1.1 Simplifying Expressions

If a cellular phone company charges \$35 per month plus \$0.25 for each minute, or fraction of a minute, that you use one of their cellular phones, then the amount of your monthly bill is given by the expression $35 + 0.25t$. To find the amount you will pay for using that phone 30 minutes in one month, you substitute 30 for t and simplify the resulting expression. This process is one of the topics we will study in this section.

As you will see in the next few sections, the first step in solving an equation is to simplify both sides as much as possible. In the first part of this section, we will practice simplifying expressions by combining what are called *similar* (or like) terms.

For our immediate purposes, a *term* is a number or a number and one or more variables multiplied together. For example, the number 5 is a term, as are the expressions $3x$, $-7y$, and $15xy$.

(def DEFINITION *similar terms*

Two or more terms with the same variable part are called ***similar*** (or ***like***) ***terms.***

The terms $3x$ and $4x$ are similar because their variable parts are identical. Likewise, the terms $18y$, $-10y$, and $6y$ are similar terms.

To simplify an algebraic expression, we simply reduce the number of terms in the expression. We accomplish this by applying the distributive property along with our knowledge of addition and subtraction of positive and negative real numbers. The following examples illustrate the procedure.

EXAMPLE 1 Simplify by combining similar terms.

a. $3x + 4x$ **b.** $7a - 10a$ **c.** $18y - 10y + 6y$

SOLUTION We combine similar terms by applying the distributive property.

a. $3x + 4x = (3 + 4)x$ Distributive property

$= 7x$ Addition of 3 and 4

b. $7a - 10a = (7 - 10)a$ Distributive property

$= -3a$ Addition of 7 and -10

c. $18y - 10y + 6y = (18 - 10 + 6)y$ Distributive property

$= 14y$ Addition of 18, -10, and 6 ■

When the expression we intend to simplify is more complicated, we use the commutative and associative properties first.

EXAMPLE 2 Simplify each expression.

a. $3x + 5 + 2x - 3$ **b.** $4a - 7 - 2a + 3$ **c.** $5x + 8 - x - 6$

SOLUTION We combine similar terms by applying the distributive property.

a.
$$\begin{aligned} 3x + 5 + 2x - 3 &= 3x + 2x + 5 - 3 && \text{Commutative property} \\ &= (3x + 2x) + (5 - 3) && \text{Associative property} \\ &= (3 + 2)x + (5 - 3) && \text{Distributive property} \\ &= 5x + 2 && \text{Addition} \end{aligned}$$

b.
$$\begin{aligned} 4a - 7 - 2a + 3 &= (4a - 2a) + (-7 + 3) && \text{Commutative and associative properties} \\ &= (4 - 2)a + (-7 + 3) && \text{Distributive property} \\ &= 2a - 4 && \text{Addition} \end{aligned}$$

c.
$$\begin{aligned} 5x + 8 - x - 6 &= (5x - x) + (8 - 6) && \text{Commutative and associative properties} \\ &= (5 - 1)x + (8 - 6) && \text{Distributive property} \\ &= 4x + 2 && \text{Addition} \end{aligned}$$

Notice that in each case the result has fewer terms than the original expression. Because there are fewer terms, the resulting expression is said to be simpler than the original expression.

Simplifying Expressions Containing Parentheses

If an expression contains parentheses, it is often necessary to apply the distributive property to remove the parentheses before combining similar terms.

EXAMPLE 3 Simplify the expression $5(2x - 8) - 3$.

SOLUTION We begin by distributing the 5 across $2x - 8$. We then combine similar terms:

$$\begin{aligned} 5(2x - 8) - 3 &= 10x - 40 - 3 && \text{Distributive property} \\ &= 10x - 43 \end{aligned}$$

EXAMPLE 4 Simplify $7 - 3(2y + 1)$.

SOLUTION By the rule for order of operations, we must multiply before we add or subtract. For that reason, it would be incorrect to subtract 3 from 7 first. Instead, we multiply -3 and $2y + 1$ to remove the parentheses and then combine similar terms:

$$\begin{aligned} 7 - 3(2y + 1) &= 7 - 6y - 3 && \text{Distributive property} \\ &= -6y + 4 \end{aligned}$$

EXAMPLE 5 Simplify $5(x - 2) - (3x + 4)$.

SOLUTION We begin by applying the distributive property to remove the parentheses. The expression $-(3x + 4)$ can be thought of as $-1(3x + 4)$. Thinking of it in this way allows us to apply the distributive property:

$$\begin{aligned} -1(3x + 4) &= -1(3x) + (-1)(4) \\ &= -3x - 4 \end{aligned}$$

The complete solution looks like this:

$$\begin{aligned} 5(x - 2) - (3x + 4) &= 5x - 10 - 3x - 4 && \text{Distributive property} \\ &= 2x - 14 && \text{Combine similar terms} \end{aligned}$$

As you can see from the explanation in Example 5, we use the distributive property to simplify expressions in which parentheses are preceded by a negative sign. In general we can write

$$\begin{aligned} -(a + b) &= -1(a + b) \\ &= -a + (-b) \\ &= -a - b \end{aligned}$$

The negative sign outside the parentheses ends up changing the sign of each term within the parentheses. In words, we say "the opposite of a sum is the sum of the opposites."

The Value of an Expression

An expression like $3x + 2$ has a certain value depending on what number we assign to x. For instance, when x is 4, $3x + 2$ becomes $3(4) + 2$, or 14. When x is -8, $3x + 2$ becomes $3(-8) + 2$, or -22. The value of an expression is found by replacing the variable with a given number.

EXAMPLE 6 Find the value of the following expressions by replacing the variable with the given number.

Expression	The Variable	Value of the Expression
a. $3x - 1$	$x = 2$	$3(2) - 1 = 6 - 1 = 5$
b. $7a + 4$	$a = -3$	$7(-3) + 4 = -21 + 4 = -17$
c. $2x - 3 + 4x$	$x = -1$	$2(-1) - 3 + 4(-1) = -2 - 3 + (-4)$ $= -9$
d. $2x - 5 - 8x$	$x = 5$	$2(5) - 5 - 8(5) = 10 - 5 - 40$ $= -35$
e. $y^2 - 6y + 9$	$y = 4$	$4^2 - 6(4) + 9 = 16 - 24 + 9 = 1$

Simplifying an expression should not change its value; that is, if an expression has a certain value when x is 5, then it will always have that value no matter how much it has been simplified as long as x is 5. If we were to simplify the expression in Example 6d first, it would look like

$$2x - 5 - 8x = -6x - 5$$

When x is 5, the simplified expression $-6x - 5$ is

$$-6(5) - 5 = -30 - 5 = -35$$

It has the same value as the original expression when x is 5.

We also can find the value of an expression that contains two variables if we know the values for both variables.

EXAMPLE 7 Find the value of the expression $2x - 3y + 4$ when x is -5 and y is 6.

SOLUTION Substituting -5 for x and 6 for y, the expression becomes

$$\begin{aligned} 2(-5) - 3(6) + 4 &= -10 - 18 + 4 \\ &= -28 + 4 \\ &= -24 \end{aligned}$$

■

EXAMPLE 8 Find the value of the expression $x^2 - 2xy + y^2$ when x is 3 and y is -4.

SOLUTION Replacing each x in the expression with the number 3 and each y in the expression with the number -4 gives us

$$\begin{aligned} 3^2 - 2(3)(-4) + (-4)^2 &= 9 - 2(3)(-4) + 16 \\ &= 9 - (-24) + 16 \\ &= 33 + 16 \\ &= 49 \end{aligned}$$

■

More About Sequences

As the next example indicates, when we substitute the counting numbers, in order, into algebraic expressions, we form some of the sequences of numbers that we studied in Chapter 1. To review, recall that the sequence of counting numbers (also called the sequence of positive integers) is

$$\text{Counting numbers} = 1, 2, 3, \ldots$$

EXAMPLE 9 Substitute 1, 2, 3, and 4 for n in the expression $2n - 1$.

SOLUTION Substituting as indicated, we have

When $n = 1$, $2n - 1 = 2 \cdot 1 - 1 = 1$

When $n = 2$, $2n - 1 = 2 \cdot 2 - 1 = 3$

When $n = 3$, $2n - 1 = 2 \cdot 3 - 1 = 5$

When $n = 4$, $2n - 1 = 2 \cdot 4 - 1 = 7$

As you can see, substituting the first four counting numbers into the formula $2n - 1$ produces the first four numbers in the sequence of odd numbers. ■

The next example is similar to Example 9 but uses tables to display the information.

EXAMPLE 10 Fill in the tables below to find the sequences formed by substituting the first four counting numbers into the expressions $2n$ and n^2.

a.

n	1	2	3	4
$2n$				

b.

n	1	2	3	4
n^2				

SOLUTION Proceeding as we did in the previous example, we substitute the numbers 1, 2, 3, and 4 into the given expressions.

a. When $n = 1$, $2n = 2 \cdot 1 = 2$
When $n = 2$, $2n = 2 \cdot 2 = 4$
When $n = 3$, $2n = 2 \cdot 3 = 6$
When $n = 4$, $2n = 2 \cdot 4 = 8$

As you can see, the expression $2n$ produces the sequence of even numbers when n is replaced by the counting numbers. Placing these results into our first table gives us

n	1	2	3	4
$2n$	2	4	6	8

b. The expression n^2 produces the sequence of squares when n is replaced by 1, 2, 3, and 4. In table form we have

n	1	2	3	4
n^2	1	4	9	16

■

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

A. What are similar terms?
B. Explain how the distributive property is used to combine similar terms.
C. What is wrong with writing $3x + 4x = 7x^2$?
D. Explain how you would find the value of $5x + 3$ when x is 6.

Problem Set 1.1

Simplify the following expressions.

1. $3x - 6x$ **2.** $7x - 5x$ **3.** $-2a + a$

4. $3a - a$ **5.** $7x + 3x + 2x$ **6.** $8x - 2x - x$

7. $3a - 2a + 5a$ **8.** $7a - a + 2a$ **9.** $4x - 3 + 2x$

10. $5x + 6 - 3x$ **11.** $3a + 4a + 5$ **12.** $6a + 7a + 8$

13. $2x - 3 + 3x - 2$ **14.** $6x + 5 - 2x + 3$ **15.** $3a - 1 + a + 3$

16. $-a + 2 + 8a - 7$ **17.** $-4x + 8 - 5x - 10$ **18.** $-9x - 1 + x - 4$

19. $7a + 3 + 2a + 3a$ **20.** $8a - 2 + a + 5a$ **21.** $5(2x - 1) + 4$

22. $2(4x - 3) + 2$ **23.** $7(3y + 2) - 8$ **24.** $6(4y + 2) - 7$

25. $-3(2x - 1) + 5$ **26.** $-4(3x - 2) - 6$ **27.** $5 - 2(a + 1)$

28. $7 - 8(2a + 3)$ **29.** $6 - 4(x - 5)$ **30.** $12 - 3(4x - 2)$

31. $-9 - 4(2 - y) + 1$ **32.** $-10 - 3(2 - y) + 3$ **33.** $-6 + 2(2 - 3x) + 1$

34. $-7 - 4(3 - x) + 1$ **35.** $(4x - 7) - (2x + 5)$ **36.** $(7x - 3) - (4x + 2)$

37. $8(2a + 4) - (6a - 1)$ **38.** $9(3a + 5) - (8a - 7)$ **39.** $3(x - 2) + (x - 3)$

40. $2(2x + 1) - (x + 4)$ **41.** $4(2y - 8) - (y + 7)$ **42.** $5(y - 3) - (y - 4)$

43. $-9(2x + 1) - (x + 5)$ **44.** $-3(3x - 2) - (2x + 3)$

Evaluate the following expressions when x is 2. (Find the value of the expressions if x is 2.)

45. $3x - 1$ **46.** $4x + 3$ **47.** $-2x - 5$ **48.** $-3x + 6$

49. $x^2 - 8x + 16$ **50.** $x^2 - 10x + 25$ **51.** $(x - 4)^2$ **52.** $(x - 5)^2$

Evaluate the following expressions when x is -5. Then simplify the expression, and check to see that it has the same value for $x = -5$.

53. $7x - 4 - x - 3$ **54.** $3x + 4 + 7x - 6$

55. $5(2x + 1) + 4$ **56.** $2(3x - 10) + 5$

Evaluate the following expressions when x is -3 and y is 5.

57. $x^2 - 2xy + y^2$ **58.** $x^2 + 2xy + y^2$ **59.** $(x - y)^2$

60. $(x + y)^2$ **61.** $x^2 + 6xy + 9y^2$ **62.** $x^2 + 10xy + 25y^2$

63. $(x + 3y)^2$ **64.** $(x + 5y)^2$

Find the value of $12x - 3$ for each of the following values of x.

65. $\frac{1}{2}$ **66.** $\frac{1}{3}$ **67.** $\frac{1}{4}$ **68.** $\frac{1}{6}$

69. $\frac{3}{2}$ **70.** $\frac{2}{3}$ **71.** $\frac{3}{4}$ **72.** $\frac{5}{6}$

1. $-3x$
2. $2x$
3. $-a$
4. $2a$
5. $12x$
6. $5x$
7. $6a$
8. $8a$
9. $6x - 3$
10. $2x + 6$
11. $7a + 5$
12. $13a + 8$
13. $5x - 5$
14. $4x + 8$
15. $4a + 2$
16. $7a - 5$
17. $-9x - 2$
18. $-8x - 5$
19. $12a + 3$
20. $14a - 2$
21. $10x - 1$
22. $8x - 4$
23. $21y + 6$
24. $24y + 5$
25. $-6x + 8$
26. $-12x + 2$
27. $-2a + 3$
28. $-16a - 17$
29. $-4x + 26$
30. $-12x + 18$
31. $4y - 16$
32. $3y - 13$
33. $-6x - 1$
34. $4x - 18$
35. $2x - 12$
36. $3x - 5$
37. $10a + 33$
38. $19a + 52$
39. $4x - 9$
40. $3x - 2$
41. $7y - 39$
42. $4y - 11$
43. $-19x - 14$
44. $-11x + 3$
45. 5
46. 11
47. -9
48. 0
49. 4
50. 9
51. 4
52. 9
53. -37
54. -52
55. -41
56. -45
57. 64
58. 4
59. 64
60. 4
61. 144

62. 484
63. 144
64. 484
65. 3
66. 1
67. 0
68. -1
69. 15
70. 5
71. 6
72. 7
75. 1, 4, 7, 10, . . .
76. -1, 1, 3, 5, . . .
77. 0, 1, 4, 9, . . .
78. 0, 1, 4, 9, . . .
79. $-6y + 4$
80. $6x - 3$
81. $0.17x$
82. $0.09x$
83. $2x$
84. $13x$
85. $5x - 4$
86. $17x - 4$
87. $7x - 5$
88. $8x + 2$
89. $-2x - 9$
90. $-x - 4$
91. $7x + 2$
92. $8x - 2$
93. $-7x + 6$
94. $-14x + 1$
95. $7x$
96. $-7y$
97. $-y$
98. x
99. $10y$
100. $-2x$
101. $0.17x + 180$
102. $0.1x + 280$
103. $0.22x + 60$
104. $0.14x + 48$
105. 49
106. 8
107. 40
108. 40
109. a. 42°F
b. 28°F
c. -14°F

73. Fill in the tables below to find the sequences formed by substituting the first four counting numbers into the expressions $3n$ and n^3.

a.

n	1	2	3	4
$3n$	3	6	9	12

b.

n	1	2	3	4
n^3	1	8	27	64

74. Fill in the tables below to find the sequences formed by substituting the first four counting numbers into the expressions $2n - 1$ and $2n + 1$.

a.

n	1	2	3	4
$2n - 1$	1	3	5	7

b.

n	1	2	3	4
$2n + 1$	3	5	7	9

Find the sequences formed by substituting the first four counting numbers, in order, into the following expressions.

75. $3n - 2$ **76.** $2n - 3$ **77.** $n^2 - 2n + 1$ **78.** $(n - 1)^2$

Here are some problems you will see later in the book. Simplify.

79. $7 - 3(2y + 1)$
80. $4(3x - 2) - (6x - 5)$
81. $0.08x + 0.09x$
82. $0.04x + 0.05x$
83. $(x + y) + (x - y)$
84. $(-12x - 20y) + (25x + 20y)$
85. $3x + 2(x - 2)$
86. $2(x - 2) + 3(5x)$
87. $4(x + 1) + 3(x - 3)$
88. $5(x + 1) + 3(x - 1)$
89. $x + (x + 3)(-3)$
90. $x - 2(x + 2)$
91. $3(4x - 2) - (5x - 8)$
92. $2(5x - 3) - (2x - 4)$
93. $-(3x + 1) - (4x - 7)$
94. $-(6x + 2) - (8x - 3)$
95. $(x + 3y) + 3(2x - y)$
96. $(2x - y) - 2(x + 3y)$
97. $3(2x + 3y) - 2(3x + 5y)$
98. $5(2x + 3y) - 3(3x + 5y)$
99. $-6\left(\frac{1}{2}x - \frac{1}{3}y\right) + 12\left(\frac{1}{4}x + \frac{2}{3}y\right)$
100. $6\left(\frac{1}{3}x + \frac{1}{2}y\right) - 4\left(x + \frac{3}{4}y\right)$
101. $0.08x + 0.09(x + 2{,}000)$
102. $0.06x + 0.04(x + 7{,}000)$
103. $0.10x + 0.12(x + 500)$
104. $0.08x + 0.06(x + 800)$

Find the value of $b^2 - 4ac$ for the given values of a, b, and c. (You will see these problems later in the book.)

105. $a = 1, b = -5, c = -6$
106. $a = 1, b = -6, c = 7$
107. $a = 2, b = 4, c = -3$
108. $a = 3, b = 4, c = -2$

Applying the Concepts

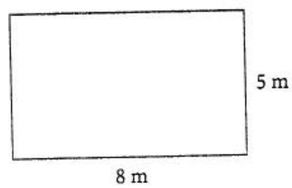
109. Temperature and Altitude If the temperature on the ground is 70°F, then the temperature at A feet above the ground can be found from the expression $-0.0035A + 70$. Find the temperature at the following altitudes.

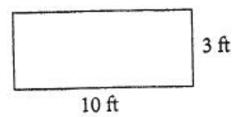
a. 8,000 feet **b.** 12,000 feet **c.** 24,000 feet

110. Perimeter of a Rectangle The expression $2l + 2w$ gives the perimeter of a rectangle with length l and width w. Find the perimeter of the rectangles with the following lengths and widths.

a. Length = 8 meters
Width = 5 meters

b. Length = 10 feet
Width = 3 feet





111. Cellular Phone Rates A cellular phone company charges \$35 per month plus \$0.25 for each minute, or fraction of a minute, that you use one of their cellular phones. The expression $35 + 0.25t$ gives the amount of money you will pay for using one of their phones for t minutes a month. Find the monthly bill for using one of their phones.

a. 10 minutes in a month **b.** 20 minutes in a month
c. 30 minutes in a month

112. Cost of Bottled Water A water bottling company charges \$7.00 per month for their water dispenser and \$1.10 for each gallon of water delivered. If you have g gallons of water delivered in a month, then the expression $7 + 1.1g$ gives the amount of your bill for that month. Find the monthly bill for each of the following deliveries.

a. 10 gallons **b.** 20 gallons **c.** 30 gallons

Getting Ready for the Next Section

These are problems that you must be able to work in order to understand the material in the next section. The problems below are exactly the type of problems you will see in the explanations and examples in the next section.

Simplify.

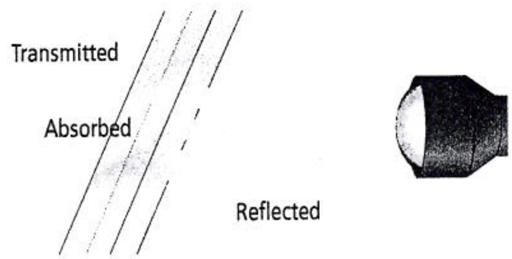
113. $17 - 5$
114. $12 + (-2)$
115. $2 - 5$
116. $25 - 20$
117. $-2.4 + (-7.3)$
118. $8.1 + 2.7$
119. $-\frac{1}{2} + \left(-\frac{3}{4}\right)$
120. $-\frac{1}{6} + \left(-\frac{2}{3}\right)$
121. $4(2 \cdot 9 - 3) - 7$
122. $5(3 \cdot 45 - 4) - 14 \cdot 45$
123. $4(2a - 3) - 7a$
124. $5(3a - 4) - 14a$
125. Find the value of $2x - 3$ when x is 5
126. Find the value of $3x + 4$ when x is -2

110. a. 26 meters
b. 26 feet
111. a. \$37.50
b. \$40.00
c. \$42.50
112. a. \$18
b. \$29
c. \$40
113. 12
114. 10
115. -3
116. 5
117. -9.7
118. 10.8
119. $-\frac{5}{4}$
120. $-\frac{5}{6}$
121. 53
122. 25
123. $a - 12$
124. $a - 20$
125. 7
126. -2

Addition Property of Equality

1.2

When light comes into contact with any object, it is reflected, absorbed, and transmitted, as shown below.



For a certain type of glass, 88% of the light hitting the glass is transmitted through to the other side, whereas 6% of the light is absorbed into the glass. To find the percent of light that is reflected by the glass, we can solve the equation

$$88 + R + 6 = 100$$

Solving equations of this type is what we study in this section. To solve an equation we must find all replacements for the variable that make the equation a true statement.

déf DEFINITION *solution set*

The ***solution set*** for an equation is the set of all numbers that when used in place of the variable make the equation a true statement

For example, the equation $x + 2 = 5$ has the solution set $\{3\}$ because when x is 3 the equation becomes the true statement $3 + 2 = 5$, or $5 = 5$.

EXAMPLE 1 Is 5 a solution to $2x - 3 = 7$?

SOLUTION We substitute 5 for x in the equation, and then simplify to see if a true statement results. A true statement means we have a solution; a false statement indicates the number we are using is not a solution.

When	$x = 5$	
the equation	$2x - 3 = 7$	
becomes	$2(5) - 3 \stackrel{?}{=} 7$	
	$10 - 3 \stackrel{?}{=} 7$	
	$7 = 7$	A true statement

Because $x = 5$ turns the equation into the true statement $7 = 7$, we know 5 is a solution to the equation. ■

Note: We can use a question mark over the equal signs to show that we don't know yet whether the two sides of the equation are equal.

EXAMPLE 6 Solve for x: $7.3 + x = -2.4$.

SOLUTION Again, we want to isolate x, so we add the opposite of 7.3 to both sides:

$$7.3 + (-7.3) + x = -2.4 + (-7.3) \qquad \text{Addition property of equality}$$
$$0 + x = -9.7$$
$$x = -9.7$$

■

Sometimes it is necessary to simplify each side of an equation before using the addition property of equality. The reason we simplify both sides first is that we want as few terms as possible on each side of the equation before we use the addition property of equality. The following examples illustrate this procedure.

EXAMPLE 7 Solve for x : $-x + 2 + 2x = 7 + 5$.

SOLUTION We begin by combining similar terms on each side of the equation. Then we use the addition property to solve the simplified equation.

$$x + 2 = 12 \qquad \text{Simplify both sides first}$$
$$x + 2 + (-2) = 12 + (-2) \qquad \text{Addition property of equality}$$
$$x + 0 = 10$$
$$x = 10$$

■

EXAMPLE 8 Solve $4(2a - 3) - 7a = 2 - 5$.

SOLUTION We must begin by applying the distributive property to separate terms on the left side of the equation. Following that, we combine similar terms and then apply the addition property of equality.

$$4(2a - 3) - 7a = 2 - 5 \qquad \text{Original equation}$$
$$8a - 12 - 7a = 2 - 5 \qquad \text{Distributive property}$$
$$a - 12 = -3 \qquad \text{Simplify each side}$$
$$a - 12 + 12 = -3 + 12 \qquad \text{Add 12 to each side}$$
$$a = 9 \qquad \text{Addition}$$

To check our solution, we replace a with 9 in the original equation.

$$4(2 \cdot 9 - 3) - 7 \cdot 9 \stackrel{?}{=} 2 - 5$$
$$4(15) - 63 \stackrel{?}{=} -3$$
$$60 - 63 \stackrel{?}{=} -3$$
$$-3 = -3 \qquad \text{A true statement}$$

■

Note: Again, we place a question mark over the equal sign because we don't know yet whether the expressions on the left and right side of the equal sign will be equal.

We can also add a term involving a variable to both sides of an equation.

EXAMPLE 9 Solve $3x - 5 = 2x + 7$.

SOLUTION We can solve this equation in two steps. First, we add $-2x$ to both sides of the equation. When this has been done, x appears on the left side only. Second, we add 5 to both sides:

$$3x + (-2x) - 5 = 2x + (-2x) + 7 \quad \text{Add } -2x \text{ to both sides}$$
$$x - 5 = 7 \quad \text{Simplify each side}$$
$$x - 5 + 5 = 7 + 5 \quad \text{Add 5 to both sides}$$
$$x = 12 \quad \text{Simplify each side}$$

Note: In my experience teaching algebra, I find that students make fewer mistakes if they think in terms of addition rather than subtraction. So, you are probably better off if you continue to use the addition property just the way we have used it in the examples in this section. But, if you are curious as to whether you can subtract the same number from both sides of an equation, the answer is yes.

$[\Delta \neq \Sigma$ PROPERTY ***A Note on Subtraction***

Although the addition property of equality is stated for addition only, we can subtract the same number from both sides of an equation as well. Because subtraction is defined as addition of the opposite, subtracting the same quantity from both sides of an equation does not change the solution.

$$x + 2 = 12 \quad \text{Original equation}$$
$$x + 2 - 2 = 12 - 2 \quad \text{Subtract 2 from each side}$$
$$x = 10 \quad \text{Subtraction}$$

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

A. What is a solution to an equation?
B. What are equivalent equations?
C. Explain in words the addition property of equality.
D. How do you check a solution to an equation?

Problem Set 1.2

Solve the following equations.

1. $x - 3 = 8$ **2.** $x - 2 = 7$ **3.** $x + 2 = 6$

4. $x + 5 = 4$ **5.** $a + \frac{1}{2} = -\frac{1}{4}$ **6.** $a + \frac{1}{3} = -\frac{5}{6}$

7. $x + 2.3 = -3.5$ **8.** $x + 7.9 = 23.4$ **9.** $y + 11 = -6$

10. $y - 3 = -1$ **11.** $x - \frac{5}{8} = -\frac{3}{4}$ **12.** $x - \frac{2}{5} = -\frac{1}{10}$

13. $m - 6 = -10$ **14.** $m - 10 = -6$ **15.** $6.9 + x = 3.3$

16. $7.5 + x = 2.2$ **17.** $5 = a + 4$ **18.** $12 = a - 3$

19. $-\frac{5}{9} = x - \frac{2}{5}$ **20.** $-\frac{7}{8} = x - \frac{4}{5}$

Simplify both sides of the following equations as much as possible, and then solve.

21. $4x + 2 - 3x = 4 + 1$ **22.** $5x + 2 - 4x = 7 - 3$

23. $8a - \frac{1}{2} - 7a = \frac{3}{4} + \frac{1}{8}$ **24.** $9a - \frac{4}{5} - 8a = \frac{3}{10} - \frac{1}{5}$

25. $-3 - 4x + 5x = 18$ **26.** $10 - 3x + 4x = 20$

27. $-11x + 2 + 10x + 2x = 9$ **28.** $-10x + 5 - 4x + 15x = 0$

29. $-2.5 + 4.8 = 8x - 1.2 - 7x$ **30.** $-4.8 + 6.3 = 7x - 2.7 - 6x$

31. $2y - 10 + 3y - 4y = 18 - 6$ **32.** $15 - 21 = 8x + 3x - 10x$

The following equations contain parentheses. Apply the distributive property to remove the parentheses, then simplify each side before using the addition property of equality.

33. $2(x + 3) - x = 4$ **34.** $5(x + 1) - 4x = 2$

35. $-3(x - 4) + 4x = 3 - 7$ **36.** $-2(x - 5) + 3x = 4 - 9$

37. $5(2a + 1) - 9a = 8 - 6$ **38.** $4(2a - 1) - 7a = 9 - 5$

39. $-(x + 3) + 2x - 1 = 6$ **40.** $-(x - 7) + 2x - 8 = 4$

41. $4y - 3(y - 6) + 2 = 8$ **42.** $7y - 6(y - 1) + 3 = 9$

43. $-3(2m - 9) + 7(m - 4) = 12 - 9$ **44.** $-5(m - 3) + 2(3m + 1) = 15 - 8$

Solve the following equations by the method used in Example 9 in this section. Check each solution in the original equation.

45. $4x = 3x + 2$ **46.** $6x = 5x - 4$ **47.** $8a = 7a - 5$

48. $9a = 8a - 3$ **49.** $2x = 3x + 1$ **50.** $4x = 3x + 5$

51. $3y + 4 = 2y + 1$ **52.** $5y + 6 = 4y + 2$ **53.** $2m - 3 = m + 5$

54. $8m - 1 = 7m - 3$ **55.** $4x - 7 = 5x + 1$ **56.** $3x - 7 = 4x - 6$

57. $5x - \frac{2}{3} = 4x + \frac{4}{3}$ **58.** $3x - \frac{5}{4} = 2x + \frac{1}{4}$ **59.** $8a - 7.1 = 7a + 3.9$

60. $10a - 4.3 = 9a + 4.7$

1. 11
2. 9
3. 4
4. -1
5. $-\frac{3}{4}$
6. $-\frac{7}{6}$
7. -5.8
8. 15.5
9. -17
10. 2
11. $-\frac{1}{8}$
12. $\frac{3}{10}$
13. -4
14. 4
15. -3.6
16. -5.3
17. 1
18. 15
19. $-\frac{7}{45}$
20. $-\frac{3}{40}$
21. 3
22. 2
23. $\frac{11}{8}$
24. $\frac{9}{10}$
25. 21
26. 10
27. 7
28. -5
29. 3.5
30. 4.2
31. 22
32. -6
33. -2
34. -3
35. -16
36. -15
37. -3
38. 8
39. 10
40. 5
41. -12
42. 0
43. 4
44. -10
45. 2
46. -4
47. -5
48. -3
49. -1
50. 5
51. -3
52. -4
53. 8
54. -2
55. -8
56. -1
57. 2

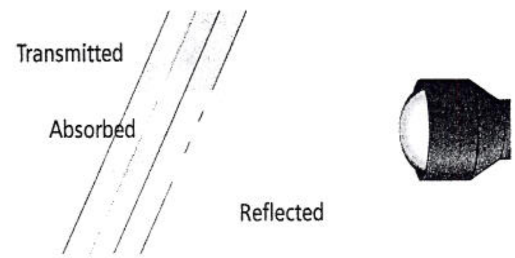
58. $\frac{3}{2}$
59. 11
60. 9
61. -5.8
62. 19.8
63. a. 6%
b. 5%
c. 2%
d. 75%

61. $11y - 2.9 = 12y + 2.9$

62. $20y + 9.9 = 21y - 9.9$

Applying the Concepts

63. Light When light comes into contact with any object, it is reflected, absorbed, and transmitted, as shown in the following figure. If T represents the percent of light transmitted, R the percent of light reflected, and A the percent of light absorbed by a surface, then the equation $T + R + A = 100$ shows one way these quantities are related.



a. For glass, $T = 88$ and $A = 6$, meaning that 88% of the light hitting the glass is transmitted and 6% is absorbed. Substitute $T = 88$ and $A = 6$ into the equation $T + R + A = 100$ and solve for R to find the percent of light that is reflected.

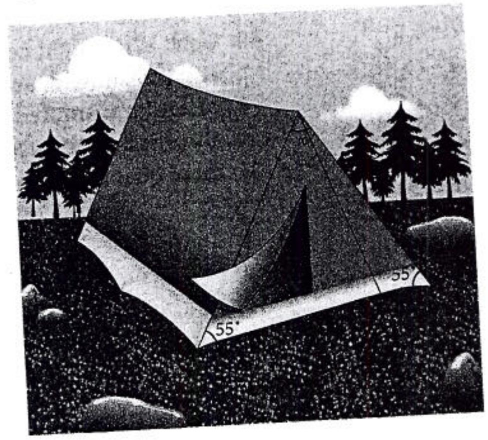
b. For flat black paint, $A = 95$ and no light is transmitted, meaning that $T = 0$. What percent of light is reflected by flat black paint?

c. A pure white surface can reflect 98% of light, so $R = 98$. If no light is transmitted, what percent of light is absorbed by the pure white surface?

d. Typically, shiny gray metals reflect 70–80% of light. Suppose a thick sheet of aluminum absorbs 25% of light. What percent of light is reflected by this shiny gray metal? (Assume no light is transmitted.)

64. $x + 55 + 55 = 180$; $70°$
65. y
66. $-y$
67. x
68. a
69. 6
70. -6
71. 6
72. $\frac{1}{2}$
73. -9
74. 6
75. $-\frac{15}{8}$
76. -2
77. 8
78. -18
79. $-\frac{5}{4}$
80. $-\frac{6}{5}$
81. $3x$
82. $3x$

64. Geometry The three angles shown in the triangle at the front of the tent in the following figure add up to 180°. Use this fact to write an equation containing x, and then solve the equation to find the number of degrees in the angle at the top of the triangle.



Getting Ready for the Next Section

To understand all of the explanations and examples in the next section you must be able to work the problems below.

Simplify.

65. $\frac{3}{2}\left(\frac{2}{3}y\right)$ **66.** $\frac{5}{2}\left(-\frac{2}{5}y\right)$ **67.** $\frac{1}{5}(5x)$ **68.** $-\frac{1}{4}(-4a)$

69. $\frac{1}{5}(30)$ **70.** $-\frac{1}{4}(24)$ **71.** $\frac{3}{2}(4)$ **72.** $\frac{1}{26}(13)$

73. $12\left(-\frac{3}{4}\right)$ **74.** $12\left(\frac{1}{2}\right)$ **75.** $\frac{3}{2}\left(-\frac{5}{4}\right)$ **76.** $\frac{5}{3}\left(-\frac{6}{5}\right)$

77. $13 + (-5)$ **78.** $-13 + (-5)$ **79.** $-\frac{3}{4} + \left(-\frac{1}{2}\right)$ **80.** $-\frac{7}{10} + \left(-\frac{1}{2}\right)$

81. $7x + (-4x)$ **82.** $5x + (-2x)$

Multiplication Property of Equality 1.3

As we have mentioned before, we all have to pay taxes. According to Figure 1, people have been paying taxes for quite a long time.

FIGURE 1 *Collection of taxes, ca. 3000 b.c. Clerks and scribes appear at the right, with pen and papyrus, and officials and taxpayers appear at the left.*

Suppose 21% of your monthly pay is withheld for federal income taxes and another 8% is withheld for Social Security, state income tax, and other miscellaneous items, leaving you with \$987.50 a month in take-home pay. The amount you earned before the deductions were removed from your check, your gross income G, is given by the equation

$$G - 0.21G - 0.08G = 987.5$$

In this section we will learn how to solve equations of this type.

In the previous section, we found that adding the same number to both sides of an equation never changed the solution set. The same idea holds for multiplication by numbers other than zero. We can multiply both sides of an equation by the same nonzero number and always be sure we have not changed the solution set. (The reason we cannot multiply both sides by zero will become apparent later.) This fact about equations is called the *multiplication property of equality*, which can be stated formally as follows.

Note: This property is also used many times throughout the book. Make every effort to understand it completely.

[Δ≠Σ PROPERTY ***Multiplication Property of Equality***

For any three algebraic expressions A, B, and C, where $C \neq 0$,

if	$A = B$
then	$AC = BC$

In words: Multiplying both sides of an equation by the same nonzero number will not change the solution set.

Suppose we want to solve the equation $5x = 30$. We have $5x$ on the left side but would like to have just x. We choose to multiply both sides by $\frac{1}{5}$ because $\left(\frac{1}{5}\right)(5) = 1$. Here is the solution:

$$5x = 30$$

$$\frac{1}{5}(5x) = \frac{1}{5}(30) \qquad \text{Multiplication property of equality}$$

$$\left(\frac{1}{5} \cdot 5\right)x = \frac{1}{5}(30) \qquad \text{Associative property of multiplication}$$

$$1x = 6$$

$$x = 6$$

We chose to multiply by $\frac{1}{5}$ because it is the reciprocal of 5. We can see that multiplication by any number except zero will not change the solution set. If, however, we were to multiply both sides by zero, the result would always be $0 = 0$ because multiplication by zero always results in zero. Although the statement $0 = 0$ is true, we have lost our variable and cannot solve the equation. This is the only restriction of the multiplication property of equality. We are free to multiply both sides of an equation by any number except zero.

Here are some more examples that use the multiplication property of equality.

EXAMPLE 1 Solve for a: $-4a = 24$.

SOLUTION Because we want a alone on the left side, we choose to multiply both sides by $-\frac{1}{4}$:

$$-\frac{1}{4}(-4a) = -\frac{1}{4}(24) \quad \text{Multiplication property of equality}$$
$$\left[-\frac{1}{4}(-4)\right]a = -\frac{1}{4}(24) \quad \text{Associative property}$$
$$a = -6$$

EXAMPLE 2 Solve for t: $-\frac{t}{3} = 5$.

SOLUTION Because division by 3 is the same as multiplication by $\frac{1}{3}$, we can write $-\frac{t}{3}$ as $-\frac{1}{3}t$. To solve the equation, we multiply each side by the reciprocal of $-\frac{1}{3}$, which is -3.

$$-\frac{t}{3} = 5 \quad \text{Original equation}$$
$$-\frac{1}{3}t = 5 \quad \text{Dividing by 3 is equivalent to multiplying by } \frac{1}{3}$$
$$-3\left(-\frac{1}{3}t\right) = -3(5) \quad \text{Multiply each side by } -3$$
$$t = -15 \quad \text{Multiplication}$$

EXAMPLE 3 Solve $\frac{2}{3}y = 4$.

SOLUTION We can multiply both sides by $\frac{3}{2}$ and have $1y$ on the left side:

$$\frac{3}{2}\left(\frac{2}{3}y\right) = \frac{3}{2}(4) \quad \text{Multiplication property of equality}$$
$$\left(\frac{3}{2} \cdot \frac{2}{3}\right)y = \frac{3}{2}(4) \quad \text{Associative property}$$
$$y = 6 \quad \text{Simplify } \frac{3}{2}(4) = \frac{3}{2}\left(\frac{4}{1}\right) = \frac{12}{2} = 6$$

Note: Notice in Examples 1 through 3 that if the variable is being multiplied by a number like -4 or $\frac{2}{3}$, we always multiply by the number's reciprocal, $-\frac{1}{4}$ or $\frac{3}{2}$, to end up with just the variable on one side of the equation.

EXAMPLE 4 Solve $5 + 8 = 10x + 20x - 4x$.

SOLUTION Our first step will be to simplify each side of the equation:

$$13 = 26x \quad \text{Simplify both sides first}$$
$$\frac{1}{26}(13) = \frac{1}{26}(26x) \quad \text{Multiplication property of equality}$$
$$\frac{13}{26} = x \quad \text{Multiplication}$$
$$\frac{1}{2} = x \quad \text{Reduce to lowest terms}$$

In the next three examples, we will use both the addition property of equality and the multiplication property of equality.

EXAMPLE 5 Solve for $x: 6x + 5 = -13$.

SOLUTION We begin by adding -5 to both sides of the equation:

$$\begin{aligned} 6x + 5 + (-5) &= -13 + (-5) && \text{Add } -5 \text{ to both sides} \\ 6x &= -18 && \text{Simplify} \\ \frac{1}{6}(6x) &= \frac{1}{6}(-18) && \text{Multiply both sides by } \tfrac{1}{6} \\ x &= -3 \end{aligned}$$

EXAMPLE 6 Solve for $x: 5x = 2x + 12$.

SOLUTION We begin by adding $-2x$ to both sides of the equation:

$$\begin{aligned} 5x + (-2x) &= 2x + (-2x) + 12 && \text{Add } -2x \text{ to both sides} \\ 3x &= 12 && \text{Simplify} \\ \frac{1}{3}(3x) &= \frac{1}{3}(12) && \text{Multiply both sides by } \tfrac{1}{3} \\ x &= 4 && \text{Simplify} \end{aligned}$$

Note: Notice that in Example 6 we used the addition property of equality first to combine all the terms containing x on the left side of the equation. Once this had been done, we used the multiplication property to isolate x on the left side.

EXAMPLE 7 Solve for $x: 3x - 4 = -2x + 6$.

SOLUTION We begin by adding $2x$ to both sides:

$$\begin{aligned} 3x + 2x - 4 &= -2x + 2x + 6 && \text{Add } 2x \text{ to both sides} \\ 5x - 4 &= 6 && \text{Simplify} \end{aligned}$$

Now we add 4 to both sides:

$$\begin{aligned} 5x - 4 + 4 &= 6 + 4 && \text{Add 4 to both sides} \\ 5x &= 10 && \text{Simplify} \\ \frac{1}{5}(5x) &= \frac{1}{5}(10) && \text{Multiply by } \tfrac{1}{5} \\ x &= 2 && \text{Simplify} \end{aligned}$$

The next example involves fractions. You will see that the properties we use to solve equations containing fractions are the same as the properties we used to solve the previous equations. Also, the LCD that we used previously to add fractions can be used with the multiplication property of equality to simplify equations containing fractions.

EXAMPLE 8 Solve $\frac{2}{3}x + \frac{1}{2} = -\frac{3}{4}$.

SOLUTION We can solve this equation by applying our properties and working with the fractions, or we can begin by eliminating the fractions.

Method 1 Working with the fractions.

$$\frac{2}{3}x + \frac{1}{2} + \left(-\frac{1}{2}\right) = -\frac{3}{4} + \left(-\frac{1}{2}\right) \quad \text{Add } -\tfrac{1}{2} \text{ to each side}$$

$$\frac{2}{3}x = -\frac{5}{4} \quad \text{Note that } -\tfrac{3}{4} + \left(-\tfrac{1}{2}\right) = -\tfrac{3}{4} + \left(-\tfrac{2}{4}\right)$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}\left(-\frac{5}{4}\right) \quad \text{Multiply each side by } \tfrac{3}{2}$$

$$x = -\frac{15}{8}$$

Method 2 Eliminating the fractions in the beginning.

$$12\left(\frac{2}{3}x + \frac{1}{2}\right) = 12\left(-\frac{3}{4}\right) \quad \text{Multiply each side by the LCD 12}$$

$$12\left(\frac{2}{3}x\right) + 12\left(\frac{1}{2}\right) = 12\left(-\frac{3}{4}\right) \quad \text{Distributive property on the left side}$$

$$8x + 6 = -9 \quad \text{Multiply}$$

$$8x = -15 \quad \text{Add } -6 \text{ to each side}$$

$$x = -\frac{15}{8} \quad \text{Multiply each side by } \tfrac{1}{8}$$

Note: Our original equation has denominators of 3, 2, and 4. The LCD for these three denominators is 12, and it has the property that all three denominators will divide it evenly. Therefore, if we multiply both sides of our equation by 12, each denominator will divide into 12 and we will be left with an equation that does not contain any denominators other than 1.

As the third line in Method 2 indicates, multiplying each side of the equation by the LCD eliminates all the fractions from the equation.

As you can see, both methods yield the same solution. ■

[Δ≠Σ PROPERTY *A Note on Division*

Because *division* is defined as multiplication by the reciprocal, multiplying both sides of an equation by the same number is equivalent to dividing both sides of the equation by the reciprocal of that number; that is, multiplying each side of an equation by $\frac{1}{3}$ and dividing each side of the equation by 3 are equivalent operations. If we were to solve the equation $3x = 18$ using division instead of multiplication, the steps would look like this:

$$3x = 18 \quad \text{Original equation}$$

$$\frac{3x}{3} = \frac{18}{3} \quad \text{Divide each side by 3}$$

$$x = 6 \quad \text{Division}$$

Using division instead of multiplication on a problem like this may save you some writing. However, with multiplication, it is easier to explain "why" we end up with just one x on the left side of the equation. (The "why" has to do with the associative property of multiplication.) My suggestion is that you continue to use multiplication to solve equations like this one until you understand the process completely. Then, if you find it more convenient, you can use division instead of multiplication.