ORDER OF OPERATIONS

When you calculate numbers in arithmetic, it is important that you use a specific **order of operations** to make sure that the calculations always result in the same answer. For example, suppose that we consider the calculation 4 + 7·2 .

IF we were to add 4 +7 first to get 11 and then multiply by 2 we would get 22. On the other hand if we were to multiply 7·2 first to get 14 and then add four we would get 18. Clearly which way we do these operations makes a difference. So which one is right? The answer can be found by what is called the hierarchy of order of operations, which you may have seen written as PEMDAS (**p**arentheses, **e**xponent, **m**ultiplication, **d**ivision, **a**ddition and **s**ubtraction.) Some people have learned the mnemonic sentence “Please Excuse My Dear Aunt Sally” to remember these. However it is important to note that in fact division is equivalent to multiplication and addition is equivalent to subtraction so a better way to write it would be like this:

P

E

MD

AS

So because multiplication is higher in the order of operations than addition, the correct way is to do the multiplication first so that

**4+ 7·2 = 4 + 14 = 18**. Any other way of computing this would not the correct.

**ADDITION AND SUBTRACTION:**

As described in section A.1, when we have multiple additions and subtractions, because in fact addition and subtraction are considered equivalent operations under the order of operations hierarchy, we must go from left to right. This is especially important in cases in which we have a subtraction followed by a subsequent addition. For example,

Example 1:

13 – 5 + 4 = 8 + 4 = **12**.

Some people if they went strictly by the order of all the letters in ‘PEMDAS’ might be fooled into adding 5 + 4 first and then subtracting that from 13 to get the wrong answer.

**MULTIPLICATION AND DIVISION:**

Just like with addition and subtraction, if there is more than one multiplication and/or division we go from left to right, and the multiplication or division sign indicates multiplication or division only among those numbers that immediately follow it. For example,

Example 2:

5 + 18÷2$·$3= 5 + 9·3 = 5 + 27 = **32**.

Note that we divide by 2 before we multiply by three because the division is written before the multiplication. Some people would erroneously multiply first and try to divide 18 by 6 and get it wrong.

Example 3:

10 - 20÷5 = 10 – 4 = 6. Note that we divide before we subtract.

Example 4:

14 + 2 ÷ 4 = 14 + $\frac{1}{2}$ = $\frac{14}{1}$ · $\frac{2}{2}$ + $\frac{1}{2}$ = $\frac{28}{2}$ + $\frac{1}{2}$ = $\frac{29}{2}$

Also be aware that a division can also be written as a fraction, so for example

Example 5: 4 – $\frac{15}{3}$ ·4 = 4 – 5·4 = 4 – 20 = **-16**.

**EXPONENTS:**

Exponents should always be evaluated before multiplication and division, as well as before adding and subtracting. It is a common mistake that some people want to multiply first, but unless the multiplication is inside parentheses you should not do so. Also note that a negative sign in front of a number is not subject to the exponent unless the sign is inside parentheses.

Example 6: 7 + 5·22 = 7 + 5·4 = 7 + 20 = **27**.

Example 7: -62 + 53 = -36 + 125 = **89**.

**PARENTHESES:**

Anytime part of an arithmetic expression is in parentheses then you must compute whatever is inside the parentheses before you compute whatever is on the outside of it.

Example 8: 8(3+4)

8(3+4) = 8·7 = **56**.

Normally we would multiply before we would add. But because the addition is inside parentheses we first add 3 + 4 and **then** we multiply by eight.

Example 9: 5 -2(2-6)2 ÷ 8 + 2

5 -2(2-6)2÷8+2 = 5-2(-4)2÷8+2 = 5-2(16)÷8+2 = 5 – 32 ÷ 8 + 2 = 5 – 4 + 2 = 1 + 2 = **3**

Note that you should compute 2 – 6 = -4, before taking the exponent. A common mistake that people sometimes make is to try to distribute the exponent over a plus or a minus inside parentheses, that is to claim that (2-6)2 was the same thing as 22 – 62 even though in fact it is different. In fact in general (a+b)2 **IS NOT** a2 + b2.

**IMPLIED PARENTHESES**:

As mentioned on the last page, a division can be written as a fraction. Sometimes the numerator and denominator of the fraction itself have to be simplified, in which case what you essentially have is (numerator)÷(denominator); that is to say that you first simplify the numerator and the denominator and then do the division. There are no parentheses expressly written in the fraction but they could be since you may be adding or subtracting within the numerator and the denominator.

Example 10: 5 + $\frac{15+3}{13-4}$ - 1

5 + $\frac{15+3}{13-4}$ – 1 = 5 + $\frac{18}{9}$ - 1 = 5 + 2 – 1 = **6**.

In this case you added 15 + 3 and subtracted 13 – 4 first even though they were not inside parentheses because there are implied parentheses around the numerator and denominator of the fraction.

**ABSOLUTE VALUE**:

If an expression contains an absolute value then you in effect treat it like parentheses, as far as prioritizing it by computing the inside, except that as an absolute value if the inside turns out to be negative you then have to evaluate the absolute value which in effect makes it positive.

Example 11: 2 + 8|22 – 7| ÷ 12

2 + 8|22 – 7| ÷ 12 = 2 + 8|4-7| ÷ 12 = 2 + 8| -3| ÷ 12 =

2 + 8(3) ÷ 12 = 2 + 24 ÷ 12 = 2 + 2 = **4**.

**PROBLEM SET A-3**:

Calculate the values:

1. 3 + 4·5 2. -6 -2·5 3. 3·5 + 8÷2 4. 16 – 9 + 4

5. 50 – 6 + 17 6. 42 ÷ 7·2 7. -72 + 5·3 8. 6 · 3 + 8

9. 6(3+8) 10. (5 + 3)2 ÷2 11. 9 -2(1-5)2 ÷ 16 12. 9 -3(3+6)2 ÷ 27

13. 2+5(3-7) 14. 4+2(2+9) 15. 3+ (5+2)2 ÷ 3 16. 5 + 6(3 + 23)

17. 3 – 5|2-4| 18. 6 + 4|3 + 4| 19. 5 + 8· $\frac{3+7}{4(5-1)}$ 20. 3 – $\frac{8 (1+3)}{(11-7)}$2

REVIEW

In problems 21- 28 give all values of the variable that satisfy the given equations.

1. |x| = -7 22 . |x| = 36 23. |x| = 25 24. |x| = 0

25. |x| = 10 26. |x| = -8 27. |x| = 16 28. |x| = 7