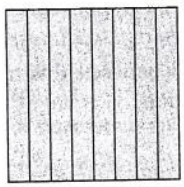
**19.** Consider the following question. Answers vary. 9 times

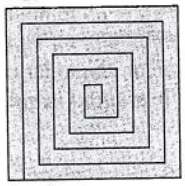
How many times between 4 A.M. and 2 P.M. will the minute hand of a clock pass the hour hand?

Describe the strategy that you used to solve this problem.

**20.** Chu has to cut a lawn that is 60 feet by 60 feet with a lawn mower that cuts a 30-inch strip. He wants to figure out whether he should cut it back and forth in parallel strips or cut it in a spiral pattern in order to minimize the distance he walks with the mower. Answers vary. One strategy is to Draw a Diagram.



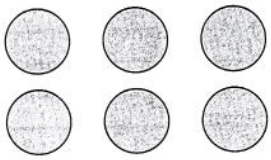
Parallel pattern



Spiral pattern

What strategies would you suggest to Chu to help him solve the problem? Explain your selection. (You need not solve the problem.)

**21.** Place the digits 1, 2, 3, 4, 5, and 6 in the circles shown so that the sum of each adjacent pair of circles (both horizontally and vertically) is different. There are seven adjacent pairs of circles. Row 1 = 1 2 3 and row 2 = 5 6 4 is a solution to the problem.



What strategy did you use in solving this problem?

**22.** Suppose that you wanted to find the whole numbers represented by each of the letters in the following addition problem:

$$\begin{array}{r} S\ E\ N\ D \\ +\ M\ O\ R\ E \\ \hline M\ O\ N\ E\ Y \end{array}$$

Which problem-solving strategy might be most helpful to you in solving this problem? (You need not solve the problem.) Answers vary. Probably Guess–Check–Review is the most popular strategy.

## C. Reasoning and Problem Solving

**23.** Create two problems based on the following situation:

Nathan is considering buying a new 42-inch plasma screen TV that costs $3,000. The bracket to put TV on the wall costs $200. If he hires someone from the store to install the TV, the installation will cost $500 but he would get a 10% discount on the TV. Answers vary.

**24.** Form a small group of classmates and exchange the problems you created in Exercise 23. Solve each others' problems. Answers vary.

**25.** Solve the problems stated in Exercises 7–12.

**26.** Write a paragraph or two telling which problem you solved in Exercise 25 was easiest and which was most difficult. Explain your reasons for each answer.

**27.** Consider the following problem:

There are five more children in Ms. Brown's room than in Mr. White's room. Ten children move from Ms. Brown's room to Mr. White's room. Now there are twice as many children in Mr. White's room as in Ms. Brown's room. How many children were in Ms. Brown's room before the move?

Suppose that Angela tries to solve the problem by making the table below. She is discouraged because this method doesn't seem to be generating a solution. What advice would you give her for solving the problem? Answers vary.

**Table for Exercise 27**

| Brown | White | Brown | White | Brown | White | Brown | White |
|---|---|---|---|---|---|---|---|
| 16 | 11 | 17 | 12 | 18 | 13 | 19 | 14 |
| 6 | 21 | 7 | 22 | 8 | 23 | 9 | 24 |

**28. The Baby Problem.** Use two different strategies to solve the following problem:

According to a book of records, a woman gave birth 40 times, bearing a total of 53 children. Eight of the births were twins or triplets. How many single babies, twins, and triplets were born? 32 singles, 3 sets of twins, 5 sets of triplets.

Describe the strategies. Strategies vary.

**29. The Balance Problem.** Eight balls all look alike. One of them is heavier than the other seven, all of which weigh the same. How can you find the heavier ball if you use a balance scale only two times? Answers vary.

**30. a.** Use a computer spreadsheet to explore the problem in Example 1.12. (*Hint:* Set up columns for Brenda, Tina, and Gene.) Answers vary.

**b.** Write a paragraph describing how you used spreadsheet formulas for the number of Brenda's slices in order to calculate the number of slices eaten by Tina and Gene.

## D. Communicating and Connecting Ideas

**31.** Consider the following problem:

A rectangular garden 16 meters wide and 20 meters long is to be fenced. Posts should be set no more than 4 meters apart. How many posts are needed? Answers vary.

Create a series of questions based on the problem-solving model that you think would be helpful to someone trying to solve the problem.

32. Work with a partner to solve the following problem:

    A beetle is at the bottom of a 10-foot well. Each day the beetle climbs up 5 feet. But at night when it sleeps, the beetle slips back 4 feet. On what day (fourth, fifth, and so on) will the beetle get out of the well? Sixth day

    Now compare your strategy for solving the problem with that of other pairs of students. In what way are the strategies the same? different?

33. **Making Connections.** Consider the following two problems: (a) 10 (b) Answers vary.

    Five people come to a party and shake hands with each other. If each person shakes the hand of every other person, how many handshakes occur?

    If each of five dots on a circle is connected to each of the other dots with a line, how many lines will be drawn?

    **a.** Solve the two problems.
    **b.** Write a short paragraph about the connection(s) that you see between the problems.

34. **Historical Pathways.** Henderson and Pingry (1953) published a well-known article, "Problem Solving in Mathematics," in which they describe the following three steps for the "pre-solution period of problem solving": Answers vary.
    **a.** orientation to the problem,
    **b.** producing relevant ideas for solving the problem,
    **c.** forming and testing hypotheses for solving problems.

    Describe the relationship between this model and the one presented in this section.

## Chapter Summary

### Reflections On The Big Ideas And Essential Understandings: Questions and Answers

#### SECTION 1.1

- **What role do communicating and representing play in mathematics?** (*pp. 2–4*) People communicate through the use of words, symbols, and graphs that represent various mathematical ideas in a variety of ways.
- **What role does technology play in communicating and representing mathematical ideas?** (*pp. 4–9*) Technology allows the exploration of mathematical topics that would otherwise be difficult. The use of computer spreadsheets, various types of calculators, and geometry exploration software are some of the technologies available for mathematical exploration and problem solving.

#### SECTION 1.2

- **What is inductive reasoning and how can it be used to find patterns?** (*pp. 13–23*) Inductive reasoning involves the use of information from specific examples to draw a general conclusion, called a generalization. You can use inductive reasoning to find patterns involving examples from numerical or geometric sequences, tables of real-world data, or sequences of number sentences.
- **What is deductive reasoning?** (*pp. 23–30*) Deductive reasoning involves drawing conclusions from given true statements by using rules of logic.

#### SECTION 1.3

- **What is the role of problem solving in mathematics?** (*pp. 40–43*) Mathematics is a process of solving problems, which emphasizes the notion that mathematics is a method of thinking. Problem solving is the key to doing mathematics.
- **Is there a problem-solving model?** (*pp. 43–47*) A model for problem solving exists only in the sense that various strategies and skills for solving problems can be very helpful. Essentially, this approach involves understanding the problem, developing a plan, implementing the plan, and looking back. Within these four phases of problem solving, specific strategies and problem-solving skills can be identified.
- **How are problem-solving strategies used?** (*pp. 47–49*) Problem solving involves considering the range of possible strategies and then determining which strategy or strategies might be best for solving a particular problem.

### Terms, Concepts, and Generalizations

#### SECTION 1.1

Mathematical communication (p. 2)
Technology (p. 4)
Spreadsheet (p. 5)
Calculators (p. 7)
Geometry exploration software (p. 8)

#### SECTION 1.2

Inductive reasoning (p. 14)
Generalization (p. 14)

Counterexample (p. 16)
Patterns (p. 17)
Sequence (p. 17)
Terms of the sequence (p. 17)
Arithmetic sequence (p. 18)
Common difference (p. 18)
Geometric sequence (p. 18)
Common ratio (p. 18)
Table of values (p. 20)
Column extension pattern (p. 20)
Fibonacci sequence (p. 20)
Row relationship pattern (p. 20)
Statement (p. 23)
Negation (p. 24)
Conditional statements (p. 24)
Hypothesis (p. 24)
Conclusion (p. 25)
Truth of conditional statement (p. 26)
Rules of logic (p. 27)
Affirming the hypothesis (p. 27)
Modus ponens (p. 27)
Denying the conclusion (p. 27)
Modus tollens (p. 27)
Deductive reasoning (p. 28)
Assuming the converse (p. 29)
Assuming the inverse (p. 29)
Converse (p. 30)
Inverse (p. 31)
Contrapositive (p. 31)
Logically equivalent statement (p. 31)
Conjunction (p. 32)
Disjunction (p. 33)
Biconditional statement (p. 34)

**SECTION 1.3**
Problem (p. 40)
Problem solving (p. 41)
Answer (p. 42)
Solution (p. 42)
Problem-solving skills (p. 43)
Problem-solving model (p. 43)
Problem-solving strategies (p. 44)
Estimation (p. 45)

## Chapter Review

A ☛ icon indicates the answer is at the back of the book.

### Concepts and Skills

1. Choose two symbols and two words that you have used in mathematics and explain what they represent. ☛
2. What formulas would you have to enter to instruct the computer to calculate the numbers in cells C2 and C3 in the spreadsheet shown? ☛

| | A | B | C |
|---|---|---|---|
| 1 | Number of Items | Cost per Item (12) | Total Cost ($) |
| 2 | 7 | 8 | 56 |
| 3 | 9 | 12 | 108 |

3. Produce a counterexample to each of the following conditional statements: ☛
   a. If 10 is divided by another number, then the result will be less than 10.
   b. If a geometric figure is a rectangle, then it is a square.
   c. If a number is multiplied by 7, then the product is an odd number.
4. Give the hypothesis and the conclusion in each of the following statements: ☛
   a. If Morgan Freeman gives a good performance, then he will win the Oscar.
   b. An actress will be in high demand if she wins an Oscar.

*State the appropriate conclusion in Exercises 5 and 6 and whether the reasoning is affirming the hypothesis or denying the conclusion.*

5. If a four-sided geometric figure has three right angles, then its fourth angle is a right angle. In figure *ABCD*, angles *A*, *B*, and *D* are right angles. Therefore . . . ☛
6. If you lay in the hot sun for 4 hours, you will become sunburned. Nicole is not sunburned. Therefore . . . ☛
7. Use the following conditional statement: If a person lives in Austin, then the person lives in Texas. ☛
   a. Create a valid reasoning situation in which affirming the hypothesis is used.
   b. Create a valid reasoning situation in which denying the conclusion is used.
   c. Create an invalid reasoning situation in which the converse is assumed.
   d. Create an invalid reasoning situation in which the inverse is assumed.
8. Create a truth table using $p$, $q$, and $p \rightarrow q$ and describe a situation in which each of the following statements is false: ☛
   a. If a person lives in Texas, then the person lives in Austin.
   b. I can't go to the game if I spend my money for concert tickets.
9. Suppose $p$ is true. Argue whether the conjunction of $p$ and its negation is true or false. ☛
10. Suppose $p$ is false. Argue whether the disjunction of $p$ and its negation is true or false. ☛
11. Consider the following false conditional statement: ☛

    If you live in California, then you live in Los Angeles.

    Are any of the converse, inverse, or contrapositive statements true? Write the statements.
12. If Ellen works overtime on Sunday or Wednesday night and Thursday night, she will earn enough money to buy a tennis racket. Does this give her two or three chances to earn enough money? Explain your reasoning. ☛
13. Study the following sequences: ☛
    i. 22, 33, 44, 55, 66, 77, 88, 99, . . .
    ii. 1, 11, 12, 22, 23, 33, 34, 44, . . .

iii. 2, 6, 18, 54, 162, 486, 1458, . . .

a. Classify each sequence as arithmetic, geometric, or neither.

b. Give the next three terms in each sequence.

14. Give the next statement in each sequence of statements: ☛

a. $1 \div 0.5 = 2$. $2 \div 0.5 = 4$. $3 \div 0.5 = 6$. $4 \div 0.5 = 8$. $5 \div 0.5 = 10$. $6 \div 0.5 = 12$. ?

b. $5 \times 4 = 20$. $5 \times 14 = 70$. $5 \times 24 = 120$. $5 \times 34 = 170$. $5 \times 44 = 220$. $5 \times 54 = 270$. ?

15. Consider the following table of values. ☛

a. Look for a column extension pattern and write three more rows for the table.

b. Look for a row relationship pattern and show the 25th row of the table. (*Hint:* Thinking about doubling the number of items might help you determine a row relationship pattern.)

| Time (weeks) | Growth of Plant (inches) |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |
| 5 | 9 |
| 6 | 11 |

16. Name five problem-solving strategies that are frequently used. Answers vary.

17. Consider the following problem: ☛

A concert ticket and parking cost $90. If the concert ticket costs four times the cost of parking, how much does each cost?

a. What problem-solving strategy would you use to solve the problem?

b. Use the strategy to obtain the answer.

18. Give a verbal, visual, numerical, and graphic representation of the idea *three-fourths.* ☛

19. What connection do you see between the following two problems? ☛

a. How many different bike rides can be taken by four people if everyone rides only once with everyone else?

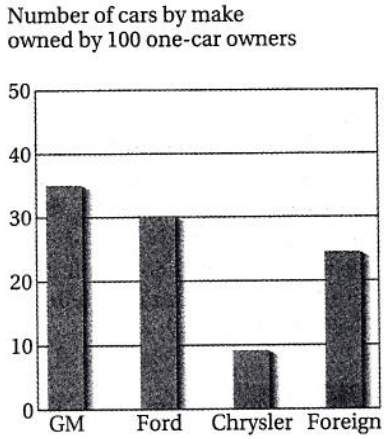
b. How many computer cables are needed to connect four desks so that each pair of desks is connected by only one cable?

20. Describe two different ways that you could represent the number 0.80. ☛

21. What geometric model would you use to represent three highways that intersect pairwise in three different cities? Why? Answers vary.

## Reasoning and Problem Solving

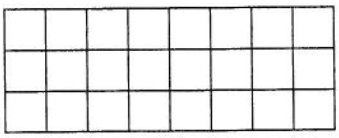
22. Write a short paragraph that communicates the message of the following bar graph: Answers vary.



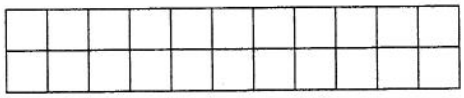

23. Create a graph different from the one above that could represent the same information about cars. Answers vary.

24. Consider the three following rectangles: ☛

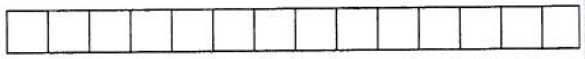
i.



ii.



iii.



a. For each rectangle, determine the area (number of squares) and the perimeter (distance around).

b. What happens to the areas as you go from i to ii to iii?

c. What happens to the perimeters as you go from i to ii to iii?

d. Write a statement that describes the connection between the perimeters and areas of these three rectangles.

25. Consider the following ways to communicate directions for drawing a certain figure: ☛

i. Draw a square. Draw a circle inside the square.

ii. Draw a square. Draw a circle inside the square that touches each side of the square.

iii. Draw a square. Draw a circle inside the square that touches the square.

a. In what way are the directions the same?

b. In what way might the directions result in the drawing of different figures? Show the resulting figures.

26. **The Double Cheeseburger Problem.** Fast Fried Foods has a special on double cheeseburgers. The first double cheeseburger costs \$3, the second double cheeseburger costs \$2, and any other cheeseburgers purchased by the same person cost \$1.50 each. Mrs. Cooper wants to buy 4 double cheeseburgers for her family and a \$1.00 soft drink for each double cheeseburger purchased. Tax on the purchase is 7%. How much change will she receive if she gives the cashier \$20? \$7.16

27. **The Tennis Problem.** A store that sells tennis clothing offers the following discount plan. If you pay \$100 up front, you may purchase any of the shirts or shorts for \$20. If the shirts or shorts all cost \$40 each, how many would you have to buy in order to break even? Explain your strategy. ☛

28. Give a convincing argument that the advertisement encouraged the reader to engage in invalid reasoning.

    *Advertisement:* If you want your skin to stay smooth, use Wrinkleflee cream morning and night. People who use Wrinkleflee treat their skin with loving care.

    *Reader's interpretation:* If you use Wrinkleflee cream morning and night, then your skin will stay smooth, but if you don't use Wrinkleflee, then you don't treat your skin with loving care. Answers vary.

## Alternative Assessment

29. Study the problem-solving model in Figure 1.6 *(p. 43)* and write a paragraph generally explaining the connection between the *problem-solving process*, *problem-solving skills*, and *problem-solving strategies*. Answers vary.

30. Work in a small group to decide the best way to explain to someone the difference between inductive and deductive reasoning. ☛

31. Caitlin has solved the following problem (Exercise 8 in Section 1.3).

    Bella is in a race with 20 other racers. Bella is fifth from last when the race begins. At the end of the race she is third. How many racers did she pass? ☛

    She obtained an answer of 14 racers, which she found out was the correct answer. However, Caitlin thinks that this answer depends on assumptions made about the race. For example, she claims that the number of times one runner can pass another and who finishes first and second make a difference. Consider her claim and discuss how these assumptions might affect the solution to the problem.

32. Make a table for the following problem:

    An underwater search expert took 14 days to find an old sunken treasure. The person who hired the expert agreed to pay him by putting \$1 in a safe the first day, \$2 in the safe the second day, \$4 the third day, \$8 the fourth day, and so on, doubling the amount put in the safe each day. When the expert opened the safe after 14 days, how much money did he get for the job? Answers vary.

    a. Use this problem as an example to help you write an explanation of what is meant by *column extension patterns* and *row relationship patterns*.

    b. In your written response to part (a), include a discussion of the advantages and disadvantages of the two types of table patterns for solving the problem.