**SET NOTATION:** A **set** is a collection of individual members of the set. These members are actually called **elements** of the set. A set must be clearly defined in that there is no question about whether a specific item is an element of the set or not. The set itself is sometimes given a letter name, and one way you know that you are looking at a set is that set brackets, { } are written around the set. So for example the set of vowels (which we will name V) which we will define as vowels which are only used as vowels in English (in other words, not including ‘y’) could be written V = {a,e,i,o,u}; When it written this way (in a list) that is called the **roster** method of writing a set. If we are talking about elements of a set we can use the notation ∈ to indicate that something is an element of a set. For example, u ∈ V because u is an element of set V. We could also write k ∉ V because the letter k is not an element of set V. If you see a problem that begins with, for example, the statement x ∈ {3,6,9,12,15,18} then what the problem is saying is that x must take on one of those values. So for example, 12 is a legitimate value of x because it is in the set, but 13 is not.

A set can alternatively be written using **set-builder notation** in which the set is written beginning with {x| in which a variable name (generally x, though in some cases other variables can be used for a reason) and the vertical line means ‘such that.’ What follows is a verbal or verbal/symbolic representation of the set. For example, we could write the set V from above using set-builder notation as V = {x|x is a vowel.}

When we talk about ‘numbers,’ we generally know what we mean. But in math, it is important to talk about which numbers we mean.

The first set of numbers we know are the **natural**, or counting numbers. The natural numbers are the set, {1,2,3,4,5,6,…} The three dots indicate that once the pattern has been established it continues on forever. So, for example, the

number 7,241,607,793 is an example of a natural number because it is possible that you could count that high if you took enough time.

The next set of numbers is the **whole numbers**. The only difference between the natural numbers and the whole numbers is that the set of whole numbers includes the number zero. So if we wrote the whole numbers out as a set then it would look like {0,1,2,3,4,5,6,….} Notice the set of whole numbers includes the set of natural numbers, so that if a number is a natural number then it must also be a whole number.

The next set up from the whole numbers is the set of **integers**. This set includes the whole numbers as well as all of their negatives. So the set of integers could be written using roster notation as {…,-3,-2,-1,0,1,2,3,…} Note that the set of integers continues for ever in both the positive and negative direction. The negative of zero is still zero so that there is only one zero, with no sign. While in general a set that has a natural order to it does not have to be written in that natural order (so for example the set of vowels could have been written in a different order like V = {o,i,e,u,a} when we are establishing a pattern order becomes relevant so that we do write the sets of natural numbers, whole numbers and integers in order.

Beyond the set of integers is the set of **rational numbers**, or numbers that can be written as a ratio (fraction) of two integers. Because while there is a natural order to rational numbers it is not possible to write out a portion of it (between any two fractions one can always find another fraction) we use set-builder notation to write out the set of rational numbers rather than roster notation. So, we define the set of rational numbers as {$\frac{p}{q}$ | p,q ∈ integers, and q ≠ 0}. The restriction on q is because a fraction really represents a division of its numerator by its denominator and remember that division by zero is undefined.

Note that all integers are rational numbers because they can always be written as themselves divided by the integer one. So for example the integer 5 = $\frac{5}{1}$ if we wanted to think of it that way.

**NOTE**: unlike with integers, there are multiple ways to write a number as a rational number. To avoid confusion between equivalent ways of writing the number, you should always reduce fractions. So for example, if the answer to a problem is $\frac{-8}{6}$ then you should reduce it by cancelling out a factor of 2 and writing $\frac{-4}{3}$ which is now reduced. This is still an improper fraction, of course. In mathematical equations it is generally easier to work with improper fractions than mixed fractions, so if this is just a computational problem or an answer which might be used in future calculations then it would be best to leave it as an improper fraction. However, if there is a unit associated with it like percent, degrees or inches then you should write it as a mixed fraction (in this case the mixed fraction -1$\frac{1}{3}$ ) because it is easier for most people to visualize a quantity with a unit on it if it is written as a mixed fraction.

The final set of numbers that we will refer to in this course is the set of **real numbers**. The set of real numbers is roughly speaking, the set of all numbers that could exist as a result of some measurement of nature. All rational numbers are also real numbers, but there are also real numbers that can’t be written as a ratio of two integers, such as π or $\sqrt{5}$. Sometimes real numbers which are not rational are referred to as **irrational** numbers. But either way, they are all real numbers.

**THE REAL NUMBER LINE:**

One way of locating real numbers is on the real number line. We can use it to show the location of any real number. For convenience, we mark it in intervals generally of integers. The real number line looks like this:

 | | | | | | | | | | |

 -5 -4 -3 -2 -1 0 1 2 3 4 5

The arrows on each end indicate that it keeps on going. The positive or larger end is to the right. We can locate real numbers by their position on the real number line. For example, we could locate π, which is approximately 3.14 here:

 | | | | | | | | | | |

 -5 -4 -3 -2 -1 0 1 2 3 4 5

Unless otherwise labeled we assume that the increments between marked intervals is one, and the number zero should always be labeled.

We can define inequalities in terms of the number line.

We say a < b (a **is less than** b) if a is to the left of b on the number line. We also say a > b (a **is greater than** b) if a is to the right of b on the number line. So for example, 3 > 1 because 3 is to the right of 1 on the number line and -4 < -2 because -4 is to the left of -2 on the number line. Note that this means that ALL positive numbers are larger than ALL negative numbers.

Also, a number equals itself, meaning that it is neither less than nor greater than itself, so for example the statement 2 < 2 would be **FALSE.** We can however define the symbols < and> as **strict inequalities** and then also define the symbols < (less than or equal to) and > (greater than or equal to) as **weak inequalities** which allow equality as well as inequality. So for example while 2 < 2 was false the statement 2 < 2 is **TRUE**.

Example 1: Put the right symbol, < or > between the numbers:

-30 - 14

The solution is -30 < -14 because of you extended the number line far enough to the left - 30 would be to the left of - 14 on the number line.

Example 2: Let n ∈ {-12, -6, 0, 12} For which values of n is the statement n > -6 true?

Solution: The original statement restricts n to being an element of the set, in other words it is one of the four numbers -12, -6, 0 and 12. The statement -12 > -6 is false but the other three numbers are either equal to or greater than -6. So the values that satisfy the original inequality are -6, 0 and 12. Your answer should be these three numbers, -6, 0 and 12. You may write them as a set {-6,0,12} but you don’t have to as long as you make it clear that these three numbers are the solutions.

The absolute value of a number, |x| is the distance that the number is from zero on the number line. Distance is always a positive number (or zero if there is no distance to measure,) regardless of what direction you are counting in. So for example, |2| = 2 but |-5| = 5 because it is five units from zero to -5 on the number line.

Example 3: What is | -11| ?

| -11| = 11

Example 4: What is - | - 3| ?

-| -3| = - (3) = -3 because you have to evaluate the inside of the absolute value first (i.e. |-3| = 3) and THEN consider the negative sign which is on the outside.

Example 5: | 3 – 7| = | -4| = 4 (note that we compute what is inside the absolute value first and then evaluate it.)

Example 6: Which number(s) satisfy the equation |x| = 10?

There are two solutions, x = 10 and x = -10 because |10| = 10 and |-10| = 10.

Example 7: Which number(s) satisfy the equation |x| = -14?

This has NO SOLUTION because an absolute value is a distance and it cannot be negative.

**COMPUTATIONS INVOLVING INTEGERS**

When you add integers the result is the net sum of positive values minus the net sum of negative values. So for example, 4 + (-5) = 4 – 5 = -1 because we have four positives and five negatives for a net of one more negative than positive. As another example, – 3 – 8 = –11 because we have only negative numbers, a total of – 11. Keep in mind (+)(-) is negative, so for example -5 + (-7) = – 5 – 7 = – 12, and that (–)(–) is positive, so –8 – (–13) = – 8 + 13 = 5.

If we have longer sequences of additions and subtractions, then we should first simplify by combining adjacent signs and then go from left to right. For example, if we have – 6 + (5) – (–8) = – 6 – 5 + 8 = – 11 + 8 = – 3.

Example 8: – 51 – (– 16) + (– 40) – 24 =

* 51 + 16 – 40 – 24 = – 36 – 40 – 24 = – 76 – 24 = – 100.

When we multiply or divide then we still have that two negatives is a positive (whether they are multiplied together or divided into each other and that a positive and a negative multiplied or divided together is a negative.

Example 9: (-4)(-5) = 20 because two negatives multiply together to be a positive.

Also note that a fraction means the numerator divided by the denominator, just as if a ÷ sign had been used openly.

For example, -6 ÷ -3 = 2, which could also have been written $\frac{-6}{-3}$ = 2. Keep in mind also that division into zero is still zero ( for example if you divide zero dollars among 10 people each of them still gets zero dollars.) On the other hand division by zero makes no sense (try dividing some money among no people, and it

doesn’t even make sense to ask how much per person.) Therefore division by zero is **undefined**, indicating there is no meaningful way to define it.

Example 10: 0 ÷ 46 = 0 but 46 ÷ 0 is undefined.

Finally note that if we are looking at a difference between two numbers that have a physical meaning, the word ‘difference’ indicates subtract. But if we are then subtracting a signed number with a negative sign the result will be a positive number.

For example, suppose you are jumping off of a diving board which is 12 feet above the surface of the water. The water is 12 feet deep. However it is **not true** (as some people would write) that the distance you are above the bottom of the pool is zero. In fact both the height of the diving board and the depth of the pool are measured against a set standard, the surface of the water. The height of the diving board is 12 feet above the surface and the depth is measured in the opposite direction, so it is effectively – 12 feet. So the actual distance between the high dive and the bottom of the pool is 12 – (– 12) = 12 + 12 = 24 feet.

PROBLEMS:

In problems 1- 12, put the right symbol (< or >) between the numbers.

1. 5 9 2. -12 -5 3. 5 -6 4. -3 -11

5. -5 0 6. 0 17 7. -9 -4 8. | -5| 3

9. | 6| | -5| 10. - | -9| 4 11. 2 | -5| 12. | 0| -4

13. Suppose that v ∈ {-5, -2, 0, 8} For which values of v is the statement v > 0 a true statement?

14. Suppose that v ∈ {-5, -2, 0, 8} For which values of v is the statement v> 0 a true statement?

15. Suppose that r ∈ { -20, -15, -12, 0} For which values of r is the statement that r > -14 a true statement?

16. Suppose that z ∈ { -6, -2, 3, 8} For which values of z is the statement z < 3 a true statement?

17. Suppose that x ∈ {-12, -4, 5, 10} For which values of x is the statement |x|> 5 a true statement?

18. Suppose that n ∈ { -20, -11, 9, 16 } For which values of x is the statement that |n| > 16 a true statement?

In problems 19 - 26 Simplify:

19. |-11| 20. | -3| 21. | 32 | 22. | 0|

23. - |-9| 24. - |-5| 25. | -2 + 5| 26. | 5 -9|

In problems 27- 38 give all values of the variable that satisfy the given equations.

27. |x| = 8 28. | x | = 13 29. | x | = -5 30. |x | = 0

31. |m| = 47 32. |r | = -1 33. | y | = 22 34. |x| = $\frac{3}{5}$

35. |x| = 11 36. |x | = -9 37. | x | = $\frac{1}{8}$ 38. |y| = $\frac{-3}{5}$

Perform the additions and subtractions:

39. 4 + 15 40. 31 – 19 41. 11 – 6 42. 8 – 8

43. -3 + 19 44. -5 + 12 45. -9 + 0 46. 13 – 16

47. 20 – 24 48. 3 – 25 49. 16 – 68 50. 50 – 150

51. -9 – 5 52. -3 – 7 53. -6 – 12 54. -37 – 43

55. 3 – (-4) 56. 4 – (-11) 57. -6 – (-2) 58. -30 – (-10)

59. 18 + (-4) 60. 25 + (-7) 61. -11 + (-9) 62. -40 + (-7)

63. 3 + 2 + 5 64. 8 + 23 + 6 65. 3 + 9 – 5 66. 12 + 20 – 8

67. -5 + 10 – 3 68. 4 – 12 – 5 69. 30 – 55 + 4 70. -8 + 13 – 9

71. -9 – 12 – 6 72. 32 – 7 – 9 73. 51 – 29 – 22 74. 20 – 6 + 4

75. -40 + 8 – 9 76. -3 – (-6) + 14 77. -7 – (-2) + 28 78. 6 + (-8) – 3

79. 8 – (-5) – (-2) 80. 19 + (-5) + (-8) 81. 13 + (-4) – 0 82. 64 + (-9) – (-7)

Perform the multiplications and divisions:

83. (8)(12) 84. (13)(7) 85. (-2)(7) 86. (-11)(12)

87. (4)(0) 88. (-9)(0) 89. (-4)(-5) 90. (-9)(-12)

91. (-5)(-6) 92. (-1)(-10) 93. 28 ÷ 7 94. $\frac{63}{9}$

95. $\frac{-8}{2}$ 96. $\frac{-48}{8}$ 97. $\frac{56}{-7}$ 98. (-60) ÷ (-12)

99. $\frac{-35}{-7}$ 100. $\frac{81}{-9}$ 101. $\frac{-54}{-9}$ 102. 0 ÷ 4

103. 4 ÷ 0 104. $\frac{19}{0}$ 105. $\frac{-12}{0}$ 106. $\frac{0}{8}$

107. 14 ÷0 108. 12 ÷ 0 109. $\frac{-16}{0}$ 110. $\frac{25}{0}$

111. 0 ÷ (-9) 112. $\frac{1}{0}$ 113. - $\frac{8}{-4}$ 114. - $\frac{-72}{-12}$

115. Suppose that the high temperature in Bellemont, Arizona was 37 ᵒ F and the low temperature was – 11 ᵒ F on the same day. Find the difference between the high and the low temperature.

116. A 90 foot tall building (measured from street level to the top of the building) has a 12 foot deep underground basement. How far above the bottom of the basement is the top of the building?

117. The top of Mauna Kea volcano in Hawaii is 2.7 miles above sea level. The sea floor that the island it is on rises from is 2.8 miles deep. How many miles is the top of Mauna Kea from the sea floor it stand on?