**SECTION A-5: SETS AND INTERVALS**

Recall that a **set** is written using set brackets {,}. You can, if the set is small enough, just list all the elements (the roster method) or you can write it out using set-builder notation.

For example, if we take the set of notes in a musical scale (a though g) and name the set ‘M’ we would have M = {a,b,c,d,e,f,g}, if we wrote it in the roster method

Set-builder notation for the same would be M = {x|x is a musical note} OR you could write M = {x|x is a letter of the English alphabet that comes before H.}

Example 1: Write the set (and name it ‘A’) of even positive integers less than 20, both using the roster method and using set-builder notation.

A = {2,4,6,8,10,12,14,16,18} (roster method)

A = {x|x <20, x ∈ positive integers}

Note that sometimes there is more than one way you could write set-builder notation, for example A = {x|0<x<20, x ∈ integers} would say the same thing because 0<x<20 is called a **between** statement, and we will look more specifically at between statements in chapter B.

Also, note that we specified that x was an integer. Unless some other set of numbers is specified (such as the integers) if x is a number then it is assumed to be a **real number**.

**THE EMPTY SET**

The set with no elements, { } is sometimes referred to as the **empty set**. The empty set is sometimes also written as ∅ . **NOTE:** Now that this notation for the empty set has been defined, do NOT write the number zero that way because the symbol ∅ is now reserved for the empty set.

Example 2: Write the set S of negative integers which are larger than 10.

S = { } or S = ∅ because there are no negative integers which are larger than ten.

**INTERSECTION OF SETS**:

 The **intersection** of two sets A and B, written A ∩ B consists of those elements which appear in both set A **AND also** in set B. Note that by definition the intersection of two sets cannot be larger than either of the sets individually.

Example 3: Let set N = {1,2,3,4,5,6} and set M = {2,4,6,8,10,12} Then find A ∩ B.

A ∩ B = {2,4,6} because the elements 2, 4 and 6 are the only things which are elements of set A and also are elements of set B.

Example 4: Let set S = {-3, 0, 4,6,9} and set T = {-1, 2, 3, 5, 7} Find S ∩ T.

S ∩ T = ∅ because there are **no** elements of S which are also elements of T.

**UNION OF SETS**:

The **union** of two sets A and B, written A ∪ B, consists of everything which is **either** an element of set A or of set B. Note that this certainly includes anything which is an element of both sets (the intersection) but you only list each element of a union **one time**. Because of the way a union of sets is defined, a union will necessarily be at least as large a set as the larger of A and B.

Example 5: Let set N = {1,2,3,4,5,6} and set M = {2,4,6,8,10,12} Then find A ∪ B.

A ∪ B = {1,2,3,4,5,6,8,10,12} because all of these elements are either in A or in B.

Also note that you only write 2,4 and 6 once each in the union.

**VARIABLE INEQUALITIES**:

Suppose that you have a statement like x < 2. Note that since no other set is specified, x is a **real number** less than two. In fact, there is no way we could list the set of all real numbers less than two using the roster method because there are infinitely many numbers less than four (don’t just think of integers either; for example, $\frac{7}{5}$ is a number which is less than two so it satisfies this inequality.) You could write this using set-builder notation, as {x|x<2} . However, we often find it more convenient to illustrate a set like this using a number line. For example in this case,

 -4 -3 -2 -1 0 1 2 3 4 5

Numbers which are **less than** two lie to the left of two. It could be any number less than 2 so you shade the entire number line to the left of two, including the arrow at the left end of the line to indicate that this set continues indefinitely to the left. Note that since 2 is not less than itself, we draw a parenthese right on two. This indicates an **open set**, or a set which does not include its ending point at two.

Let’s make one minor adjustment. Suppose that instead we are looking at the set {x|x ≤ 2} . The only difference is in the inequality, and specifically we now include the value 2 which could equal itself. We draw the number line as we did before:

 -4 -3 -2 -1 0 1 2 3 4 5

Note the square bracket on two indicating a closed set. This is because the number two itself is in the set.

Sometimes we combine sets, for example if we have {x|x<-4} ∩{x|x> 1}. In this case, we see that there is no number which is both less than -4 and larger than one at the same time. Therefore the intersection of these two sets is the empty set, and on a number line we would show this by shading nothing (leaving a blank number line.)

**PROBLEM SET A-5:**

In problems 1-8, give the set using the roster method and also using set-builder notation (unless the answer is the empty set, in which case you should write it.)

1. The set S of all letters in the word, ‘education’
2. The set S of all vowels in the word, ‘facetious’
3. The set V of all vowels in the word, ‘sequoia’
4. The set A of all even integers between 5 and 11.
5. The set A of all whole numbers less than 12.
6. The set N of all negative integers greater than -4.
7. The set N of all negative integers greater than 5.
8. The set B of all unicorns presently living on the earth.

In problems 9-12, the sets are infinite and so cannot be written using the roster method. Use set builder notation to describe them.

1. The set of all rational numbers between 0 and 5.
2. The set of all positive real numbers less than or equal to 9.
3. The set of all negative integers less than -13.
4. The set of all irrational numbers (recall that irrational numbers are real numbers which are not rational.)

In problems 13- 20 two sets are given, A and B. In each problem find A ∩ B and also A ∪ B

1. A = {3,5,6,9} B = {0, 2, 4, 6} 14. A = { h, i, j, k} B = {e, f, g, h, i, j }

15. A = { 2, 3, 4} B = { 3, 5, 7 } 16. A = { a, e, i, o, u} B = {s, a, f, e}

17. A = {q, r, s, t, u} B = {r, s, t} 18. A = {5, 10, 15, 20} B = {1, 2, 4, 8}

19. A = {K, L, M, N, O, P} B = {1, 2, 3 } 20. A = {5, 7, 10, 13} B = ∅

In problems 21-30, draw and label a number line and then shade the regions covered by the following sets.

21. {x|x > -1 } 22. {x|x ≤ 3} 23. {x|x ≥ 0}

24. {x|-2 < x } 25. {x| x <5} ∩ {x| x > 2} 26. {x|x ≤ 3} ∩ {x|x ≥ 1}

27. {x|x> 1} ∪ {x|x < -3} 28. {x|x< 0} ∪ {x|x≥ 2} 29. {x|x ≥ 2} ∩ {x|x ≤ -3}

30. {x|x > -1} ∩ {x|x < -4} 31. {x|x < 4} ∪ {x|x > -1} 32. {x|x ≥-3} ∪ {x|x < 0}