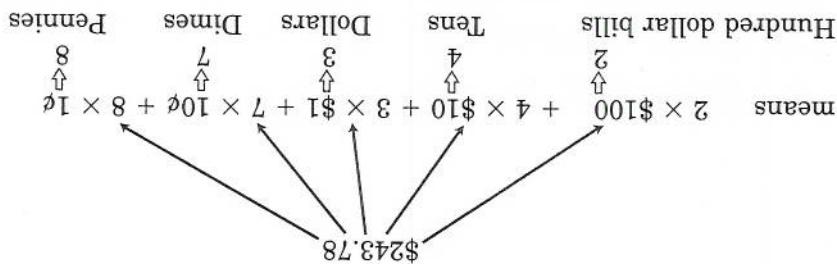
(d) 0.07 (e) 5.064 (f) 0.0018

(a) 0.56 (b) 19.278 (c) 6.4

Write the following decimal numbers in words.

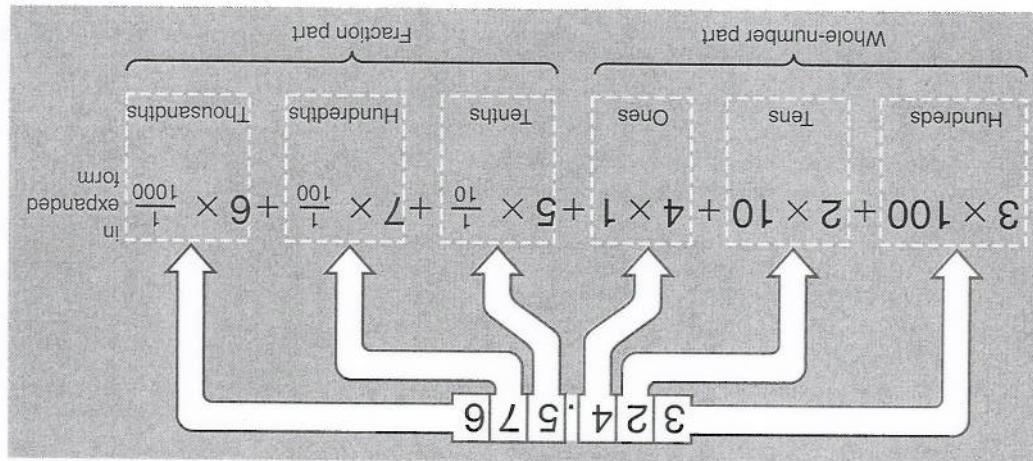
YOUR TURN

To write a decimal number in words, remember this diagram:



ing with money.

You are already familiar with this way of interpreting decimal numbers from work-



- (d) Seven hundredths
 - (e) Five and sixty-four thousandths
 - (f) Eighteen ten-thousandths
-

Learning Help ►

The word **and** represents the decimal point. Everything preceding **and** is the whole-number part, and everything after **and** is the decimal part of the number. ◀

It is also important to be able to perform the reverse process—that is, write a decimal in numerical form if it is given in words.

EXAMPLE

To write “six and twenty-four thousandths” as a numeral,

First, write the whole-number part (six). This is the part before the “and.” Follow this by a decimal point—the “and.”

6.

Next, draw as many blanks as the decimal part indicates. In this case, the decimal part is “thousandths,” so we allow for three decimal places. Draw three blanks.

6. ___ ___ ___

Finally, write a number giving the decimal (twenty-four). Write it so it *ends* on the far right blank. Fill in any blank decimal places with zeros.

6. ___ 2 4

6.024

YOUR TURN

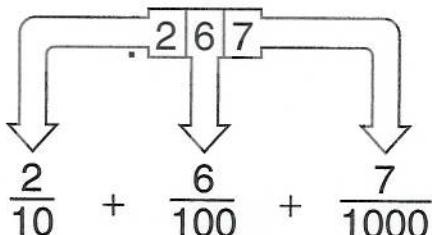
Now you try it. Write each of the following as decimal numbers.

- (a) Five thousandths
- (b) One hundred and six tenths
- (c) Two and twenty-eight hundredths
- (d) Seventy-one and sixty-two thousandths
- (e) Three and five hundred eighty-nine ten-thousandths

ANSWERS

- (a) 0.005
 - (b) 100.6
 - (c) 2.28
 - (d) 71.062
 - (e) 3.0589
-

Expanded Form The decimal number .267 can be written in expanded form as

$$\frac{2}{10} + \frac{6}{100} + \frac{7}{1000}$$


The number 43.6708 has four decimal digits: 6, 7, 0, and 8.

Decimal Digits In the decimal number 86.423 the digits 4, 2, and 3 are called decimal digits.

				Each row changes by a factor of ten
7 ×	1	ones	7 . 8	tenths
4 ×	10	tens	40 . 09	hundredths
2 ×	100	hundreds	200 . 005	thousandths
3 ×	1000	thousands	3000 . 0006	ten-thousandths
			3247 . 8956	6 × 0.0001

example,

Notice that the denominators in the decimal fractions change by a factor of 10. For

Learning Help

$$\begin{aligned}
 \text{(d)} \quad 235.22267 &= 200 + 30 + 5 + \frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + \frac{6}{10000} + \frac{7}{100000} \\
 \text{(c)} \quad 14.5060 &= 10 + 4 + \frac{5}{10} + \frac{6}{100} + \frac{0}{1000} + \frac{0}{10000} \\
 &= 40 + 3 + \frac{6}{10} + \frac{0}{100} + \frac{7}{1000} \\
 \text{(b)} \quad 43.607 &= 4 \times 10 + 3 \times 1 + 6 \times \frac{1}{10} + 0 \times \frac{1}{100} + 7 \times \frac{1}{1000} \\
 &= 80 + 6 + \frac{4}{10} + \frac{2}{100} \\
 \text{(a)} \quad 86.42 &= 8 \times 10 + 6 \times 1 + 4 \times \frac{1}{10} + 2 \times \frac{1}{100}
 \end{aligned}$$

ANSWERS

- (a) 86.42 (b) 43.607 (c) 14.5060 (d) 235.22267

To help get these ideas clear in your mind, write the following in expanded form.

YOUR TURN

It is easy to mistake .4 for 4, but the decimal point in 0.4 cannot be overlooked. That zero out front will help you remember where the decimal point is located.

.001 would be written 0.001

.4 would be written 0.4

.526 would be written 0.526

decimal point.

We usually write a decimal number less than 1 with a zero to the left of the deci-

$$.526 = \frac{5}{10} + \frac{2}{100} + \frac{6}{1000}$$

Write the decimal number .526 in expanded form.

EXAMPLE

The number 5376.2 has one decimal digit: 2.

All digits to the right of the decimal point, those that name the fractional part of the number, are decimal digits.

YOUR TURN

How many decimal digits are included in each of these numbers?

- (a) 1.4 (b) 315.7 (c) 0.425 (d) 324.0075

ANSWERS

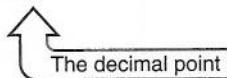
- (a) one (b) one (c) three (d) four

We will use the idea of decimal digits often in doing arithmetic with decimal numbers.

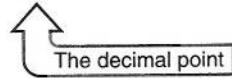
The decimal point is simply a way of separating the whole-number part from the fraction part. It is a place marker. In whole numbers the decimal point usually is not written, but it is understood to be there.

The whole number 2 is written 2. as a decimal.

$$2 = 2.$$



$$\text{or} \quad 324 = 324.$$



This is very important. Many people make big mistakes in arithmetic because they do not know where that decimal point should go.

Very often, additional zeros are attached to the decimal number without changing its value. For example,

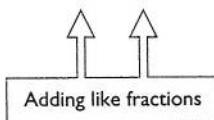
$$8.5 = 8.50 = 8.5000 \quad \text{and so on}$$

$$6 = 6. = 6.0 = 6.000 \quad \text{and so on}$$

The value of the number is not changed, but the additional zeros may be useful, as we shall see.

Addition Because decimal numbers represent fractions with denominators equal to multiples of ten, addition is very simple.

$$\begin{aligned} 2.34 &= 2 + \frac{3}{10} + \frac{4}{100} \\ + 5.23 &= 5 + \frac{2}{10} + \frac{3}{100} \\ &\qquad\qquad\qquad 7 + \frac{5}{10} + \frac{7}{100} = 7.57 \end{aligned}$$



Of course, we do not need this clumsy business in order to add decimal numbers. As with whole numbers, we may arrange the digits in vertical columns and add directly.

Arrange each sum vertically, placing the decimal points in the same column, then add as with whole numbers.

0.6 ounces of cleaning concentrate on successive jobs. What total amount of concentrate has he used during this time?

- (a) \$4.02 + \$3.67 = _____
 (b) 13.2 + 1.57 = _____
 (c) 23.007 + 1.12 = _____
 (d) 14.6 + 1.2 + 3.15 = _____
 (e) 5.7 + 3.4 = _____
 (f) \$9 + \$0.72 + \$6.09 = _____
 (g) 0.07 + 6.79 + 0.3 + 3 = _____
 (h) **Sheet Metal Technology** A sheet metal worker uses 2.36, 7, 3.9, and

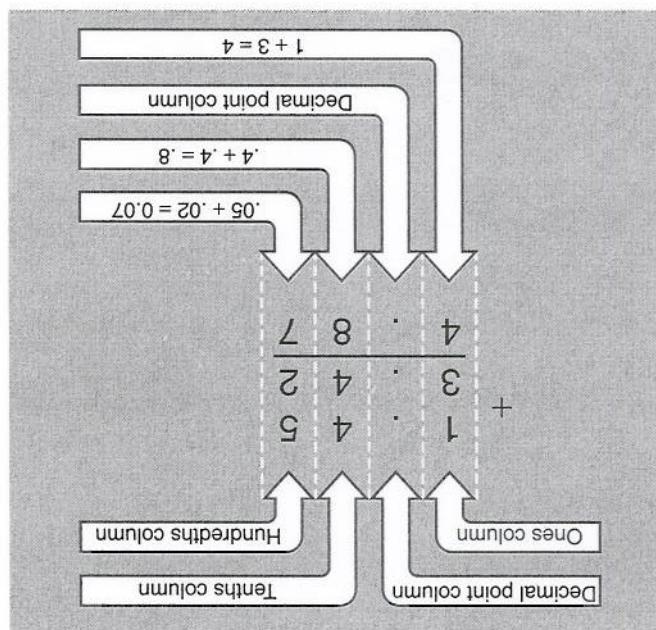
Add the following decimal numbers.

YOUR TURN

Except for the preliminary step of lining up decimal points, addition of decimal numbers is exactly the same process as addition of whole numbers.

If one of the numbers is written with fewer decimal digits than the other, attach as many zeros as needed so that both have the same number of decimal digits.

Digits of the same power of ten are placed in the same vertical column. Decimal points are always lined up vertically.



Let's add 1.45 + 3.42.

EXAMPLE

What do you mean by "one decimal place"?

One decimal digit or one digit to the right of the decimal point. 4.2 has one decimal digit, 4.23 has two, 4.234 has three, ... Got it?

SOLUTIONS

Decimal points are lined up vertically.

$$\begin{array}{r}
 \$4.02 \\
 \$3.67 \\
 \hline
 \$7.69
 \end{array}$$

0.02 + 0.07 = 0.09 Add cents.
 0.0 + 0.6 = 0.6 Add 10-cent units.
 4 + 3 = 7 Add dollars.

As a check, notice that the sum is roughly \$4 + \$4 or \$8, which agrees with the actual answer. Always check your answer by first estimating it, then comparing your estimate or rough guess with the final answer.

Decimal points are in line.

$$\begin{array}{r}
 13.20 \\
 1.57 \\
 \hline
 14.77
 \end{array}$$

Attach a zero to provide the same number of decimal digits as in the other addend.
 Place answer decimal point in the same vertical line.

$13 + 2 = 15$, which agrees roughly with the answer.

$$\begin{array}{r}
 23.007 \\
 + 1.120 \\
 \hline
 24.127
 \end{array}$$

← Attach extra zero.

$$\begin{array}{r}
 14.60 \\
 1.20 \\
 + 3.15 \\
 \hline
 18.95
 \end{array}$$

← Attach extra zeros.

$$\begin{array}{r}
 5.7 \\
 + 3.4 \\
 \hline
 9.1
 \end{array}$$

$0.7 + 0.4 = 1.1$ Write 0.1.
Carry 1.
Add: $1 + 5 + 3 = 9$

$$\begin{array}{r}
 1 \\
 \$9.00 \\
 0.72 \\
 \hline
 \$15.81
 \end{array}$$

$$\begin{array}{r}
 0.07 \\
 6.79 \\
 0.30 \\
 3.00 \\
 \hline
 10.16
 \end{array}$$

← Attach extra zeros.

$$\begin{array}{r}
 1 \\
 2.36 \\
 7.00 \\
 3.90 \\
 0.60 \\
 \hline
 13.86
 \end{array}$$

← Attach extra zeros.

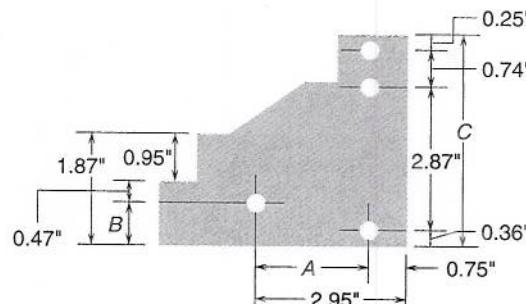
Careful ► You must line up the decimal points vertically to be certain of getting a correct answer. ◀

- B. Add or subtract as shown.
17. One hundred sixteen ten-thousandths
 18. Forty-seven ten-thousandths
 19. Two and three hundred seventy-four ten-thousandths
 20. Ten and two hundred twenty-two ten-thousandths
- C. Practical Problems
1. Machine Technology What is the combined thickness of these five shims: 0.008, 0.125, 0.150, 0.185, and 0.005 in.?
 2. Electrical Technology The combined weight of a spool and the wire it carries is 13.6 lb. If the weight of the spool is 1.75 lb, what is the weight of the wire?
 3. Electrical Technology The following are diameters of some common household wires: No. 10 is 0.102 in., No. 11 is 0.090 in., No. 12 is 0.081 in., No. 14 is 0.064 in., and No. 16 is 0.051 in.
- (a) The diameter of No. 16 wire is how much smaller than the diameter of No. 14 wire?
- (b) Is No. 12 wire larger or smaller than No. 10 wire? What is the difference in their diameters?
- (c) John measured the thickness of a wire with a micrometer as 0.059 in. Assuming that the manufacturer was slightly off, what wire size did John have?

4. **Painting and Decorating** In estimating a finishing job, a painter included the following items:

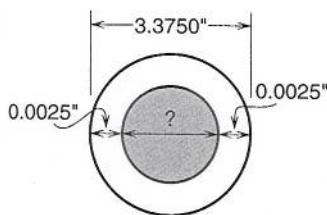
Material	\$ 677.85
Trucking	\$ 32.80
Profit	\$ 250
Labor	\$1645.50
Overhead	\$ 67.75

What was his total estimate for the job?



Problem 5

5. **Metalworking** Find A , B , and C .
6. **Plumbing** A piece of pipe 8.4 in. long is cut from a piece 40.8 in. long. What is the length of the longer piece remaining if the width of the saw cut is 0.2 in.?
7. **Machine Technology** A certain machine part is 2.345 in. thick. What is its thickness after 0.078 in. is ground off?
8. **Masonry** Find the total cost of the materials for a certain masonry job if sand cost \$43.65, cement cost \$114.47, a steel lintel cost \$32.65, and brick cost \$1476.28.
9. **Masonry** The specifications for a reinforced masonry wall called for 1.5 sq in. of reinforcing steel per square foot of cross-sectional area. If the three pieces of steel being used had cross sections of 0.125, 0.200, and 1.017 sq in., did they meet the specifications for a 1-sq-ft area?
10. **Building Construction** A plot plan of a building site showed that the east side of the house was 46.35 ft from the east lot line, and the west side of the house was 41.65 ft from the west lot line. If the lot was 156.00 ft wide along the front, how wide is the house?
11. **Auto Mechanics** A mechanic must estimate the total time for a particular servicing. He figures 0.3 hour for an oil change, 1.5 hours for a tune-up, 0.4 hour for a brake check, and 1.2 hours for air-conditioning service. What is the total number of hours of his estimate?
12. **Auto Mechanics** A heated piston measures 8.586 cm in diameter. When cold it measures 8.573 cm in diameter. How much does it expand when heated?
13. **Auto Mechanics** A piston must fit in a bore with a diameter of 3.3750 in. What must be the diameter of the piston given the clearance shown in the diagram?
14. **Auto Mechanics** In squaring a damaged car frame, an autobody worker measured the diagonals between the two front cross-members. One diagonal was 196.1 cm and the other was 176.8 cm. What is the difference that must be adjusted?
15. **Machine Technology** The diameter of a steel shaft is reduced 0.007 in. The original diameter of the shaft was 0.850 in. Calculate the reduced diameter of the shaft.



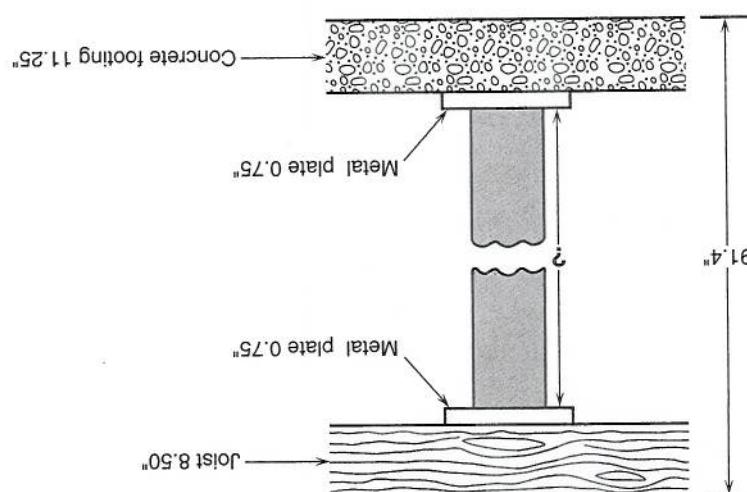
Problem 13

"Sticker price"	\$21,325.00		
Leather interior	875.40		
Destination and delivery	635.00		
5-speed automatic transmission	900.00		
CD player	575.95		
Moonroof	735.50		
Tax and license	1602.34		
Less trade-in	\$ 1780.00		

2. What is the actual cost of the following car?

Date	Balance	Withdrawals	Deposits	No. 4	?
Oct. 8				\$357.41	
Oct. 16		87.50		364.85	
Oct. 18		9.43		100.84	
Oct. 20		30.09		21.88	
Oct. 22				Nov. 2	
Oct. 27		259.47		Nov. 4	
Oct. 30		25.94			

1. Balance this checkbook by finding the closing balance as of November 4.
 D. Calculator Problems (If you need help using a calculator with decimals, turn to page 141.)



16. **Building Construction** A structural steel Lally column is mounted on a concrete footing as shown. Find the length of the Lally column.

3. Add as shown.

(a) 0.0067	(b) 1379.4	(c) 14.07
0.032	204.5	67.81
0.0012	16.75	132.99
0.0179	300.04	225.04
0.045	2070.08	38.02
0.5	167.99	4
0.05	43.255	16.899
0.0831	38.81	7.007
0.004	19.95	4.6

(d) $0.002 + 17.1 + 4.806 + 9.9981 - 3.1 + 0.701 - 1.001 - 14 - 8.09 + 1.0101$

Check your answers in the Appendix, then continue in Section 3-2.

3-2 MULTIPLICATION AND DIVISION OF DECIMAL NUMBERS

A decimal number is really a fraction with a multiple of 10 as denominator. For example,

$$0.5 = \frac{5}{10} \quad 0.3 = \frac{3}{10} \quad \text{and} \quad 0.85 = \frac{85}{100}$$

Multiplication of decimals is easy to understand if we think of it in this way:

$$0.5 \times 0.3 = \frac{5}{10} \times \frac{3}{10} = \frac{15}{100} = 0.15$$

Learning Help ► To estimate the product of 0.5 and 0.3, remember that if two numbers are both less than 1, their product must be less than 1. ◀

Multiplication Of course it would be very, very clumsy and time consuming to calculate every decimal multiplication this way. We need a simpler method. Here is the procedure most often used:

Step 1 Multiply the two decimal numbers as if they were whole numbers. Pay no attention to the decimal points.

Step 2 The sum of the decimal digits in the two numbers being multiplied will give you the number of decimal digits in the answer.

EXAMPLE

Multiply 3.2 by 0.41.

Step 1 Multiply, ignoring the decimal points.

$$\begin{array}{r} 32 \\ \times 41 \\ \hline 1312 \end{array}$$

Step 2 Count decimal digits in each number: 3.2 has *one* decimal digit (the 2), and 0.41 has *two* decimal digits (the 4 and the 1). The total number of decimal digits in the two factors is three. The answer will have *three* decimal digits. Count over *three* digits from right to left in the answer.

1.312 *three decimal digits*



3.2×0.41 is roughly $3 \times \frac{1}{2}$ or about $1\frac{1}{2}$. The answer 1.312 agrees with our rough guess. Remember, even if you use a calculator to do the actual work of arithmetic, you must *always* estimate your answer first and check it afterward.

$$2 \times 0.4 = 0.8$$

Count over one decimal digit: 8

$$(d) 2 \times 0.4 \quad 2 \times 4 = 8$$

$$10 \times 8.123 = 81.23 \quad \text{and so on} \blacktriangleright$$

$$10 \times 0.075 = 0.75$$

$$10 \times 6.2 = 62$$

right.

Learning Help Notice that multiplication by 10 simply shifts the decimal place one digit to the right.

$$\text{So } 10 \times 0.6 = 6.0.$$

Count over one decimal digit from the right: 6.0

$$(e) 10 \times 0.6 \quad 10 \times 6 = 60$$

$$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}.$$

$$\text{So } 0.1 \times 0.1 = 0.01.$$

Digits in the product, attach a zero on the left: 0.01

Count over two decimal digits from the right. Since there are not two decimal

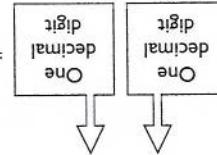
$$(b) 0.1 \times 0.1 \quad 1 \times 1 = 1$$

$2 \times \frac{1}{2}$ is about 1, so the answer seems reasonable.

The product is 1.25.

Count over two decimal digits from the right: 1.25 two decimal digits.

Second, count decimal digits = a total of two decimal digits.



Second, count decimal digits.

First, multiply $25 \times 5 = 125$.

$$(a) 2.5 \times 0.5 = \underline{\hspace{2cm}}$$

SOLUTIONS

(h) **Roofing** If asphalt tile weighs 1.1 lb per square foot, what is the weight of tile covering 8 sq ft?

$$(g) 0.04 \times 0.005 = \underline{\hspace{2cm}}$$

$$(f) 0.01 \times 0.02 = \underline{\hspace{2cm}}$$

$$(e) 2 \times 0.003 = \underline{\hspace{2cm}}$$

$$(d) 2 \times 0.4 = \underline{\hspace{2cm}}$$

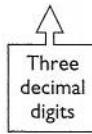
$$(c) 10 \times 0.6 = \underline{\hspace{2cm}}$$

$$(b) 0.1 \times 0.1 = \underline{\hspace{2cm}}$$

Try these simple decimal multiplications.

YOUR TURN

(e)

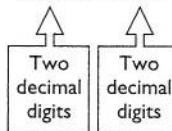


$$2 \times 0.003$$

$$2 \times 3 = 6$$

Count over three decimal digits. Attach two zeros as place holders.

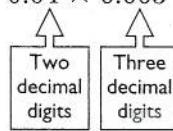
$$\underline{0.006} \quad 2 \times 0.003 = 0.006$$

(f) 0.01×0.02 

$$1 \times 2 = 2$$

Count over four decimal digits. Attach three zeros.

$$\underline{0.0002} \quad 0.01 \times 0.02 = 0.0002$$

(g) 0.04×0.005 

$$4 \times 5 = 20$$

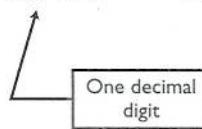
Count over five decimal digits. Attach three zeros.

$$\underline{0.00020} \quad 0.04 \times 0.005 = 0.00020 = 0.0002$$



- (h) We are given the weight of 1 sq ft and we need to calculate the weight of 8 sq ft. Multiply

$$1.1 \times 8 \qquad 11 \times 8 = 88$$



The answer will have *one* decimal digit: $1.1 \times 8 = 8.8$

The weight of 8 sq ft of tile is 8.8 lb.

Learning Help ►

1. Do not try to do this entire process mentally until you are certain you will not misplace zeros.
2. Always estimate before you begin the arithmetic, and check your answer against your estimate. ◀

Multiplication of larger decimal numbers is performed in exactly the same manner.

PROBLEMS

Try these:

(a) $4.302 \times 12.05 =$ _____

(b) $6.715 \times 2.002 =$ _____

(c) $3.144 \times 0.00125 =$ _____

- (d) **Construction Technology** How much does 25.8 sq ft of heavy plastic sheeting weigh if 1 sq ft weighs 2.37 lb?

One nice thing about a calculator is that it counts decimal digits and automatically adjusts the answer in any calculation. But to get an estimate of the answer, you must still understand the process.

Note When multiplying, you need not line up the decimal points. ▶

The answer and estimate are roughly equal. 

$$\begin{array}{r}
 & & & \text{three decimal digits} \\
 & & 61.146 \\
 & \times 2.37 & & \text{two decimal digits} \\
 \hline
 & & & \text{one decimal digit}
 \end{array}$$

Multiply: 25.8 Estimate: 25.8×2.37 is about 30×2 or 60 lb.

(d) Estimate: 25.8×2.37 is about 60 lb.

The answer and estimate are roughly equal. 

$$3.144 \times 0.00125 = 0.00393$$

In order to count over 8 digits we had to attach 2 zeros on the left. Eight decimal digits

$$\begin{array}{r}
 & & & \text{a total of eight decimal digits} \\
 & & .00393000 & \\
 & \times 0.00125 & & \text{0.00125 has five decimal digits} \\
 \hline
 & & & \text{3.144 has three decimal digits}
 \end{array}$$

Multiply: 3.144 Estimate: 3×0.001 is about 0.003.

(c) Estimate: 3×0.001 is about 0.003.

The answer and estimate are roughly equal. 

$$6.715 \times 2.002 = 13.44343$$

$$\begin{array}{r}
 & & & \text{six decimal digits} \\
 & & 13.443430 & \\
 & \times 2.002 & & \text{2.002 has three decimal digits} \\
 \hline
 & & & \text{a total of six decimal digits}
 \end{array}$$

Multiply: 6.715 Estimate: 6.7×2 is about 7×2 or 14.

The answer 51.8391 is approximately equal to the estimate of 48. 

$$\text{So that } 4.302 \times 12.05 = 51.83910 = 51.8391$$

51.83910,

answer.

The two factors being multiplied have a total of five decimal digits (three in the 4.302 and two in 12.05). Count over five decimal digits from the right in the

with the multiplication of whole numbers.)

(If you cannot do this multiplication correctly, turn to Section 1-3 for help

$$\begin{array}{r}
 & & & \text{5183910} \\
 & & \times 1205 & \\
 \hline
 & & & \text{4302}
 \end{array}$$

(a) Estimate: $4 \times 12 = 48$ The answer will be about 48.

Division Division of decimal numbers is very similar to the division of whole numbers. For example,

$$6.8 \div 1.7 \quad \text{can be written} \quad \frac{6.8}{1.7}$$

and if we multiply both top and bottom of the fraction by 10 we have

$$\frac{6.8}{1.7} = \frac{6.8 \times 10}{1.7 \times 10} = \frac{68}{17}$$

But you should know how to divide these whole numbers.

$$68 \div 17 = 4$$

$$\text{Therefore, } 6.8 \div 1.7 = 4.$$

To divide decimal numbers, use the following procedure.

Step 1 Write the divisor and dividend in standard long-division form.

Example:

$$6.8 \div 1.7$$

$$1.7) \overline{6.8}$$

Step 2 Shift the decimal point in the divisor to the right so as to make the divisor a whole number.

$$1.7) \overline{6.8}$$

Step 3 Shift the decimal point in the dividend the same amount. (Attach zeros if necessary.)

$$17.) \overline{68}$$

Step 4 Place the decimal point in the answer space directly above the new decimal position in the dividend.

$$17.) \overline{68}$$

Step 5 Now divide exactly as you would with whole numbers. The decimal points in divisor and dividend may now be ignored.

$$17.) \overline{68}$$

$$6.8 \div 1.7 = 4$$

$$0$$

Notice in steps 2 and 3 that we have simply multiplied both divisor and dividend by 10.

If there is a remainder in step 5, we must add additional zeros to complete the division. This will be necessary in the next example.

EXAMPLE

Divide: $1.38 \div 2.4$

Let's do it step by step.

$$2.4) \overline{1.38}$$

$$2.4) \overline{13.8}$$

Shift both decimal points one digit to the right to make the divisor (2.4) a whole number (24).

$$24.) \overline{138}$$

Place the decimal point in the answer space.

$$24.) \overline{138}$$

Divide as usual. 24 goes into 138 five times.

$$\begin{array}{r} 120 \\ \hline 18 \end{array}$$

There is a remainder (18), so we must keep going.

$$24.) \overline{13.80}$$

Attach a zero to the dividend and bring it down.

$$\begin{array}{r} 120 \\ \hline 180 \end{array}$$

Divide 180 by 24. 24 goes into 180 seven times.

$$\begin{array}{r} 168 \\ \hline 12 \end{array}$$

Now the remainder is 12.

$$(a) \underline{\quad} \div 0.001 = \underline{\quad}$$

$$(b) 9 \div 0.02 = \underline{\quad}$$

$$(c) 0.365 \div 18.25 = \underline{\quad}$$

$$(d) 8.8 \div 3.2 = \underline{\quad}$$

Try these problems.

PROBLEMS

In this last example inexperienced students think the problem is finished after the first division because they get a remainder of zero. You must keep dividing at least until all the places up to the decimal point in the answer are filled. The answer here is 50, not 5. ▶

Note ▶

Shifting the decimal point three digits and attaching zeros to the right of the decimal point in this way is equivalent to multiplying both divisor and dividend by 1000.

$$2.6 \div 0.052 = 50 \quad \boxed{A} \quad 0.052 \times 50 = 2.6$$

$$\begin{array}{r} & & 0 \\ & & 0 \\ \overline{52.)} & 260 & \rightarrow \\ 260 & & \\ \hline 50. & & \end{array}$$

$$5 \times 52 = 260$$

Now place the decimal point in the answer space above that in the dividend.

To shift the decimal place three digits in the dividend, we must attach two zeros to its right.

$$\overbrace{0.052)}^{\text{2.6}} \overbrace{2.6}^{\text{0}}$$

SOLUTION

$$2.6 \div 0.052 = ?$$

How would you do this one?

YOUR TURN

Note ▶ In some problems, we may continue to attach zeros and divide yet never get a remainder of zero. We will examine problems like this later. ▶

Double-Check: $2.4 \times 0.575 = 1.38$. Always multiply the answer by the divisor to double-check the work. Do this even if you are using a calculator.

$$1.38 \div 2.4 \text{ is roughly } 1 \div 2 \text{ or } 0.5. \quad \boxed{A}$$

$$\begin{array}{r} & & 0 \\ & & 120 \\ \overline{120)} & 168 & \\ 120 & & \\ \hline 48 & & \\ 48 & & \\ \hline 0 & & \end{array}$$

The remainder is zero. $1.38 \div 2.4 = 0.575$

24 goes into 120 exactly five times.

Attach another zero and bring it down.

24.) 13.800

.575

(e) $7.230 \div 6 =$ _____ (f) $3 \div 4 =$ _____

(g) $30.24 \div 0.42 =$ _____ (h) $273.6 \div 0.057 =$ _____

- (i) **Construction Technology** How many pieces of plywood 0.375 in. thick are in a stack 30 in. high?

SOLUTIONS

(a) $0.001 \overline{)3.500}$

$$\begin{array}{r} 3500 \\ 1.\overline{)3500} \\ \quad 35 \\ \hline \quad 0 \end{array}$$

$3.5 \div 0.001 = 3500$
 $0.001 \times 3500 = 3.5$

(b) $0.02 \overline{)9.00}$ Shift the decimal point two places to the right. Divide 900 by 2.

$$\begin{array}{r} 450 \\ 2.\overline{)900} \\ \quad 90 \\ \hline \quad 0 \end{array}$$

$9 \div 0.02 = 450$
 $0.02 \times 450 = 9$

Dividing a whole number by a decimal is very troublesome for most people.

(c) $18.25 \overline{)0.365}$

$$\begin{array}{r} .02 \\ 1825.\overline{)36.50} \\ \quad 36 \\ \hline \quad 50 \\ \quad 0 \end{array}$$

1825 does not go into 365, so place a zero above the 5. Attach a zero after the 5.

1825 goes into 3650 twice. Place a 2 in the answer space above the zero.

$$0.365 \div 18.25 = 0.02$$

$18.25 \times 0.02 = 0.365$

(d) $3.2 \overline{)8.8}$

$$\begin{array}{r} 2.75 \\ 32.\overline{)88.00} \\ \quad 64 \\ \quad 240 \\ \quad 224 \\ \quad 160 \\ \quad 160 \\ \hline \quad 0 \end{array}$$

$$2 \times 32 = 64$$

Subtract, attach a zero, and bring it down.

$$7 \times 32 = 224$$

Subtract, attach another zero, and bring it down.

$$5 \times 32 = 160$$

$$8.8 \div 3.2 = 2.75$$

The estimated answer is $9 \div 3$ or 3.

Double-check: $3.2 \times 2.75 = 8.8$

(e) $6\overline{)7.230}$

$$\begin{array}{r} 1.205 \\ -6\downarrow \\ \quad 12 \\ -12\downarrow \\ \quad 03 \\ \quad 0\downarrow \\ \quad 30 \\ -30 \\ \hline \quad 0 \end{array}$$

The divisor 6 is a whole number, so we can bring the decimal point in 7.23 up to the answer space.

$1.205 \times 6 = 7.230$

$$7.230 \div 6 = 1.205$$

When doing "mental arithmetic" like this, it is important that you start with a rough estimate of the answer.

Example: $73 \times 20 = 73 \times 2 \times 10 = 146 \times 10 = 1460$

4. To multiply a number by 20, first multiply by 2, then multiply by 10, shifting the decimal point one place to the right.

Example: $86 \div 20 = 86 \div 2 \div 10 = 43 \div 10 = 4.3$

3. To divide a number by 20, first divide by 2, then divide by 10.

Example: $304 \div 25 = 304 \times 4 \div 100 = 1216 \div 100 = 12.16$

2. To divide a number by 25, first multiply by 4, then divide by 100, shifting the decimal point two places left.

Example: $64 \div 5 = 64 \times 2 \div 10 = 128 \div 10 = 12.8$

1. To divide a number by 5, use the fact that 5 is one-half of 10. First, multiply the number by 2, then divide by 10 by shifting the decimal point one place to the left.

ARITHMETIC "TRICKS OF THE TRADE"

If the dividend is not exactly divisible by the divisor, either stop the process after some preset number of decimal places in the answer, or round the answer. We do not generally indicate a remainder in decimal division.

There are 80 pieces of plywood in the stack.

$$\square \quad 80 \times 0.375 = 30$$

$$(i) \quad \begin{array}{r} 0.375 \\ \times 30.000 \\ \hline 3000 \\ 0 \end{array}$$

$$\square \quad 4800 \times 0.057 = 273.6$$

$$(h) \quad \begin{array}{r} 0.057 \\ \times 273.600 \\ \hline 228 \\ 456 \\ \hline 273.600 \end{array}$$

$$\square \quad 72 \times 0.42 = 30.24$$

$$(g) \quad \begin{array}{r} 0.42 \\ \times 30.24 \\ \hline 72 \\ 84 \\ 294 \\ \hline 30.24 \end{array}$$

$$\square \quad 3 \div 4 = 0.75$$

$$(f) \quad \begin{array}{r} 4)3.00 \\ \hline 28 \\ 20 \\ \hline 0.75 \end{array} \quad 0.75 \times 4 = 3.00$$

Rounding Decimal Numbers

Calculations involving decimal numbers often result in answers having more decimal digits than are justified. When this occurs, we must round our answer. The process of rounding a decimal number is very similar to the procedure for rounding a whole number. The only difference is that after rounding to a given decimal place, all digits to the right of that place are dropped. The following examples illustrate this difference.

Round 35782.462 to the

nearest thousand nearest hundredth

Step 1 Place a \wedge mark to the right of the place to which the number is to be rounded.

35 \wedge 782.462 35782.46 \wedge 2

Step 2 If the digit to the right of the mark is less than 5, replace all digits to the right of the mark with zeros.

35782.460

Drop this decimal digit zero.

If these zeros are decimal digits, discard them.

The rounded number is 35782.46

Step 3 If the digit to the right of the mark is equal to or larger than 5, increase the digit to the left by 1 and replace all digits to the right with zeros.

36 000.000

Drop these right-end decimal zeros

Drop all right-end decimal zeros, but keep zero placeholders.

36000

Keep these zeros as placeholders.

The rounded number is 36000.

Careful ►

Drop only the decimal zeros that are to the right of the \wedge mark. For example, to round 6.4086 to three decimal digits, we write 6.408 \wedge 6, which becomes 6.409 or 6.409. We dropped the end zero because it was a decimal zero to the *right* of the mark. But we retained the other zero because it is needed as a placeholder. ◀

YOUR TURN

Try rounding these numbers.

- Round 74.238 to two decimal places.
- Round 8.043 to two decimal places.
- Round 0.07354 to three decimal places.
- Round 7.98 to the nearest tenth.

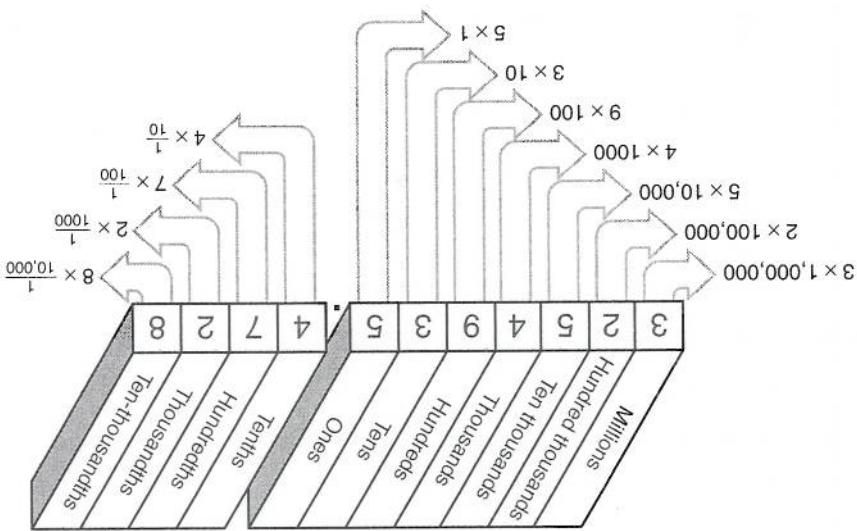
ANSWERS

- 74.238 is 74.24 to two decimal places.
(Write 74.23 \wedge 8 and note that 8 is larger than 5, so increase the 3 to 4 and drop the last digit because it is a decimal digit.)
- 8.043 is 8.04 to two decimal places.
(Write 8.04 \wedge 3 and note that 3 is less than 5 so drop it.)

It is useful to recognize that, for example, the digit 8 represents 8 tens-thousands or $\frac{8}{10,000}$, and the digit 7 represents 7 hundreds or $\frac{7}{100}$. Most often, however, this number is read more simply as "three million, two hundred fifty-four thousand, nine hundred thirty-five, point four, seven, two, eight." This way of reading the number is easiest to write, to say, and to understand.

Notice that the decimal point is read "and" thousandths."

It may be read "three million, two hundred fifty-four thousand, nine hundred thirty-five, and four thousand seven hundred twenty-eight ten-thousandths."



The decimal number 3,254,935.4728 should be interpreted as

HOW TO NAME DECIMAL NUMBERS

Our rule will be quite satisfactory for most of your work in arithmetic.

1. Some engineers use a more complex rule when rounding a number that ends in 5. There are a few very specialized situations where this rounding rule is not used:
2. In business, fractions of a cent are usually rounded up to determine selling price. Three items for 25 cents or $\frac{8}{3}$ cents each is rounded to 9 cents each.

- (Write 7.9 $\sqrt{8}$ and note that 8 is greater than 5, so increase the 9 to 0 and the 7 to 8. Drop the digit in the hundredths place.)
- (Write 0.073 $\sqrt{54}$ and note that the digit to the right of the mark is 5; therefore, change the 3 to a 4 to get 0.074 $\sqrt{00}$. Finally, drop the digits on the right to get 0.074.)
- (c) 0.07354 is 0.074 to three decimal places.
- (d) 7.98 is 8.0 to the nearest tenth.