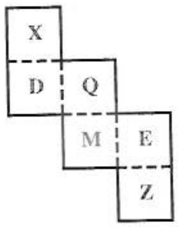
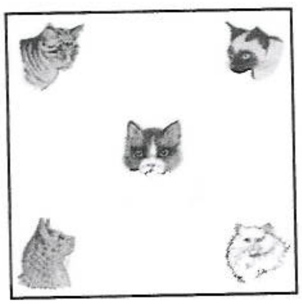
58. *Weighing Coins* You have eight coins. Seven are genuine and one is a fake, which weighs a little less than the other seven. You have a balance scale, which you may use only three times. Tell how to locate the bad coin in three weighings. (Then show how to detect the bad coin in only *two* weighings.)

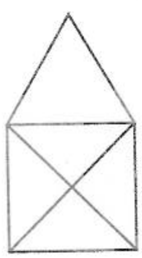
59. *Geometry Puzzle* When the diagram shown is folded to form a cube, what letter is opposite the face marked Z? Q



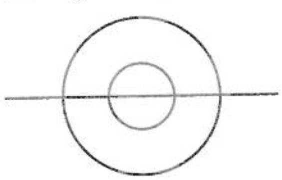
60. *Picture Puzzle* Draw a square in the following figure so that no two cats share the same region.



61. *Geometry Puzzle* Draw the following figure without picking up your pencil from the paper and without tracing over a line you have already drawn.



62. *Geometry Puzzle* Repeat Exercise 61 for this figure.



63. *Decimal Digit* What is the 100th digit in the decimal representation for $\frac{1}{7}$? 8

64. *Books on a Shelf* Volumes 1 and 2 of *The Complete Works of Wally Smart* are standing in numerical order from left to right on your bookshelf. Volume 1 has 450 pages and Volume 2 has 475 pages. Excluding the covers, how many pages are between page 1 of Volume 1 and page 475 of Volume 2? none

65. *Oh Brother!* The brother of the chief executive officer (CEO) of a major industrial firm died. The man who died had no brother. How is this possible? The CEO is a woman.

66. *Teenager's Age* A teenager's age increased by 2 gives a perfect square. Her age decreased by 10 gives the square root of that perfect square. She is 5 years older than her brother. How old is her brother? 9 years old

67. *Ages* James, Dan, Jessica, and Cathy form a pair of married couples. Their ages are 36, 31, 30, and 29. Jessica is married to the oldest person in the group. James is older than Jessica but younger than Cathy. Who is married to whom, and what are their ages? Dan (36) is married to Jessica (29); James (30) is married to Cathy (31).

68. *Making Change* In how many different ways can you make change for a half dollar using currently minted U.S. coins, if cents (pennies) are not allowed? 10

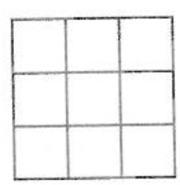
69. *Days in a Month* Some months have 30 days and some have 31 days. How many months have 28 days? 12; All months have 28 days.

70. *Dirt in a Hole* How much dirt is there in a cubical hole, 6 feet on each side? None, because there is no dirt in a hole.

71. *Fibonacci Property* Refer to Example 1, and observe the sequence of numbers in color. Choose any four successive terms. Multiply the first one chosen by the fourth; then multiply the two middle terms. Repeat this process. What do you notice when the two products are compared? The products always differ by 1.

72. *Palindromic Greeting* The first man introduced himself to the first woman with a brief "palindromic" greeting. What was the greeting? (*Hint:* See Exercises 31, 48, and 51.) "Madam, I'm Adam."

73. *Geometry Puzzle* What is the maximum number of small squares in which we may place crosses (×) and not have any row, column, or diagonal completely filled with crosses? Illustrate your answer.

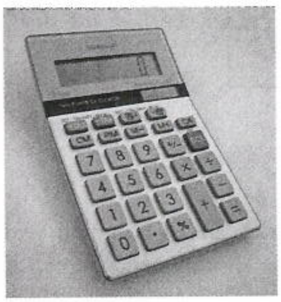


74. ***Determining Operations*** Place one of the arithmetic operations $+$, $-$, $\times$, or $\div$ between each pair of successive numbers on the left side of this equation to make it true. Any operation may be used more than once or not at all. Use parentheses as necessary.

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 = 100$$

1.4 Calculating, Estimating, and Reading Graphs

Calculation • Estimation • Interpretation of Graphs



The photograph shows the **Sharp Elsimate EL-330M,** a typical four-function calculator.

Since the introduction of hand-held calculators in the early 1970s, the methods of everyday arithmetic have been drastically altered. One of the first consumer models available was the Texas Instruments SR-10, which sold for nearly \$150 in 1973. It could perform the four operations of arithmetic and take square roots, but could do very little more.

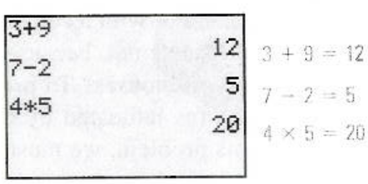
Calculation

The search for easier ways to calculate and compute has culminated in the development of hand-held calculators and computers. This text assumes that all students have access to calculators, allowing them to spend more time on the conceptual nature of mathematics and less time on computation with paper and pencil. For the general population, a calculator that performs the operations of arithmetic and a few other functions is sufficient. These are known as **four-function calculators.** Students who take higher mathematics courses (engineers, for example) usually need the added power of **scientific calculators. Graphing calculators,** which actually plot graphs on small screens, are also available. Remember the following.

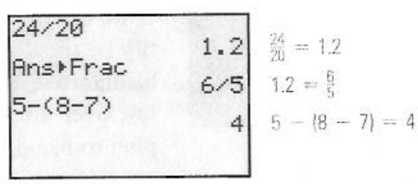
Always refer to your owner's manual if you need assistance in performing an operation with your calculator. If you need further help, ask your instructor or another student who is using the same model.

Graphing calculators have become the standard in the world of advanced hand-held calculators. One of the main advantages of a graphing calculator is that both the information the user inputs into the calculator and the result generated by that calculator can be viewed on the same screen. In this way, the user can verify that the information entered into the calculator is correct. Although it is not necessary to have a graphing calculator to study the material presented in this text, we occasionally include graphing calculator screens to support results obtained or to provide supplemental information.*

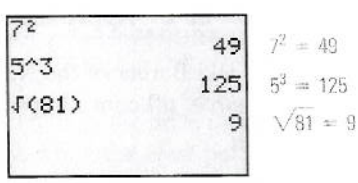
The screens that follow illustrate some common entries and operations.



A



B

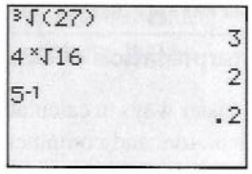


C

*Because they are the most popular models of graphing calculators, we include screens generated by TI-83 Plus and TI-84 Plus models from Texas Instruments.

Any calculator (particularly a graphing calculator) consists of two components: the electronic "box" and the owner's manual that explains how to use it. The **TI-84 Plus** graphing calculator is shown.

Screen A illustrates how two numbers can be added, subtracted, or multiplied. Screen B shows how two numbers can be divided, how the decimal quotient (stored in the memory cell Ans) can be converted into a fraction, and how parentheses can be used in a computation. Screen C shows how a number can be squared, how it can be cubed, and how its square root can be taken.

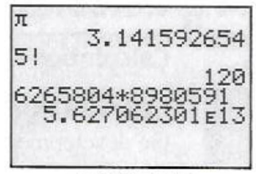


$\sqrt[3]{27} = 3$

$\sqrt[4]{16} = 2$

5^{-1} (or $\frac{1}{5}$) $= .2$

D



$\pi \approx 3.141592654$

$5!$ (or $1 \times 2 \times 3 \times 4 \times 5$) $= 120$

$6{,}265{,}804 \times 8{,}980{,}591 \approx 5.627062301 \times 10^{13}$

$\approx$ indicates "is approximately equal to"

E

Screen D shows how other roots (cube root and fourth root) can be found, and how the reciprocal of a number can be found using -1 as an exponent. Screen E shows how π can be accessed with its own special key, how a *factorial* (as indicated by !) can be found and how a result might be displayed in *scientific notation*. (The "E13" following 5.627062301 means that this number is multiplied by 10^{13}. This answer is still only an approximation, because the product $6{,}265{,}804 \times 8{,}980{,}591$ contains more digits than the calculator can display.)

Estimation Although calculators can make life easier when it comes to computations, many times we need only estimate an answer to a problem, and in these cases a calculator may not be necessary or appropriate.

EXAMPLE 1 Estimating an Appropriate Number of Birdhouses

A birdhouse for swallows can accommodate up to 8 nests. How many birdhouses would be necessary to accommodate 58 nests?

SOLUTION

If we divide 58 by 8 either by hand or with a calculator, we get 7.25. Can this possibly be the desired number? Of course not, because we cannot consider fractions of birdhouses. Do we need 7 or 8 birdhouses? To provide nesting space for the nests left over after the 7 birdhouses (as indicated by the decimal fraction), we should plan to use 8 birdhouses. In this problem, we must round our answer *up* to the next counting number. ■

EXAMPLE 2 Approximating Average Number of Yards per Carry

In 2004, Tiki Barber of the New York Giants carried the football 322 times for 1518 yards (*Source:* nfl.com). Approximate his average number of yards per carry.

SOLUTION

Because we are are asked only to find Tiki's approximate average, we can say that he carried about 300 times for about 1500 yards, and his average was about $\frac{1500}{300} =$ 5 yards per carry. (A calculator shows that his average to the nearest tenth was 4.7 yards per carry. Verify this.) ■

EXAMPLE 3 Comparing Proportions of Workers by Age Groups

In a recent year, there were approximately 127,000 males in the 25–29-year age bracket working on farms. This represented part of the total of 238,000 farm workers in that age bracket. Of the 331,000 farm workers in the 40–44-year age bracket, 160,000 were males. Without using a calculator, determine which age bracket had a larger proportion of males.

SOLUTION

Here, it is best to think in terms of thousands instead of dealing with all the zeros. First, let us analyze the age bracket 25–29 years. Because there were a total of 238 thousand workers, of which 127 thousand were males, there were $238 - 127 = 111$ thousand female workers. Here, more than half of the workers were males. In the 40–44-year age bracket, of the 331 thousand workers, there were 160 thousand males, giving $331 - 160 = 171$ thousand females, meaning fewer than half were males. A comparison, then, shows that the 25–29-year age bracket had the larger proportion of males.

Interpretation of Graphs

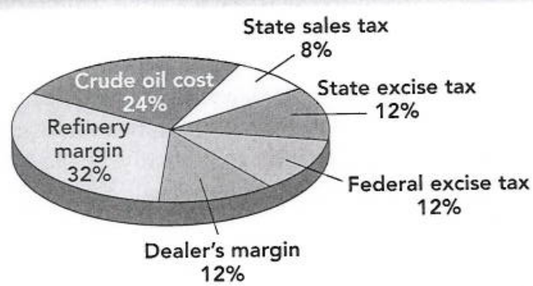
Using graphs is an efficient means of transmitting information in a concise way. Any issue of the newspaper *USA Today* will verify this. *Circle graphs* or *pie charts, bar graphs*, and *line graphs* are the most common.

A **circle graph** or **pie chart** is used to give a pictorial representation of data. A circle is used to indicate the total of all the categories represented. The circle is divided into sectors, or wedges (like pieces of a pie), whose sizes show the relative magnitudes of the categories. The sum of all the fractional parts must be 1 (for 1 whole circle).

EXAMPLE 4 Interpreting Information in a Circle Graph

Use the circle graph in Figure 9 to determine how much of the amount spent for a \$3.50 gallon of gasoline in California goes to refinery margin and to crude oil cost.

BREAKING DOWN THE PRICE OF A GALLON OF GAS



Source: California Energy Commission.

FIGURE 9

SOLUTION

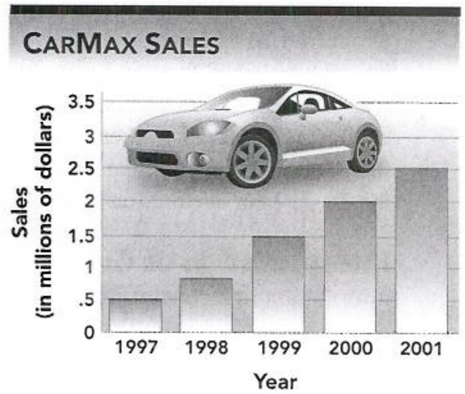
The sectors are sized to match how the price is divided. For example, most of the price (32%) goes to the refinery, while the least portion (8%) goes for state sales tax. As expected, the percents total 100%. If the price of gasoline is \$3.50 per gallon,

Refinery margin: $\$3.50 \times \underbrace{.32}_{\text{32\% converted to a decimal}} = \$1.12$ Crude oil cost: $\$3.50 \times \underbrace{.24}_{\text{24\% converted to a decimal}} = \$.84.$

A **bar graph** is used to show comparisons. We illustrate with a bar graph where we must estimate the heights of the bars.

EXAMPLE 5 Interpreting Information in a Bar Graph

The bar graph in Figure 10 shows sales in millions of dollars for CarMax Auto Superstores, Inc. The graph compares sales for 5 years.



Source: Circuit City CarMax Group.

FIGURE 10

(a) Estimate sales in 1998.
(b) In what years were sales greater than $1 million?
(c) As the years progress, describe the change in sales.

SOLUTION

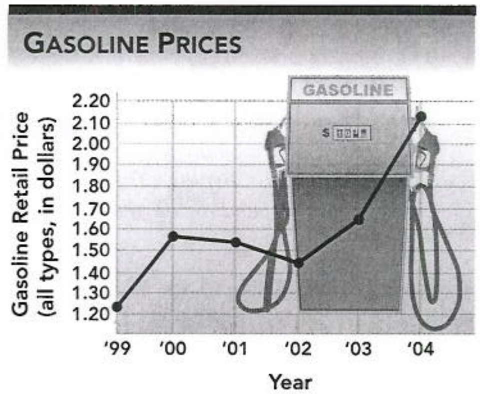
(a) Move horizontally from the top of the bar for 1998 to the scale on the left to see that sales in 1998 were about $.8 million.
(b) Locate 1 on the vertical scale and follow the line across to the right. Three years—1999, 2000, and 2001—have bars that extend above the line for 1, so sales were greater than $1 million in those years.
(c) Sales increase steadily as the years progress, from about $.5 million to $2.5 million.

A **line graph** is used to show changes or trends in data over time. To form a line graph, we connect a series of points representing data with line segments.

EXAMPLE 6 Interpreting Information in a Line Graph

The line graph in Figure 11 shows average prices for all types of gasoline in the U.S. for the years 1999 through 2004.

(a) In which years shown did the average price decrease from the previous year?
(b) What was the general trend in gasoline prices from 1999 to 2004?
(c) Estimate the average prices for 2002 and 2004. About how much did gasoline price rise from 2002 to 2004?



Source: Energy Information Administration.

FIGURE 11

SOLUTION

(a) The line segments joining the points for the years 2000, 2001, and 2002 fall from left to right. This indicates that average prices decreased from the previous years in 2001 and 2002.

(b) Although the prices fell in 2001 and 2002, the general trend is that prices rose from 1999 to 2004, as indicated by the overall rise of the line graph from left to right.

(c) It appears that in 2002 the average price was about \$1.44 and in 2004 about \$2.12. Thus, the price rose about $\$2.12 - \$1.44 = \$.68$ per gallon.

For Further Thought

Are You "Numerate"?

Letter is to *number* as *literacy* is to *numeracy*. Much has been written about how important it is that the general population be "numerate." The essay "Quantity" by James T. Fey in *On the Shoulders of Giants: New Approaches to Numeracy* contains this description of an approach to numeracy.

> Given the fundamental role of quantitative reasoning in applications of mathematics as well as the innate human attraction to numbers, it is not surprising that number concepts and skills form the core of school mathematics. In the earliest grades all children start on a mathematical path designed to develop computational procedures of arithmetic together with corresponding conceptual understanding that is required to solve quantitative problems and make informed decisions. Children learn many ways to describe quantitative data and relationships using numerical, graphic, and symbolic representations; to plan arithmetic and algebraic operations and to execute those plans using effective procedures; and to interpret quantitative information, to draw inferences, and to test the conclusions for reasonableness.

For Group Discussion or Individual Investigation

With calculator in hand, fill in the boxes with the digits 3, 4, 5, 6, 7, or 8, using each digit at most once. See how close you can come to the "goal number." You are allowed 1 minute per round. Good luck!

Round I	$\square \times \square\square\square\square = 30{,}000$
Round II	$\square \times \square\square\square\square = 40{,}000$
Round III	$\square \times \square\square\square\square = 50{,}000$
Round IV	$\square\square \times \square\square\square\square = 30{,}000$
Round V	$\square\square \times \square\square\square\square = 60{,}000$

1.4 Exercises

Exercises 1–20 are designed to give you practice in learning how to do some basic operations on your calculator. Perform the indicated operations and give as many digits in your answer as shown on your calculator display. (The number of displayed digits may vary depending on the model used.)

1. $39.7 + (8.2 - 4.1)$ 43.8 **2.** $2.8 \times (3.2 - 1.1)$ 5.88

3. $\sqrt{5.56440921}$ 2.3589 **4.** $\sqrt{37.38711025}$ 6.1145

5. $\sqrt[3]{418.508992}$ 7.48 **6.** $\sqrt[3]{700.227072}$ 8.88

7. 2.67^2 7.1289 **8.** 3.49^3 42.508549

9. 5.76^5 6340.338097 **10.** 1.48^6 10.50921537

11. $\dfrac{14.32 - 8.1}{2 \times 3.11}$ 1 **12.** $\dfrac{12.3 + 18.276}{3 \times 1.04}$ 9.8

13. $\sqrt[5]{1.35}$ 1.061858759 **14.** $\sqrt[6]{3.21}$ 1.214555893

15. $\dfrac{\pi}{\sqrt{2}}$ 2.221441469 **16.** $\dfrac{2\pi}{\sqrt{3}}$ 3.627598728

17. $\sqrt[4]{\dfrac{2143}{22}}$ 3.141592653 **18.** $\dfrac{12{,}345{,}679 \times 72}{\sqrt[3]{27}}$ 296296296

19. $\dfrac{\sqrt{2}}{\sqrt[3]{6}}$ .7782717162 **20.** $\dfrac{\sqrt[3]{12}}{\sqrt{3}}$ 1.321802152

21. Choose any number consisting of five digits. Multiply it by 9 on your calculator. Now add the digits in the answer. If the sum is more than 9, add the digits of this sum, and repeat until the sum is less than 10. Your answer will always be 9. Repeat the exercise with a number consisting of six digits. Does the same result hold? yes

22. Use your calculator to *square* the following two-digit numbers ending in 5: 15, 25, 35, 45, 55, 65, 75, 85. Write down your results, and examine the pattern that develops. Then use inductive reasoning to predict the value of 95^2. Write an explanation of how you can mentally square a two-digit number ending in 5. $95^2 = 9025$; explanations will vary.

By examining several similar computation problems and their answers obtained on a calculator, we can use inductive reasoning to make conjectures about certain rules, laws, properties, and definitions in mathematics. Perform each calculation and observe the answers. Then fill in the blank with the appropriate response. (Justification of these results will be discussed later in the book.)

23. $(-3) \times (-8)$; $(-5) \times (-4)$; $(-2.7) \times (-4.3)$
Multiplying a negative number by another negative number gives a __positive__ product.
(negative/positive)

24. $5 \times (-4)$; -3×8; $2.7 \times (-4.3)$
Multiplying a negative number by a positive number gives a __negative__ product.
(negative/positive)

25. 5.6^0; π^0; 2^0; 120^0; $.5^0$
Raising a nonzero number to the power 0 gives a result of __1__.

26. 1^2; 1^3; 1^{-3}; 1^0; 1^{13}
Raising 1 to any power gives a result of __1__.

27. $1/7$; $1/(-9)$; $1/3$; $1/(-8)$
The sign of the reciprocal of a number is __the same as__ the sign of the number.
(the same as/different from)

28. $5/0$; $9/0$; $\pi/0$; $-3/0$; $0/0$
Dividing a number by 0 gives a(n) __error message__ on a calculator.

29. $0/8$; $0/2$; $0/(-3)$; $0/\pi$
Zero divided by a nonzero number gives a quotient of __0__.

30. $(-3) \times (-4) \times (-5)$; $(-3) \times (-4) \times (-5) \times (-6) \times (-7)$; $(-3) \times (-4) \times (-5) \times (-6) \times (-7) \times (-8) \times (-9)$
Multiplying an *odd* number of negative numbers gives a __negative__ product.
(positive/negative)

31. $(-3) \times (-4)$; $(-3) \times (-4) \times (-5) \times (-6)$; $(-3) \times (-4) \times (-5) \times (-6) \times (-7) \times (-8)$
Multiplying an *even* number of negative numbers gives a __positive__ product.
(positive/negative)

32. $\sqrt{-3}$; $\sqrt{-5}$; $\sqrt{-6}$; $\sqrt{-10}$
Taking the square root of a negative number gives a(n) __error message__ on a calculator.

33. Find the decimal representation of $1/6$ on your calculator. Following the decimal point will be a 1 and a string of 6s. The final digit will be a 7 if your calculator *rounds off* or a 6 if it *truncates*. Which kind of calculator do you have? Answers will vary.

34. Choose any three-digit number and enter the digits into a calculator. Then enter them again to get a six-digit number. Divide this six-digit number by 7. Divide the result by 13. Divide the result by 11. What is your answer? Explain why this happens. Answers will vary.

35. Choose any digit except 0. Multiply it by 429. Now multiply the result by 259. What is your answer? Explain why this happens. Answers will vary.

36. Choose two natural numbers. Add 1 to the second and divide by the first to get a third. Add 1 to the third and divide by the second to get a fourth. Add 1 to the fourth and divide by the third to get a fifth. Continue this process until you discover a pattern. What is the pattern?

When a four-function or scientific calculator (not a graphing calculator, however) is turned upside down, the digits in the display correspond to letters of the English alphabet as follows:

$0 \leftrightarrow O$ $3 \leftrightarrow E$ $7 \leftrightarrow L$
$1 \leftrightarrow I$ $4 \leftrightarrow h$ $8 \leftrightarrow B$
$2 \leftrightarrow Z$ $5 \leftrightarrow S$ $9 \leftrightarrow G.$

Perform the indicated calculation on a four-function or scientific calculator. Then turn your calculator upside down to read the word that belongs in the blank in the accompanying sentence.

37. $(100 \div 20) \times 14{,}215{,}469$
I filled my tank with gasoline from the ShELLOIL station.

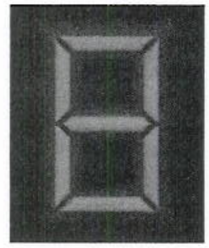
38. $\dfrac{10 \times 10{,}609}{\sqrt{4}}$
"It's got to be the ShOES."

39. $60^2 - \dfrac{368}{4}$
The electronics manufacturer BOSE produces the Wave Radio.

40. $187^2 + \sqrt{1600}$
Have you ever read *Mother* GOOSE nursery rhymes?

41. Make up your own exercise similar to Exercises 37–40. Answers will vary.

42. Displayed digits on some calculators show some or all of the parts in the pattern as in the figure at the top of the next column. For the digits 0 through 9:
(a) Which part is used most frequently?
(b) Which part is used the least?



(c) Which digit uses the most parts? 8 (all seven parts)
(d) Which digit uses the fewest parts? 1 (two parts)

Give an appropriate counting number answer to each question in Exercises 43–46. (Find the least counting number that will work.)

43. ***Pages to Store Trading Cards*** A plastic page designed to hold trading cards will hold up to 9 cards. How many pages will be needed to store 563 cards? 63

44. ***Drawers for Videocassettes*** A sliding drawer designed to hold videocassettes has 20 compartments. If Chris wants to house his collection of 408 Disney videotapes, how many such drawers will he need? 21

45. ***Containers for African Violets*** A gardener wants to fertilize 800 African violets. Each container of fertilizer will supply up to 60 plants. How many containers will she need to do the job? 14

46. ***Fifth-Grade Teachers Needed*** False River Academy has 155 fifth-grade students. The principal, Butch LeBeau, has decided that each fifth-grade teacher should have a maximum of 24 students. How many fifth-grade teachers does he need? 7

In Exercises 47–52, use estimation to determine the choice closest to the correct answer.

47. ***Price per Acre of Land*** To build a "millennium clock" on Mount Washington in Nevada that would tick once each year, chime once each century, and last at least 10,000 years, the nonprofit Long Now Foundation

purchased 80 acres of land for $140,000. Which one of the following is the closest estimate to the price per acre? B

A. $1000 **B.** $2000 **C.** $4000 **D.** $11,200

48. ***Time of a Round Trip*** The distance from Seattle, Washington, to Sprinfield, Missouri, is 2009 miles. About how many hours would a roundtrip from Seattle to Springfield and back take a bus that averages 50 miles per hour for the entire trip? C

A. 60 **B.** 70 **C.** 80 **D.** 90

49. ***People per Square Mile*** Buffalo County in Nebraska has a population of 40,249 and covers 968 square miles. About how many people per square mile live in Buffalo County? A

A. 40 **B.** 400 **C.** 4000 **D.** 40,000

50. ***Revolutions of Mercury*** The planet Mercury takes 88.0 Earth days to revolve around the sun once. Pluto takes 90,824.2 days to do the same. When Pluto has revolved around the sun once, about how many times will Mercury have revolved around the sun? C

A. 100,000 **B.** 10,000 **C.** 1000 **D.** 100

51. ***Rushing Average*** In 2004, Muhsin Muhammad of the Carolina Panthers caught 93 passes for 1405 yards. His approximate number of yards gained per catch was __D__.

A. $\frac{9}{14}$ **B.** 140 **C.** 1.4 **D.** 14

52. ***Area of the Sistine Chapel*** The Sistine Chapel in Vatican City measures 40.5 meters by 13.5 meters.

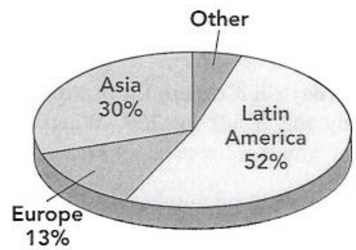
Which is the closest approximation to its area? D

A. 110 meters **B.** 55 meters
C. 110 square meters **D.** 600 square meters

The circle graph at the top of the next column shows the approximate percent of immigrants admitted into the United States during the 1990s. Use the graph to answer the questions in Exercises 53–56.

53. What percent of the immigrants were from the "Other" group of countries? 5%

U.S. IMMIGRANTS BY REGION OF BIRTH



Source: U.S. Bureau of the Census.

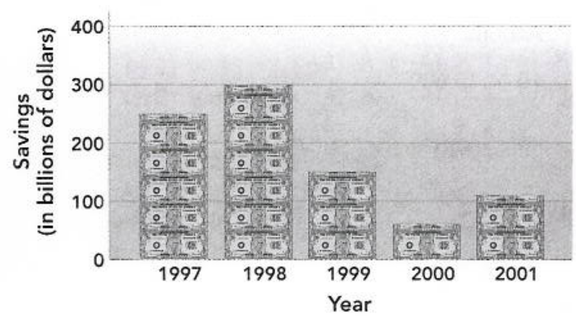
54. What percent of the immigrants were not from Asia? 70%

55. In a group of 2,000,000 immigrants, how many would you expect to be from Europe? 260,000

56. In a group of 4,000,000 immigrants, how many more would there be from Latin America than all the other regions combined? 160,000

The bar graph shows the amount of personal savings, in billions of dollars, accumulated during the years 1997 through 2001. Use the graph to answer the questions in Exercises 57–60.

PERSONAL SAVINGS IN THE UNITED STATES



Source: U.S. Bureau of Economic Analysis.

57. Which year had the greatest amount of savings? Which had the least? 1998; 2000

58. Which years had amounts greater than $200 billion? 1997, 1998

59. Approximately how much was the amount for 1997? for 1998? $250 billion; $300 billion

60. Approximately how much more was saved in 1998 than 1997? $50 billion

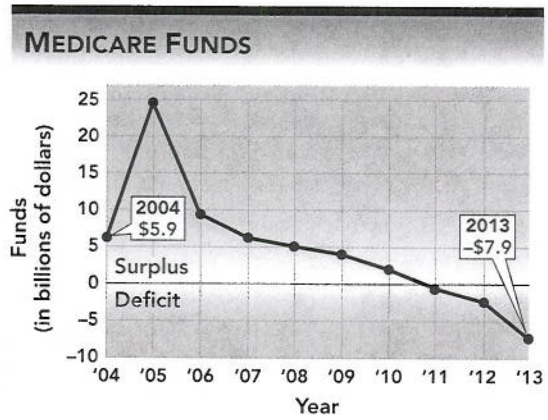
The line graph indicates that current projections for Medicare funding will not cover its costs unless the program changes. Use the graph to answer the questions in Exercises 61–64.

61. Which is the only period in which Medicare funds are predicted to increase? from 2004 to 2005

62. By approximately how much will the funds decrease between the years 2005 and 2006? \$15 billion

63. How do the amounts for 2004 and 2007 compare? They are approximately the same.

64. In which year will funds first show a deficit? 2011



Source: Centers for Medicare and Medicaid Services.

EXTENSION

Using Writing to Learn About Mathematics

Research has indicated that the ability to express mathematical observations in writing can serve as a positive force in one's continued development as a mathematics student. The implementation of writing in the mathematics class can use several approaches.

Journals One way of using writing in mathematics is to keep a journal in which you spend a few minutes explaining what happened in class that day. The journal entries may be general or specific, depending on the topic covered, the degree to which you understand the topic, your interest level at the time, and so on. Journal entries are usually written in informal language, and are often an effective means of communicating to yourself, your classmates, and your instructor what feelings, perceptions, and concerns you are having at the time.

Learning Logs Although journal entries are for the most part unstructured writings in which the student's thoughts are allowed to roam freely, entries in learning logs are typically more structured. An instructor may pose a specific question for a student to answer in a learning log. In this text, we intersperse writing exercises in each exercise set that are appropriate for answering in a learning log. For example, consider Exercise 13 in the exercise set for the opening section in this chapter.

Discuss the differences between inductive and deductive reasoning. Give an example of each.

(continued)

Here is a possible response to this exercise.

Deductive reasoning occurs when you go from general ideas to specific ones. For example, I know that I can multiply both sides of $\frac{1}{2}x = 6$ by 2 to get $x = 12$, because I can multiply both sides of any equation by whatever I want (except 0). Inductive reasoning goes the other way. If I make a general conclusion from specific observations, that's inductive reasoning. Example – in the numbers 4, 8, 12, 16, and so on, I can conclude that the next number is 20, since I always add 4 to get the next number.

Mathematical writing takes many forms. One of the most famous author/mathematicians was **Charles Dodgson** (1832–1898), who used the pen name **Lewis Carroll.**

Dodgson was a mathematics lecturer at Oxford University in England. Queen Victoria told Dodgson how much she enjoyed *Alice's Adventures in Wonderland* and how much she wanted to read his next book; he is said to have sent her *Symbolic Logic*, his most famous mathematical work.

The *Alice* books made Carroll famous. Late in life, however, Dodgson shunned attention and denied that he and Carroll were the same person, even though he gave away hundreds of signed copies to children and children's hospitals.

Reports on Articles from Mathematics Publications The motto "Publish or perish" has long been around, implying that a scholar in pursuit of an academic position must publish in a journal in his or her field. There are numerous journals that publish papers in mathematics research and/or mathematics education. In Activity 3 at the end of this section, we provide some suggestions of articles that have appeared within the last few years. A report on such an article can help you understand what mathematicians do and what ideas mathematics teachers use to convey concepts to their students.

Term Papers Professors in mathematics survey courses are, in increasing numbers, requiring short term papers of their students. In this way, you can become aware of the plethora of books and articles on mathematics and mathematicians, many written specifically for the layperson. In Activities 5 and 6 at the end of this section, we provide a list of possible term paper topics.

EXTENSION ACTIVITIES

Rather than include a typical exercise set, we list some suggested activities in which writing can be used to enhance awareness and learning of mathematics.

Activity 1 Keep a journal. After each class, write for a few minutes on your perceptions about the class, the topics covered, or whatever you feel is appropriate. You may want to use the following guidelines.

Journal Writing*

1. *WHO should write in your journal?* You should.
2. *WHAT should you write in your journal?* New words, ideas, formulas, or concepts; profound thoughts; wonderings, musings, problems to solve; reflections on the class; questions—both answerable and unanswerable; writing ideas
3. *WHEN should you write in your journal?* After class each day; as you are preparing, reading, or studying for class; anytime an insight or question hits you.
4. *WHERE should you write in your journal?* Anywhere—so keep it with you when possible.
5. *WHY should you write in your journal?* It will help you record ideas that you might otherwise forget. It will be worthwhile for you to read later on so that you can note your growth. It will facilitate your learning, problem solving, writing, reading, and discussion in class.
6. *HOW should you write in your journal?* In wonderful, long, flowing sentences with perfect punctuation and perfect spelling and in perfect handwriting; or in single words that express your ideas, in short phrases, in sketches, in numbers, in maps, in diagrams, in sentences. (You may even prefer to organize your journal entries on your desktop, notebook, or palmtop computer.)

Activity 2 Keep a learning log, answering at least one writing exercise from each exercise set covered in your class syllabus. Ask your teacher for suggestions of other types of specific writing assignments. For example, you might want to choose a numbered example from a section in the text and write your own solution to the problem, or comment on the method that the authors use to solve the problem. Don't be afraid to be critical of the method used in the text.

Activity 3 The National Council of Teachers of Mathematics publishes journals in mathematics education: *Teaching Children Mathematics* (formerly called *Arithmetic Teacher*) and *Mathematics Teacher* are two of them. These journals can be found in the periodicals section of most college and university libraries. We have chosen several recent articles in each of these journals. There are thousands of other articles from which to choose. Write a short report on one of these articles according to guidelines specified by your instructor.

From *Mathematics Teacher*

2001

Johnson, Craig M. "Functions of Number Theory in Music." Vol. 94, No. 8, November 2001, p. 700.

Lightner, James E. "Mathematics Didn't Just Happen." Vol. 94, No. 9, December 2001, p. 780.

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*"Journal Writing" from "No Time for Writing in Your Class?" by Margaret E. McIntosh in *Mathematics Teacher,* September 1991, p. 431. Reprinted by permission.

(continued)

McNeill, Sheila A. "The Mayan Zeros." Vol. 94, No. 7, October 2001, p. 590.

Socha, Susan. "Less Is Sometimes More." Vol. 94, No. 6, September 2001, p. 450.

2002

Houser, Don. "Roots in Music." Vol. 95, No. 1, January 2002, p. 16.

Howe, Roger. "Hermione Granger's Solution." Vol. 95, No. 2, February 2002, p. 86.

Kolpas, Sidney J. "Let Your Fingers Do the Multiplying." Vol. 95, No. 4, April 2002, p. 246.

Van Dresar, Vickie J. "Opening Young Minds to Closure Properties." Vol. 95, No. 5, May 2002, p. 326.

2003

McDaniel, Michael. "Not Just Another Theorem: A Cultural and Historical Event." Vol. 96, No. 4, April 2003, p. 282.

Nelson, Joanne E., Margaret Coffey, and Edie Huffman. "Stop This Runaway Truck, Please." Vol. 96, No. 8, November 2003, p. 548.

Roberts, David L., and Angela L. E. Walmsley. "The Original New Math: Storytelling versus History." Vol. 96, No. 7, October 2003, p. 468.

Yoshinobu, Stan T. "Mathematics, Politics, and Greenhouse Gas Intensity: An Example of Using Polya's Problem-Solving Strategy." Vol. 96, No. 9, December 2003, p. 646.

2004

Devaney, Robert L. "Fractal Patterns and Chaos Games." Vol. 98, No. 4, November 2004, p. 228.

Francis, Richard L. "New Worlds to Conquer." Vol. 98, No. 3, October 2004, p. 166.

Hansen, Will. "War and Pieces." Vol. 98, No. 2, September 2004, p. 70.

Mahoney, John F. "How Many Votes Are Needed to Be Elected President?" Vol. 98, No. 3, October 2004, p. 154.

From *Teaching Children Mathematics*

2001

Karp, Karen S., and E. Todd Brown. "Geo-Dolls: Traveling in a Mathematical World." Vol. 8, No. 3, November 2001, p. 132.

Randolph, Tamela D., and Helene J. Sherman. "Alternative Algorithms: Increasing Options, Reducing Errors." Vol. 7, No. 8, April 2001, p. 480.

Sun, Wei, and Joanne Y. Zhang. "Teaching Addition and Subtraction Facts: A Chinese Perspective." Vol. 8, No. 1, September 2001, p. 28.

Whitenack, Joy W., et. al. "Second Graders Circumvent Addition and Subtraction Difficulties." Vol. 8, No. 4, December 2001, p. 228.

2002

Agosto, Melinda. "Cool Mathematics for Kids." Vol. 8, No. 7, March 2002, p. 397.

Huniker, DeAnn. "Calculators as Learning Tools for Young Children's Explorations of Number." Vol. 8, No. 6, February 2002, p. 316.

Strutchens, Marilyn E. "Multicultural Literature as a Context for Problem Solving: Children and Parents Learning Together." Vol. 8, No. 8, April 2002, p. 448.

Whitin, David J. "The Potentials and Pitfalls of Integrating Literature into the Mathematics Program." Vol. 8, No. 9, May 2002, p. 503.

2003

Arvold, Bridget, Gina Stone, and Lynn Carter. "What Do You Get When You Cross a Math Professor and a Body Builder?" Vol. 9, No. 7, March 2003, p. 408.

Edelson, R. Jill, and Gretchen L. Johnson. "Integrating Music and Mathematics in the Elementary Classroom." Vol. 9, No. 8, April 2003, p. 474.

Phillips, Linda J. "When Flash Cards Are Not Enough." Vol. 9, No. 6, February 2003, p. 358.

Uy, Frederick L. "The Chinese Numeration System and Place Value." Vol. 9, No. 5, January 2003, p. 243.

2004

Anthony, Glenda J., and Margaret A. Walshaw. "Zero: A 'None' Number?" Vol. 11, No. 1, August 2004, p. 38.

Buschman, Larry. "Teaching Problem Solving in Mathematics." Vol. 10, No. 6, February 2004, p. 302.

Joram, Elana, Christina Hartman, and Paul R. Trafton. "'As People Get Older, They Get Taller': An Integrated Unit on Measurement, Linear Relationships, and Data Analysis." Vol. 10, No. 7, March 2004, p. 344.

Mann, Rebecca L. "Balancing Act: The Truth Behind the Equals Sign." Vol. 11, No. 2, September 2004, p. 65.

Activity 4 One of the most popular mathematical films of all time is *Donald in Mathmagic Land*, produced by Disney in 1959. Spend an entertaining half-hour watching this film, and write a report on it according to the guidelines of your instructor.

Activity 5 Write a report according to the guidelines of your instructor on one of the following mathematicians, philosophers, and scientists.

Abel, N.
Agnesi, M. G.
Agnesi, M. T.
Al-Khowârizmi
Apollonius
Archimedes
Aristotle
Babbage, C.
Bernoulli, Jakob
Bernoulli, Johann
Cantor, G.
Cardano, G.
Copernicus, N.
De Morgan, A.
Descartes, R.
Euler, L.
Fermat, P.
Fibonacci (Leonardo of Pisa)
Galileo (Galileo Galilei)
Galois, E.
Gauss, C.
Hilbert, D.
Kepler, J.
Kronecker, L.
Lagrange, J.
Leibniz, G.
L'Hospital, G.
Lobachevsky, N.
Mandelbrot, B.
Napier, J.
Nash, J.
Newton, I.
Noether, E.
Pascal, B.
Plato
Polya, G.
Pythagoras
Ramanujan, S.
Riemann, G.
Russell, B.
Somerville, M.
Tartaglia, N.
Whitehead, A.
Wiles, A.

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