**SECTION A-4**

A **variable** is an algebraic quantity which represents a number whose value is unknown. Typically a variable is represented by a letter or other symbol. While the value of a variable is unknown (and may even change over time) it is still a number so it obeys all the rules which apply to any other number. For example, if you add any number to itself the result will be twice that number. Similarly, if we add x + x we get 2x. Or, 14y – 5y = 9y.

**EVALUATING A VARIABLE EXPRESSION:**

In some cases we have expressions containing one or more variables in which we do learn the values of the variables. When this is true we can simplify it to an arithmetic expression and compute the value just by replacing each variable with its actual value.

For example, recall that the distance around a rectangle (its perimeter) is given by the formula 2L + 2W. Now, suppose we were to measure the length and width of the rectangle and find that L = 3 feet and W = 2 feet. In that case we could **evaluate** the variable expression by actually replacing L with 3 and W with 2 in the expression. Note that the units of ‘feet’ still exist and because they are multiplied by unitless numbers and then added the result would be in feet. So in this case, we get 2L + 2W = 2(3) + 2(2) = 6 + 4 = 10 feet.

Example 1: Evaluate 5a + 3bc2 when a = 10, b = -3, and c = 2.

5a + 3bc2 = 5(10) + 3(-3)(2)2 = 50 + 3(-3)(4) = 50 – 36 = **14**.

Example 2: Evaluate –x2 + 3xy when x = -2 and y = -5.

-x2 + 3xy = -(-2)2 +3(-2)(-5) = -4 +30 = **26**.

**COMBINING LIKE TERMS:**

A **term** is the product (multiplication) of a real number **coefficient** and possibly one or more variables, possibly to different powers.

**Like terms** are terms that include exactly the same powers of the same variables. They differ only in terms of their coefficient. So for example 5x and 3x are like terms, but 5x and 3x2 are not because they have different powers of the variable. 5x and 3y are also not like terms because they have different variables entirely.

To combine like terms you must add the coefficients of the terms together. Also note that because the commutative and associative laws of addition the terms which you are combining need not be written next to each other before you combine them, as long as they have the same powers of the same variables.

A **constant term** is a term which consists only of a number, with no variables. Constant terms may be combined with each other by just adding the numbers.

Example 3: Simplify 5x + 8x – 7y

5x + 8x – 7y = **13x – 7y**. Note that the last term contains a different variable so it cannot be combined with the first two.

Example 4: Simplify 5 + 4x – 9x + 12 + 15x

5 + 4x -9x + 12 + 15x = (4-9+15)x + (5+12) = **10x + 17**.

Example 5: Simplify 2t + $\frac{1}{2}$ t2 – 9t + $\frac{1}{4}$ t2 .

2t + $\frac{1}{2}$ t2 – 9t + $\frac{1}{4}$ t2 = (2-9)t + ($\frac{1}{2}$ + $\frac{1}{4}$ )t2 = -7t + ($\frac{1}{2}$ · $\frac{2}{2}$ + $\frac{1}{4}$)t2 = -7t + ($\frac{2}{4}$ + $\frac{1}{4}$)t2 = **-7t +** $\frac{3}{4}$ **t2** .

**MULTIPLICATION OF TERMS BY A CONSTANT:**

When you multiply a constant (a number with no variables attached to it) times a term then you multiply the constant times the coefficient only. So for example, if you have -2(5x) then you have (-2·5)x = -10x.

Example 6: 8(6t) = **48t**

Example 7: $\frac{3}{8}$ · 16x = $\frac{3}{8}$ · (8$·2) $x = **6x**.

Example 8: -4(9y2) = **-36y2** .

If you multiply a constant times a number of terms in parentheses then you must recall the **distributive law**, that a(b+c) = ab + ac. This extends to a constant times any number of terms, in other words that the constant multiplies by each term individually. IF the constant is negative then the sign also distributes, and the effect is to change each sign.

Example 9: 5(x2 -2x + 4) = 5·x2 - 5·2x + 5·4 = **5x2 – 10x + 20**.

Example 10: -6(5x + 7y – 9) = (-6)·5x + (-6)·7y – (-6)·9 =

(-6)·5x – 6·7y + 6·9 = **-30x – 42y + 54**.

Sometimes we have more complex groupings of variable expressions to simplify. Just as with numbers, if we have more than one set of parentheses we should always work **from the inside out**.

Example 11: 5 + 6[2x – 3(4 –x)] = 5 + 6[2x –12 + 3x] = 5 + 6[5x – 12] =

5 + 6·5x – 6·12 = 5 + 30x – 72 = **30x – 67**.

**VERBAL EXPRESSIONS:**

In mathematics, rarely do we first encounter a problem already written out in terms of variables. Mathematics was developed to model real-world situations and solve real-world problems, and so words have been developed to describe relations between numbers so that we can write them into an expression (and later on, into equations.) You should become familiar with these words and what they mean.

**Note: Assume each of the following examples is preceded by “the number: x” (except where other variables are specifically named)**

**WORDS THAT MEAN ADDITION**

word example example written out

sum the sum of a number and 5 x + 5

more than 6 more than a number x + 6

 a more than b b + a

greater than 12 greater than a number x + 12

increased by 4 increased by a number 4 + x

total the total of a number and 7 x + 7

 the total of a,b and c a + b + c

 [the total of a number and

 two more than the number] x + (x+2) [ which would further simplify as 2x + 2]

plus 8 plus a number 8 + x

**WORDS THAT MEAN SUBTRACTION**

word example example written out

difference the difference of a number and 5 x – 5

less than 10 less than a number x – 10

 a number less than 14 14 – y

fewer than a number fewer than 20 20 – x

decreased by a number decreased by 9 x – 9

minus a number minus 100 x – 100

**WORDS THAT MEAN MULTIPLICATION**

word example example written out

times three times a number 3x

product the product of -9 and a number -9x

of $\frac{6}{7}$ of a number $\frac{6}{7}$ x

 8 percent of a number .08x [Note the percent is written as a decimal]

twice twice a number 2x [‘twice’ means specifically to multiply by 2]

**WORDS THAT MEAN DIVISION**

word example example written out

divided by twenty divided by a number $\frac{20}{x}$

quotient the quotient of a number and 11 $\frac{x}{11}$

per 12 miles per hour $\frac{12 miles}{hour}$

**WORDS THAT MEAN POWERS**

word example example written out

to the power a number to the third power x3

squared a number squared x2

the square of the square of a number x2

square root the square root of a number $\sqrt{x}$ [ in fact, the square root is really the

 power ½ ]

NOTE: You should pay special attention to the **subtraction** words. This is because there is a commutative law of addition (so for example while ‘17 more than x’ specifically means x + 17, if you instead wrote 17 + x it would not be wrong) but there is no commutative law of subtraction. So, if a problems says, ‘x less than 15’ it specifically means **15 - x**, NOT x -15 !

Also, many of these words are followed eventually by the word, ‘and.’ When that happens the word ‘and’ marks the specific location of the operator (+, - , X, ÷ ) and what precedes the word ‘and’ will be written before the operator and what follows it will be written after the operator (or in the case of a division written in fractional form, what precedes the word ‘and’ is the numerator and what follows the word ‘and’ is the denominator.

When we start doing full word problems we will require that you first identify and name what you are looking for. In order to get in the habit of doing so reflexively, please ALWAYS write, “the number” at the beginning of any problem that involves a number and then give it a variable name (in the examples above I used ‘x’ but you can use y, n, or some other name, such as ‘(Charles)’ as long as it is clearly defined by being written after the wording, ‘the number: ‘

Example 12: Write a variable expression for ‘the difference between 45 and a number’ and simplify if you can.

The number: x

45 - x

which cannot be simplified. Note that the word ‘difference’ is a subtraction word, so that the 45 **had** to precede the minus sign and the number x had to follow it. No other order would be correct.

If there is more than one word which represents a relation, the one that appears **first** is the central feature of the expression. So, for example, if we have ‘the sum of twice a number and five’ then we have the word ‘sum’ written before the word ‘twice’ so that the whole thing is a sum, but the word ‘twice’ only refers to that part of the sum that precedes the word ‘and’ (recall that the word ‘and’ will move you to the other side of the sum.) So, therefore, ‘the sum of twice a number and five’ means 2x + 5. On the other hand if we instead had ‘twice the sum of a number and five’ we now have the word ‘twice’ written first, so that ‘twice’ (two times) applies to the whole expression. Hence it will be 2(x+5). Note that the parentheses are required because without them you would be back to only taking twice times the number, but not twice times the five. Finally, 2(x+5) would simplify to 2x + 10.

Example 13: Write a variable expression for ‘the difference between a number and five more than the number’ and simplify if you can.

The number: x

 x – (x+5)

Note that the word ‘difference’ (a subtraction word) is written before ‘more than’ (meaning addition) so the whole expression is a subtraction, while only the second part is an addition. Also to make sure we are subtracting the entire second part of the expression from the first part, we have to put the x+5 in parentheses.

Finally, we can simplify as: x – (x+5) = x – x – 5 = **-5**.

Example 14: Write a variable expression for ‘nine less than one half of the sum of a number and seven’ and simplify if you can.

First, note that this begins with ‘nine less than’ so nine will be subtracted at the end. After the ‘nine less than’ meaning –9 at the end, we have ‘one half of’ meaning that everything except for the –9 must be multiplied by one half. Since it will be multiplying a sum, we must have the sum in parentheses to make sure the whole sum is multiplied.

The number: x

 $\frac{1}{2}$(x+7) – 9

This can then be simplified as: $\frac{1}{2}$(x+7) – 9 = $\frac{1}{2}$ · x + $\frac{1}{2}$ · 7 – 9 =

 $\frac{x}{2}$ + $\frac{7}{2}$ – 9 = $\frac{x}{2}$ + $\frac{7}{2}$ – 9· $\frac{2}{2}$ = $\frac{x}{2}$ + $\frac{7}{2}$ – $\frac{18}{2}$ = $\frac{x}{2}$ **–** $\frac{11}{2}$

Sometimes, you need to name more than one quantity using the same variable. When this happens you cannot just name both quantities x (or whatever variable you are using) because that would imply they are equal. You can name one of them x and the other must be named **in terms of x**.

Sometimes, if one quantity is not expressed directly in terms of the other, it does not matter which one gets the variable name. A common example of this is when a total is divided between two quantities. For example, if you have 500 dollars and spend some of it and save the rest then you could either name the amount saved or the amount spent first. However, whichever it was, the other amount would be what remained out of $500. So for example, you could name:

Amount spent: x

Amount saved: 500-x

The concept here, that the second quantity is the total minus the first, is something we will see frequently because often a total is what be start out knowing in a problem.

Example 15: You have fifty pounds of sand to put into two bags. Name the amount of sand in each bag in terms of the same variable.

Amount of sand in the first bag: x

Amount of sand in the second bag: 50-x

In other cases, one quantity will be clearly expressed in terms of the other. When this happens the quantity that the first one is expressed **in terms of** should get the variable name, because the expression itself will then tell you how to name the first quantity. The quantity that the first one is expressed in terms of is virtually always listed **second** in the sentence or clause that contains the comparison.

Example 16: There are five times as many people in California as there are in Arizona. Express the population of both states in terms of the same variable.

Note that in the verbal description of the problem the number of people in California is expressed in terms of the number of people in Arizona. Since Arizona is listed last in this sentence it should receive the variable name and then the sentence itself will express the number of people in California.

Population of Arizona: x

Population of California: 5x

The concept of first identifying what you are looking for, then giving it (or sometimes multiple quantities) variable names is key to doing word problems.

**EXERCISE SET A-4:**

Evaluate the variable expressions in excercises 1-20 when a = - 3, b = 2, c = - 1 and d = 5.

1. 4b 2. a + 5b 3. b2 – 2c 4. 6bc

5. 7d – 3b 6. b2 – ad 7. a2 + cd 8. ab - c

9. a2 – b2 10. abcd 11. –b2 +5b 12. d2 +ac

13. –a + 5c 14. 3d + 2cd 15. –a2 + d 16. $\frac{3b+2c}{abc}$

17. $\frac{a+d}{b+c}$ 18. b2 – 4ac 19. ad – bc 20. a2 + b2

 21. The formula for the perimeter of a rectangle is 2L + 2W. where L is the length of the rectangle and W is its width. Evaluate this formula to find the perimeter of a rectangle of length five feet and width three feet.

 22. Using the formula for the perimeter of a rectangle given in exercise 21, find the perimeter of a rectangle of length 20 centimeters and width 12 centimeters.

 Simplify the variable expressions.

23. 5x + 7x 24. 9y – 2y 25. 12x – 4x 26. 6z + 3z

27. 34x + 8x 28. 50x – 25x 29. 9x + 24x 30. 8r + r

31. 6x + 5x + 3x 32. 10m – 2m +7m 33. $\frac{4}{5}$ t + $\frac{1}{10}$ t 34. $\frac{1}{3}$ v – $\frac{1}{4}$ v

35. 7y2 – 2y2 36. 8n2 + 7n2 37. a2 + 7a2 38. 5x2 +7x2

39. $\frac{-6}{7}$ r2 – $\frac{3}{14}$ r2 40. 12u + 6v – 19u – 5v 41. 4a + 5b – 2a 42. 3 + x + 4

43. 5 – 3x – 6x 44. 6 + 2x + 5x 45. 4y – 6z + 3y 46. 5(3x)

47. 6· 5x 48. -2(4x) 49. $\frac{1}{2}$ (18x) 50. $\frac{3}{8}$ (16y)

51. $\frac{1}{3}$ (3y) 52. $\frac{1}{8}$ (-4x) 53. –2(11x) 54. –(–3x)

55. 2(x+5) 56. – (x–7) 57. 3(x + 11) 58. –4(x–5)

59. 8(7x2 – 3x) 60. 2(5x2 + 3x) 61. (x2 – 4x)·3 62. $\frac{4}{3}$ (3x – 6)

63. 4(4x2 + 3x – 2) 64. –3(2x2 – 5x – 4) 65. 5(4x + 3y + 7) 66. 0(3x –9)

67. 3x – (x –3) 68. 5x + 2(x – 4) 69. x – (4–2x) 70. 4x –(11x – 2)

71. 17 – (12n – 4) 72. 9 – (3x + 4) 73. 28 – 3(x+4) 74. 40 + 5(x – 6)

75. 5(y – 3) – 2(y + 4) 76. 7(y – 1) + 2(y+5) 77. (x–y) – (x+y) 78. 2+3[x+4(x-1)]

79. 4–2[3x –(5–2x)] 80. –3[2x + 4(x-1)] 81. 7[2x + 3(9+x)] 82. 4[2x–3(2x-1)]

83. -3x – 2[2x – 2(2-3x)] 84. 12 – 5[ x – 4(2x + 3)]

In problems 85-100, translate into a variable expression. Then simplify if you can.

85. Fifteen less than a number.

86. Thirty more than a number.

87. The difference between nine and a number.

88. The sum of a number and twelve.

89. The product of negative thirty-seven and a number.

90. Three less than twice a number.

91. Thirteen less than the total of a number and six.

92. Five less than three-sevenths of a number.

93. Four times the difference between eleven and a number.

94. The quotient of a number and one more than the number.

95. Six less than the difference between a number and thirty-two.

96. Six times the sum of three times a number and four.

97. Five more than one-half of the sum of a number and seven.

98. The difference between five twelfths of a number and one eighth of the number.

99. Three times the difference between a number and six.

100. Seventeen less than five times the difference between eight and a number.

In problems 101- 110 identify and write out the two quantities being compared and express them both in terms of the same variable.

101. Texas has twice the land area of Arizona.

102. James is five years older than Rhonda.

103. The speed of a bus is ten mph less than the speed of a car.

104. Thirty pounds of vegetables consist of potatoes and carrots. [Hint: In this problem one of your two quantities should be written out as “pounds of potatoes” and the other is “pounds of carrots.” Then express using a variable.]

105. 100 gallons of water is divided between two tanks.

106. Part of $70 is spent and the rest is saved.

107. There are 200 cars in the parking lot. Some were made in the USA and the rest are imported.

108. There are 674 students attending a small college. Some of them are men and the rest are women.

109. The pound has a total of 258 dogs and cats.

110. It takes a total of 7 hours for Chris to row upstream and then return.

111. What values of x satisfy the equation |x| = 6?

112. What values of x satisfy the equation |x| = 791 ?