SOLUTION

a. Using logic rule B, we can conclude that today is not Saturday.

b. Using logic rule A, we can conclude that square $ABCD$ is a rhombus.

YOUR TURN

Practice: Give your conclusion for each of the following conditional statements and indicate which rule of logic you used:

a. If two numbers are odd, the product of the numbers is odd.
The product of these two numbers is not odd.

b. If you send me a letter, then I will send you a letter.
You sent me a letter.

Reflect: Use your responses to parts (a) and (b) to explain how logic rule A differs from logic rule B. ■

Description of Deductive Reasoning

Deductive reasoning involves drawing conclusions from given true statements by using rules of logic.

Consider the following example of the use of deductive reasoning. Your counselor says, "If you achieve at least a 3.0 grade average, then you will be admitted to the program." You work hard and get a 3.2 grade average. Using deductive reasoning, you conclude that you will be admitted to the program. If someone asks how you know that you will be admitted to the program, you support your conclusion by telling her about the true statement made by your counselor and your logical conclusion. We can use this example to illustrate the steps in the deductive reasoning process.

Procedure for Using the Deductive Reasoning Process

■ Start with a true statement, often in if–then form.	If you achieve at least a 3.0 grade average, then you will be admitted to the program.
■ Note given information about the truth or falsity of the hypothesis or conclusion.	You achieve a 3.2 grade average. The hypothesis is true.
■ Use a rule of logic to determine the truth or falsity of the hypothesis or conclusion.	You use a rule of logic to conclude that the conclusion is true. You will be admitted to the program.

In the deductive reasoning process, you might use two or more rules of logic in sequence to arrive at a final conclusion. Discussion of these more detailed uses of deductive reasoning is beyond the scope of this book. Mini-Investigation 1.6 will help you think more carefully about the process of deductive reasoning and identify some misuses of this process.

Talk about the reasons for your conclusions.

MINI-INVESTIGATION 1.6 Using Mathematical Reasoning

An instructor said that if a student gets an A on the final examination, then the student will get an A in the course. Kaya did not get an A on the final. Ted did get an A in the course. What can you conclude about Ted and Kaya?

You must be careful to use accepted rules of logic, such as logic rule A and logic rule B, when using deductive reasoning. Boxes C and D use an easily understood situation to show two invalid reasoning patterns that are often used.

Invalid Reasoning C

- If p, then q (true) → If you live in Chicago (p), then you live in Illinois (q).
- q is true → You live in Illinois.
- Therefore p is true → You live in Chicago. (Invalid)

The invalid reasoning in box C is called **assuming the converse** and might have been used to draw the incorrect conclusion in Mini-Investigation 1.6 that Ted got an A on the final examination.

Invalid Reasoning D

- If p, then q (true) → If you live in Chicago (p), then you live in Illinois (q).
- p is false → You do not live in Chicago.
- Therefore q is false → You do not live in Illinois. (Invalid)

The invalid reasoning in box D is called **assuming the inverse** and might have been used to draw the incorrect conclusion in Mini-Investigation 1.6 that Kaya did not get an A in the course. Example 1.8 will help you learn to recognize invalid reasoning.

Example 1.8 Determining When Reasoning Is Valid or Invalid

Determine whether the reasoning is valid or invalid. If invalid, indicate whether it is based on assuming the converse or assuming the inverse.

a. Timothy's little sister knew that if a vehicle was a car, then it had four wheels. When given a toy wagon with four wheels, she concluded that it was a car.

b. Dan read that "if you play great basketball, then you wear Jumpup shoes." Cedric did not play great basketball, so Dan concluded that Cedric did not wear Jumpup shoes.

SOLUTION

a. The reasoning is invalid (assuming the converse).

b. The reasoning is invalid (assuming the inverse).

YOUR TURN

Practice: Indicate whether the reasoning in each case is valid or invalid. If invalid tell whether it is based on assuming the converse or assuming the inverse.

a. Jordan's coach said, "If you win the most tryout matches, then you will make the number one tennis team." Jordan did not make the number one team. So Jordan concluded that she had not won the most tryout matches.

b. Jackie read that if a dog is of show quality, then it has registration papers. A friend's dog, Newton, had registration papers, so Jackie concluded that Newton was of show quality.

Reflect: Explain how you decided whether the reasoning was valid or invalid. ■

The Converse, Inverse, and Contrapositive of a Conditional Statement. Consider the following conditional statement:

If a figure is square, then it has four right angles.

The hypothesis of the statement is "a figure is a square" and the conclusion is "it has four right angles." If we interchange the hypothesis and the conclusion, we obtain the following statement.

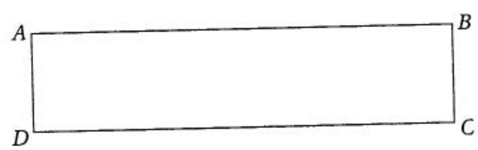
If a figure has four right angles, then it is a square.

This conditional statement is the **converse** of the original conditional statement.

Definition of Converse Statement

The **converse** of a conditional statement is formed by interchanging the hypothesis and the conclusion. That is, the converse of $p \rightarrow q$ is $q \rightarrow p$.

Note that the converse of a statement is not necessarily true even though the original statement is true. For example, figure *ABCD* has four right angles but is not a square.



Now consider the statement in which the hypothesis and the conclusion are both negated:

If a figure is not a square, then it does not have four right angles.

This conditional statement is the **inverse** of the original conditional statement.

Definition of Inverse Statement

The **inverse** of a conditional statement is formed by negating both the hypothesis and the conclusion. That is, the inverse of $p \to q$ is $\sim p \to \sim q$ (read "not p implies not q").

As is the case with the converse of a statement, the inverse of a true statement is not necessarily true. For example, figure $ABCD$ is not a square but it does have four right angles.

When the hypothesis and the conclusion are both interchanged and negated we have the **contrapositive** of the original conditional statement:

If a figure does not have four right angles, it is not a square.

Definition of Contrapositive Statement

The **contrapositive** of a conditional statement is formed by interchanging the hypothesis and the conclusion and negating both. That is, the contrapositive of $p \to q$ is $\sim q \to \sim p$.

Unlike the converse and the inverse statements, the contrapositive statement is necessarily true, assuming the original statement is true. For example, if a figure does not have four right angles, then it can not be a square.

The following truth table gives the truth values for p, q, $\sim q$, $\sim p$, $p \to q$, $\sim q \to \sim p$:

p	q	$\sim q$	$\sim p$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

Note that $\sim q \to \sim p$ is the contrapositive of $p \to q$ and that these two conditional statements have identical truth values. This demonstrates that the contrapositive of a given conditional statement has the same truth value as the original conditional statement. If the original conditional statement is true, then its contrapositive is also true.

In general, if two conditional statements have the same truth value, the two statements are **logically equivalent.**

Definition of Logical Equivalence

Two statements that have the same truth value are said to be **logically equivalent.**

We can then say that a statement and its contrapositive are logically equivalent.

Example 1.9 Forming Converse, Inverse, and Contrapositive Statements

Given the following conditional statement, form the converse, inverse, and contrapositive statements and determine whether or not each statement is true.

If you are President of the United States, then you are an American citizen.

SOLUTION

Converse: If you are an American citizen, then you are President of the United States. (not true)

Inverse: If you are not President of the United States, then you are not an American citizen. (not true)

Contrapositive: If you are not an American citizen, then you are not President of the United States. (true)

YOUR TURN

Practice: Create a true conditional statement and write the converse, inverse, and contrapositive statements. Are any of these statements true?

Reflect: If the converse of a conditional statement is true, must the inverse statement be true? Explain. ■

Conjunctions, Disjunctions, and Biconditional Statements. In mathematics we sometimes combine two statements using the word *and*, which results in what is called a **conjunction**. Consider the following statements:

p: The diagonals of a square are congruent.

q: The diagonals of a square are perpendicular.

Conjunction of p and q: The diagonals of a square are congruent and perpendicular.

The third statement is a **conjunction** formed by combining the first two statements.

Definition of a Conjunction

Given two statements p and q, the statement "p and q" (denoted $p \wedge q$) is a **conjunction.**

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The table in the margin establishes the truth of the conjunction $p \wedge q$. In other words, a conjunction $p \wedge q$ is true only when both p and q are true.

Sometimes two statements are combined by the word *or*. Consider the following statements:

p : Six is an even number.

q : Seven is an even number.

Disjunction of p *and* q: Either six is an even number or seven is an even number.

We define a **disjunction** in the following way:

Definition of a Disjunction

Given any two statements p and q, the statement "p or q" (denoted $p \vee q$) is a **disjunction.**

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The table in the margin establishes the truth of the disjunction $p \vee q$.

A disjunction $p \vee q$ is true when either or both of p and q are true.

Example 1.10 Creating Conjunctions and Disjunctions

Create a conjunction and a disjunction for each of the following pairs of statements. Indicate whether the compound statements are true or false.

The number 5 is prime.
The number 5 is odd.
The diagonals of a rectangle are congruent.
The diagonals of a rectangle are perpendicular.
The product of two odd numbers is even.
The product of two odd numbers is always greater than 4.

SOLUTION

Conjunction: The number 5 is prime and odd. (true)
Disjunction: The number 5 is prime or odd. (true)
Conjunction: The diagonals of a rectangle are congruent and perpendicular. (false)
Disjunction: The diagonals of a rectangle are congruent or perpendicular. (true)
Conjunction: The product of two odd numbers is even and always greater than 4. (false)
Disjunction: The product of two odd numbers is even or always greater than 4. (false)

YOUR TURN

Practice: Create two statements for which the conjunction is false and the disjunction is true. Explain why this is possible.

Reflect: Explain why it is impossible to have two statements for which the conjunction is true and the disjunction is false. ■

A compound statement of the form $p \leftrightarrow q$ is a conjunction of two statements: $p \rightarrow q$ and $q \rightarrow p$. Such compound conditional statements are called **biconditional statements.** Biconditional statements can also be written "p if and only if q," as illustrated in the following examples. Each example's two implied conditional statements are also given.

A quadrilateral is a rectangle if and only if it has four right angles.
If a quadrilateral is a rectangle, then it has four right angles.
If a quadrilateral has four right angles, then it is a rectangle.

A triangle has three congruent sides if and only if it has three congruent angles.
If a triangle has three congruent sides, then it has three congruent angles.
If a triangle has three congruent angles, then it has three congruent sides.

A number is odd if and only if its square is odd.
If a number is odd, then its square is odd.
If the square of a number is odd, then the number is odd.

Definition of a Biconditional Statement

A **biconditional statement** $p \leftrightarrow q$ is a conjunction of $p \rightarrow q$ and its converse $q \rightarrow p$.

The following truth table establishes the conditions in which a biconditional statement is true:

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Hence, a biconditional statement $p \rightarrow q$ is true when both p and q are true or both are false. The following two biconditional statements are both false because in each case either p or q is false:

A man lives in Ohio if and only if he lives in Cleveland. (false)
If a man lives in Ohio, then he lives in Cleveland. (false)
If a man lives in Cleveland, then he lives in Ohio. (true)

A number is negative if and only if it is not positive. (false)
If a number is negative, then it is not positive. (true)
If a number is not positive, then it is negative. (false, the number could be zero)

Example 1.11 **Analyzing Biconditional Statements**

Write the two conditional statements implied by the following biconditional statements. Indicate whether the biconditional statements are true or false.

a. A quadrilateral is a square if and only if it has four congruent sides.
b. A whole number is even if and only if it is divisible by 2.

SOLUTION

a. If a quadrilateral is a square, then it has four congruent sides. (true)
If a quadrilateral has four congruent sides, then it is a square. (false)
The given biconditional statement is false.

b. If a whole number is even, then it is divisible by 2. (true)
If a whole number is divisible by 2, then it is even. (true)
The given biconditional statement is true.

YOUR TURN

Practice: Write a biconditional statement and identify the two implied conditional statements that make up the biconditional statement.

Reflect: If both implied conditional statements of a biconditional statement are false, is the biconditional statement false? Explain. ■

Connection to the PreK–8 Classroom

One of the most important outcomes in teaching elementary school mathematics is for students to develop confidence in their ability to reason mathematically. Students should base their belief in mathematical statements on the reasoning process—not because someone says that something is true. Thus teachers need to emphasize reasoning to help them develop that skill.

Problems and Exercises for Section 1.2

A. Reinforcing Concepts and Practicing Skills

1. Use inductive reasoning and state a generalization regarding the following number sentences, which are true, about adding zero: Any number plus zero is equal to that number.

   $3 + 0 = 3$; $4 + 0 = 4$;

   $376 + 0 = 376$; $8{,}000 + 0 = 8{,}000$.

Produce a counterexample for each of the generalizations in Exercises 2–5. Answers vary.

2. If a number is even, then it ends in 2. ☛
3. If a figure has four sides, then it is a square.
4. If a figure has two right angles, then it is a rectangle.
5. If an even number is divided by an even number, the quotient is even. 24 divided by 8 is 3.
6. For which type of sequence is each term found by ☛
   a. adding a common difference to the preceding term?
   b. multiplying the preceding term by a common ratio?

Find the missing numbers in the sequences in Exercises 7–15; indicate whether the sequences are arithmetic, geometric, or neither; and give the common difference or ratio when appropriate.

7. 2, 6, 10, 14, 18, ____, ____, ____ ☛
8. 1, 2, 4, 8, 16, ____, ____, ____ ☛
9. 1, 4, 9, 16, 25, ____, ____, ____ 36, 49, 64: neither
10. 2, 4, 6, 10, 16, ____, ____, ____ 26, 42, 68: neither
11. 1, 3, 7, 15, 31, ____, ____, ____ 63, 127, 255: neither
12. 81, 27, 9, 3, ____, ____, ____ $1, \frac{1}{3}, \frac{1}{9}$: geometric ($r = \frac{1}{3}$)
13. 1, 2, 4, 7, ____, ____, ____ 11, 16, 22: neither
14. 1, 5, 9, 13, ____, ____, ____ ☛
15. 25, 21, 17, 13, ____, ____, ____ ☛
16. Consider the following example of inductive reasoning:
    The numbers 4, 9, 16, 25, 36, and 49 are all the result of squaring a number. Therefore every number squared ends in 4, 5, 6, or 9. Answers vary. One possibility: 121 is 11 squared and it ends in 1.

Find several counterexamples that show this conclusion to be false.

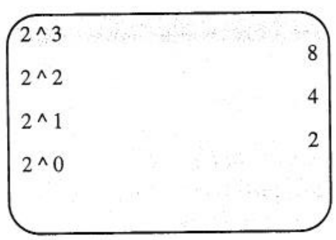
17. Give a key characteristic of each type of reasoning:
 a. Inductive reasoning
 b. Deductive reasoning Answers vary.

In Exercises 18–20, form the negation of the given statements and indicate whether the statements and their negations are true or false.

18. A car driving 90 mph on Interstate 75 in Georgia would be speeding. ☛
19. Every square is a rectangle. ☛
20. Every rectangle is a square. ☛

Give the hypothesis and the conclusion for each of the statements in Exercises 21–24.

21. If an even number is multiplied by an odd number, then the product is an even number. ☛
22. A figure is a quadrilateral if it has four connected sides.
23. If there is a high demand for a product, then its price will increase. ☛
24. A number is even if its square is even. ☛
25. Julie's calculator screen is shown. (Recall that 2 ∧ 3 represents 2^3.) Julie thinks that when she presses ENTER, a 1 will appear. What type of reasoning is Julie using? Explain. inductive reasoning



26. Suppose Jack says that if the temperature is below 50°F, he won't play tennis. Determine which of the following cases, if any, indicate that Jack did not tell the truth: ☛
 a. The temperature was below 50°, and he didn't play tennis.
 b. The temperature was below 50°, and he did play tennis. Jack didn't tell the truth in statement (b).
 c. The temperature was above 50°, and he didn't play tennis.
 d. The temperature was above 50°, and he did play tennis.
27. Create a truth table using p, q, and $p \rightarrow q$ and describe the situation in which the following statement is false:

 If Serena wins the second set of her tennis match, she will win the match. ☛

28. Describe two different situations in which the following statement could be true. ☛

 Salaries will increase if inflation remains constant.

29. Form a conjunction from the following two statements. Argue whether the conjunction is true or false. ☛

 A right triangle has one right angle.
 A right triangle has two acute angles.

30. Form a conjunction from the following two statements. Argue whether the conjunction is true or false. ☛

 A rectangle has four right angles.
 A rectangle has four congruent sides.

31. Form a disjunction from the following two statements. Argue whether the disjunction is true or false. ☛

 Nine is a prime number.
 Nine is an odd number.

32. Form a disjunction from the following two statements. Argue whether the disjunction is true or false. ☛

 Six is a multiple of three.
 Six is a multiple of twelve.

33. Write two statements implied by the following biconditional statement: ☛

 A triangle has three congruent sides if and only if it has three congruent angles.

34. Write two statements implied by the following biconditional statement. ☛

 A number is divisible by 10 if and only if the number ends in zero.

For Exercises 35–39, indicate whether the deductive reasoning used is an example of affirming the hypothesis or denying the conclusion.

35. If a number is a multiple of 10, then the square of the number is a multiple of 100. (Joe's age)^2 is not a multiple of 100. Therefore Joe's age is not a multiple of 10. denying the conclusion
36. If a four-sided figure has four right angles, it is a rectangle. Shape *ABCD* has four right angles. Therefore shape *ABCD* is a rectangle. affirming the hypothesis
37. If a woman lives in Atlanta, then she doesn't live in Chicago. Samantha lives in Chicago. Therefore Samantha doesn't live in Atlanta. denying the conclusion
38. If a man is a citizen of the United States, then he was born in the United States. Sam was not born in the United States. Therefore Sam isn't a citizen of the United States. denying the conclusion
39. If a teacher is a high school teacher, then she teaches English. Alice is a high school teacher. Therefore Alice teaches English. affirming the hypothesis

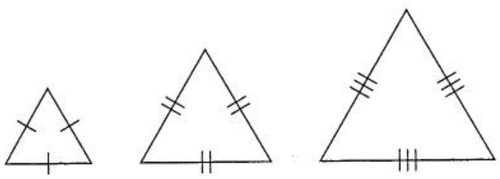
Classify the following types of reasoning in Exercises 40–43 as deductive, inductive, or neither of these:

40. The numbers 10, 20, 30, and 40 all end in zero and are divisible by 5. Therefore all numbers that end in zero are divisible by 5. inductive reasoning

41. A diagonal of a square forms two right triangles. *ABCD* is a square with diagonal *AC*. Therefore triangles *ABC* and *ADC* are both right triangles. deductive reasoning

42. All triangles have three sides. Geometric figure *ABC* is a triangle. Therefore $AB = BC = AC$. ☛

43. The three triangles shown are equilateral. Each angle of each triangle measures 60°. Hence the angles of all equilateral triangles measure 60°. inductive



State whether the reasoning in Exercises 44–47 is valid or invalid. If invalid, indicate whether it is assuming the converse or assuming the inverse.

44. If a triangle is equilateral, then all its sides are congruent. Triangle *ABC* is equilateral. Therefore all sides of triangle *ABC* are congruent. valid

45. If Beth lives in Idaho, then Beth lives in the United States. Beth lives in the United States. Therefore Beth lives in Idaho. invalid; assuming the converse

46. If a number ends in 5, then it is an odd number. Mary is a teenager whose age is an odd number. Therefore Mary is 15 years old. invalid; assuming the converse

47. If Sara makes a 90 or better on her last math test, then she is guaranteed an A in her math class. Sara made less than 90 on her last math test. Therefore Sara is not guaranteed an A in her math class. invalid; assuming the inverse

For Exercises 48–50, use the following true conditional statement:

If a number is positive, then its square is positive.

48. State the converse. Is it true? ☛

49. State the inverse. Is it true? ☛

50. State the contrapositive. Is it true? ☛

51. Write two conditional statements that are implied by the following statement: ☛

A figure is a rhombus if and only if it has four congruent sides.

For Exercises 52 and 53, use the following true conditional statement:

If a four-sided figure has equal sides and equal angles, then it is a square.

52. If a four-sided figure has equal sides but no equal angles, is it necessarily a square? No.

53. If a four-sided figure has equal angles but not equal sides, is it necessarily a square? No.

54. Consider the following true conditional statement:

The product of two numbers is positive if both numbers are positive or both numbers are negative.

Under what conditions would the product not be positive? ☛

55. Statements p and q are both true. What is the truth value of the conjunction $p \wedge \sim q$? Explain. ☛

56. Statements p and q are both true. What is the truth value of the disjunction $p \vee \sim q$? Explain. ☛

57. Suppose $p \rightarrow q$ is a true conditional statement and $q \rightarrow p$ is a false conditional statement. Will the biconditional statement $p \leftrightarrow q$ be true or false? Explain. ☛

58. Create a truth that shows $p \rightarrow \sim q$ and $q \rightarrow \sim p$ are logically equivalent. ☛

59. Create a truth table that shows $\sim p \rightarrow q$ and $\sim q \rightarrow p$ are logically equivalent. ☛

60. Determine a pattern in the following number sentences:

$$1 \times 12 = 12.$$
$$11 \times 12 = 132.$$
$$111 \times 12 = 1332.$$
$$1111 \times 12 = 13332.$$

1333332

Based on this pattern, indicate the product for 111111×12. Use a calculator to check your work.

61. Consider the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . . Consider the following products and where the numbers come from in the sequence:

1×3 and 1×2
1×5 and 2×3
2×8 and 3×5
3×13 and 5×8

What pattern do you observe in these products? Does this pattern continue with other numbers in the sequence? Demonstrate that this is or is not the case. ☛

62. In the following products note how the boldfaced numbers are formed and how the underlined numbers are formed.

15	25	35	45	55
×15	×25	×35	×45	×55
225	**6**25	**1,2**25	**2,0**25	**3,0**25

Following this pattern, what do you think will be the products of 65×65, 75×75, and 85×85? Use a calculator to see if your conclusions are correct. ☛

B. Deepening Understanding

63. Create a mathematical situation involving deductive reasoning that leads to the following conclusion: The sides of geometric figure *ABCD* are all the same length. Answers vary.

64. Jennifer considered the numbers 4, 24, 44, and 64 and noticed that they all ended in 4 and that they can be divided evenly by 4, leaving no remainder. She concluded that all numbers that end in 4 can be divided evenly by 4. What type of reasoning did Jennifer use? Is her conclusion correct? Explain your answer. ☛

65. Create a set of examples that suggests the generalization: The square of an odd number is an odd number. ☛

66. Use a calculator to create an arithmetic sequence or a geometric sequence. Write down the keystrokes you pressed to create the sequence. Answers vary.

67. In each case, write the conclusion that follows from the information given: ☛

a. If a geometric figure is a square, then it is a rectangle. Figure *ABCD* is a square.

b. If a geometric figure is a triangle, then it is a polygon. Figure *ABC* is a triangle.

c. If a number is an odd number, then a number that is 2 greater than the number is also an odd number. Thirteen is an odd number.

68. If you write the inverse of the contrapositive of a given conditional statement, what is the result? converse statement

69. If you write the contrapositive of the converse of a given conditional statement, what is the result? inverse statement

In Exercises 70 and 71, consider the following examples and generalizations, and then determine which, if either, of these two generalizations follows from the examples and whether they are true:

$3^2 - 1 = 8$; $7^2 - 1 = 48$; $13^2 - 1 = 168$;
$17^2 - 1 = 288$; $23^2 - 1 = 528$.

70. If 1 is subtracted from the square of an odd number, the result can be divided evenly by 8, leaving no remainder. ☛

71. If 1 is subtracted from the square of an odd number, the result is a number that ends in 8. ☛

Indicate whether Exercises 72–76 illustrate correctly the use of deductive reasoning. Explain your answers.

72. If Leslie trains hard, she will win the race. Leslie won the race. Therefore Leslie trained hard. invalid reasoning

73. If Angelica buys concert tickets, then she will go to the concert. Angelica didn't go to the concert. Therefore she didn't buy tickets. valid reasoning

74. If two odd numbers differ by 2, then their average is an even number. The average of two numbers that differ by 2 is odd. Therefore the two numbers are not odd. ☛

75. If a geometric figure is a square, then its sides all have equal length. Figure *ABCD* is a square. Therefore Figure *ABCD* has all sides of equal length. ☛

76. If Alice buys a new dress, she will go to the prom. Alice didn't buy a new dress. Therefore Alice won't go to the prom. invalid reasoning

C. Reasoning and Problem Solving

77. Ryan claims that two consecutive terms of a geometric sequence are 8 and 12 and that the next term is 16. Kevin says that the next term is 18. Fred maintains that the next term cannot be determined. Who is correct? Why? Kevin

78. Create an if–then statement that would make the underlined statement a valid conclusion. Answers vary. If M[...] is a dog, then Mick can f[...]
Mick is a dog. Therefore Mick can fly.

79. Write as many if–then statements as you can that seem to be implied in the following advertisement from the Sleezy Company.

Come work for Sleezy and make a fortune. Work hard and you will get promoted fast. We train all of our employees—that's the reason they are so smart. You will like working for Sleezy. Answers vary. If you work for Sleezy, you'll make a for[...]

80. Consider the following sums of numbers and how they are formed:

1 odd number
$1 = 1^2$
2 odd numbers
$1 + 3 = 4 = 2^2$
3 odd numbers
$1 + 3 + 5 = 9 = 3^2$

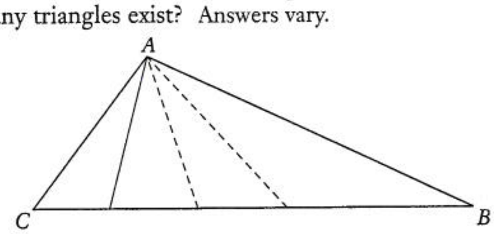
Predict the following sum and complete the generalization:

a. $1 + 3 + 5 + 7 + 9 + 11 = ?$ 36

b. $1 + 3 + 5 + \cdots + (2n - 1) = ?$ n^2

81. The Vitamin Ad Problem. Frieda read a vitamin advertisement that stated, "If you want to feel your very best, take one Vigorous Vitamin each day. People who take Vigorous Vitamins care about their health." Frieda described the ad to her friend, telling her that the ad said that "if you take a Vigorous Vitamin each day, then you'll feel your very best," and that "if you don't take Vigorous Vitamins, then you don't care about your health." Did Frieda report the main ideas of the ad correctly to her friend? Analyze this situation and use what you learned about deductive reasoning in this section to support your answer. ☛

82. As shown in the following figure, when a line segment is drawn from vertex *A* of triangle *ABC* to side $\overline{BC}$, how many triangles exist? Answers vary.



Suppose that a second line is drawn from A to side $\overline{BC}$. How many triangles exist now? Suppose that a third line is drawn from A to $\overline{BC}$. Now how many triangles are formed? Write a statement about the number of lines drawn from a vertex and the number of triangles that are formed. What kind of reasoning did you use?

83. Heather lost 4 pounds the first week of her diet. She then lost 1.5 pounds for each of the following 6 weeks. If she weighed 150 pounds when she started the diet, how much did she weigh after 7 weeks of dieting? Explain how you got your answer. 137 pounds

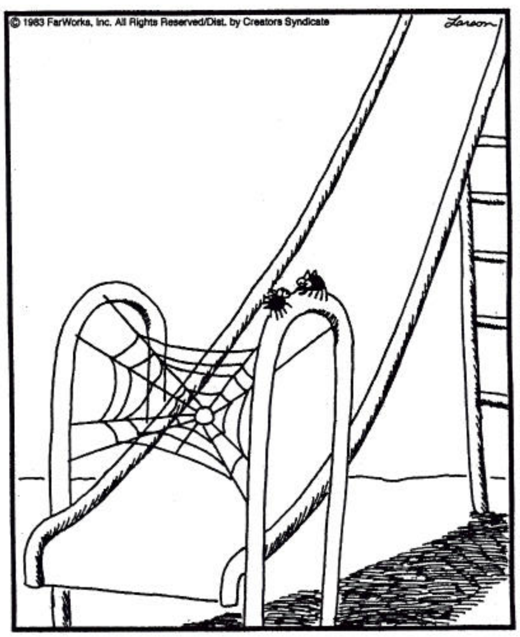
D. Communicating and Connecting Ideas

84. Work in a small group to design an advertisement for a magazine that communicates the following reasoning:

    If you use Neato Hair Spray, then you will be a very popular person.

    Exchange your group's ad with that of another group's ad. Analyze each other's ads to determine whether they accurately reflect the given statement. Answers vary.

THE FAR SIDE® **By GARY LARSON**



"If we pull this off, we'll eat like kings."

85. Consider the Far Side cartoon. Create some sort of deductive argument that is based on the if–then statement in the cartoon. Compare your argument with those of others in the class. Answers vary.

86. **Making Connections.** Consider the Waltham advertisement above at the right. What if–then statements are the creators implicitly assuming? Analyze the ad in terms of the type of mathematical reasoning they are using. Answers vary.

Figure for Exercise 86

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87. **Historical Pathways.** A magic square is an array of numbers in which the sum of each row, column, and diagonal is always a certain magic number, say, 15. The patterns in magic squares were known to the ancient Egyptians and later to the Greek mathematician Pythagoras (c. 582–507 B.C.). The sixteenth-century German artist Albert Dürer included a completed magic square, with the numbers shown, in his famous painting, *Melancholia*. Use mathematical reasoning to complete the magic square. ☛

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1