complete representation of the function. Be sure that your viewing rectangle is large enough to indicate the asymptotic behavior of the functions. You may not always be able to show all of the important behavior of the function in one viewing rectangle. (See the Graphing Calculator Power User's Corner in Section 4.2.)

7. $f(x) = \dfrac{1}{x - 4}$

8. $f(x) = \dfrac{-2}{x - 3}$

9. $f(x) = \dfrac{3}{x + 2}$

10. $f(x) = \dfrac{-1}{(x - 1)^2}$

11. $f(x) = \dfrac{1}{(x + 1)^2}$

12. $f(x) = \dfrac{-1}{x^2 + 1}$

13. $f(x) = \dfrac{x + 2}{x - 2}$

14. $f(x) = \dfrac{x}{x + 2}$

15. $f(x) = \dfrac{2x^2 + 1}{x^2 - 4}$

16. $f(x) = \dfrac{x^2 + 1}{x^2 + 2x - 3}$

17. $f(x) = \dfrac{x^2 + 2}{2x^2 - x - 6}$

18. $f(x) = \dfrac{x^2 - 1}{x + 2}$

19. $f(x) = \dfrac{x^2}{4x - 4}$

20. $f(x) = \dfrac{x - 1}{2x^3 - 2x}$

21. $f(x) = \dfrac{x^3 + 4x^2 + 3x}{x^2 - 25}$

In Exercises 22–27, determine the domain and sketch the graph of the reducible function. Furthermore, determine appropriate WINDOW values, and check your answer using your graphing calculator. Choose the viewing rectangle carefully to display the "holes." (Select the x-values from some multiple of the EQUAL viewing rectangle and choose appropriate y-values.)

22. $f(x) = \dfrac{x^2 - 25}{2x - 10}$

23. $f(x) = \dfrac{2x^2 - 8}{x + 2}$

24. $f(x) = \dfrac{2x^2 + 2x - 12}{3x - 6}$

25. $f(x) = \dfrac{x^2 + 2x - 8}{2x^2 - 8x + 8}$

26. $f(x) = \dfrac{x + 2}{x^2 - x - 6}$

27. $f(x) = \dfrac{2x}{x^2 + x}$

In Exercises 28 and 29, $f(x) = \frac{P(x)}{Q(x)}$ is an irreducible rational function. Provide a proof for the stated theorem.

28. If the polynomials $P(x)$ and $Q(x)$ are of the same degree, then there is a horizontal asymptote $y = k$, where k is the ratio of the leading coefficients of $P(x)$ and $Q(x)$.

29. If the degree of $P(x)$ is less than the degree of $Q(x)$, then there is a horizontal asymptote at $y = 0$.

30. Suppose you are a manufacturer of graphing calculators. Your fixed or start-up cost is \$10,000. Each calculator costs you \$50 to manufacture.

    a. What is the average cost to you of producing your first 100 calculators?

    b. What formula will give you the average cost of the first x calculators?

31. The average cost of producing n units of a product is given by

$$A(n) = \frac{C(n)}{n}$$

Given the cost function

$$C(n) = 0.01n^3 - 0.1n^2 + 100n + 1000$$

find

    a. The average cost of producing n units

    b. The average cost of a function of x, if the number of units produced is $n = x - 2$

    c. GRAPH the average cost formulas you found in parts a and b, and use the graphs to determine when the cost is increasing or decreasing. (You will need to find an appropriate viewing WINDOW.)

5.2 The Circle

The conic sections provide us with an opportunity to illustrate the power of analytic geometry. We shall see that a geometric figure defined as a set of points can often be described analytically by an algebraic equation. Furthermore, we can start with an algebraic equation and use graphing procedures to study the properties of the curve, as well as the equation.

First, consider how the term "conic section" originates. If we pass a plane through a cone at various angles, as shown in Figure 14, the intersections are called **conic sections.** In exceptional cases, the intersection of a plane and a cone may be a point, a line or a pair of lines.

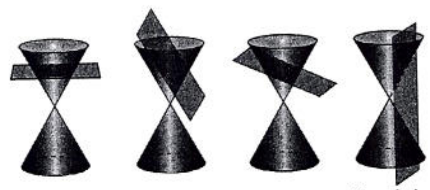


FIGURE 14 Examples of Conic Sections

We begin with the geometric definition of a circle.

Definition of a Circle

A **circle** is the set of all points in a plane that are at a given distance from a fixed point in the plane. The fixed point is called the **center** of the circle, and the given distance is called the **radius.**

Recall from Section 3.1 the formula for the length d of the line segment joining points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

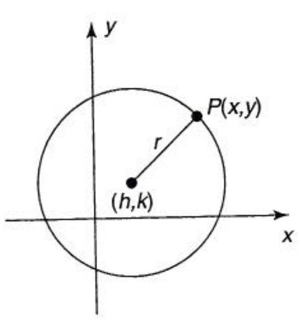


FIGURE 15 Deriving the Equation of a Circle

Using the methods of analytic geometry, we place the center at a point (h, k) as shown in Figure 15. If $P(x, y)$ is a point on the circle, then by the distance formula, the distance from P to the center (h, k) is

$$\sqrt{(x - h)^2 + (y - k)^2}$$

Since this distance is equal to the radius r, we can write

$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

or

$$(x - h)^2 + (y - k)^2 = r^2$$

Since $P(x, y)$ is any point on the circle, we say that

Standard Form of the Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

is the standard form of the equation of the circle with center (h, k) and radius r.

Note the special case of

$$x^2 + y^2 = r^2$$

which is the equation of the circle of radius r, centered at the origin.

EXAMPLE 1 **FINDING THE EQUATION OF A CIRCLE**

Write the equation of the circle with center at $(2, -5)$ and radius 3.

SOLUTION

Substituting $h = 2$, $k = -5$, and $r = 3$ into the equation

$$(x - h)^2 + (y - k)^2 = r^2$$

yields

$$(x - 2)^2 + (y + 5)^2 = 9$$

EXAMPLE 2 **FINDING THE CENTER AND RADIUS OF A CIRCLE**

Find the center and radius of the circle whose equation is

$$(x + 1)^2 + (y - 3)^2 = 4$$

SOLUTION

If we compare this equation with the standard form

$$(x - h)^2 + (y - k)^2 = r^2$$

we have

$$h = -1 \qquad k = 3 \qquad r = 2$$

The center is at $(-1, 3)$, and the radius is 2.

✔ Progress Check

Find the center and radius of the circle whose equation is

$$\left(x - \frac{1}{2}\right)^2 + (y + 5)^2 = 15$$

Answers

center $(\frac{1}{2}, -5)$, radius $\sqrt{15}$

General Form

If we are given the equation of a circle in the general form

$$Ax^2 + Ay^2 + Dx + Ey + F = 0, \qquad A \neq 0$$

in which the coefficients of x^2 and y^2 are the same, we may rewrite the equation in standard form. The process involves completing the square in each variable.

Recall from Section 2.3 that if we have the expression

$$x^2 + dx$$

we add $\left(\frac{d}{2}\right)^2$ to form

$$x^2 + dx + \frac{d^2}{4} = \left(x + \frac{d}{2}\right)^2$$

For example, starting with the expressions

$$x^2 + 4x \quad \text{and} \quad y^2 - 10y$$

we complete the squares in this way:

$$x^2 + 4x + 4 = (x + 2)^2 \quad \text{and} \quad y^2 - 10y + 25 = (y - 5)^2$$

EXAMPLE 3 STANDARD FORM OF THE EQUATION OF A CIRCLE

Write the equation of the circle $2x^2 + 2y^2 - 12x + 16y - 31 = 0$ in standard form.

SOLUTION

Grouping the terms in x and y and factoring produces

$$2(x^2 - 6x) + 2(y^2 + 8y) = 31$$

Completing the square in both x and y, we have

$$2(x^2 - 6x + 9) + 2(y^2 + 8y + 16) = 31 + 18 + 32$$
$$2(x - 3)^2 + 2(y + 4)^2 = 81$$

Note that the quantities 18 and 32 were added to the right-hand side to maintain equality. The last equation can be written as

$$(x-3)^2+(y+4)^2=\frac{81}{2}$$

This is the standard form of the equation of the circle with center at $(3, -4)$ and radius

$$r=\sqrt{\frac{81}{2}}=\frac{9\sqrt{2}}{2}$$

✔ Progress Check

Write the equation of the circle $4x^2+4y^2-8x+4y=103$ in standard form, and determine the center and radius.

Answers

$(x-1)^2+(y+\frac{1}{2})^2=27$, center $(1, -\frac{1}{2})$, radius $3\sqrt{3}$

EXAMPLE 4 STANDARD FORM OF THE EQUATION OF A CIRCLE

Write the equation $3x^2+3y^2-6x+15=0$ in standard form.

SOLUTION

Regrouping, we have

$$3(x^2-2x)+3y^2=-15$$

We then complete the square in x and y:

$$\begin{aligned}3(x^2-2x+1)+3y^2&=-15+3\\3(x-1)^2+3y^2&=-12\\(x-1)^2+y^2&=-4\end{aligned}$$

Since $r^2=-4$ is impossible, the graph of the equation is not a circle. (Note that the left-hand side of the equation in standard form is a sum of squares and hence nonnegative. However, the right-hand side is negative.) Thus, there are no real values of x and y that satisfy the equation. This is an example of an equation that does not have a graph.

EXAMPLE 5 STANDARD FORM OF THE EQUATION OF A CIRCLE

Find an equation of the circle that has its center at $C(-1, 2)$ and that passes through the point $P(3, 4)$.

SOLUTION

Since the distance from the center to any point on the circle determines the radius, we can use the distance formula to find

$$r=\overline{PC}=\sqrt{20}$$

Then we can write the equation of the circle in standard form as

$$(x + 1)^2 + (y - 2)^2 = 20$$

✔ Progress Check

Write the equation $x^2 + y^2 - 12y + 36 = 0$ in standard form, and analyze its graph.

Answers

The standard form is $x^2 + (y - 6)^2 = 0$. The equation is that of a "circle" with center at (0, 6) and radius 0. The "circle" is actually the point (0, 6).

Graphing Calculator Power User's Corner

Graphing Circles

For these graphs, it is worthwhile to use WINDOW values that are multiples of the EQUAL viewing rectangle. They will produce "better shaped" circles.

To display the circle $(x - h)^2 + (y - k)^2 = r^2$ on your graphing calculator, graph $y = \sqrt{r^2 - (x - h)^2} + k$ and $y = -\sqrt{r^2 - (x - h)^2} + k$. For example, we display in Figure 16

$$(x + 1)^2 + (y - 3)^2 = 4$$

That is, we GRAPH

$$y = \sqrt{(4 - (x + 1)^2)} + 3 \quad \text{and} \quad y = -\sqrt{(4 - (x + 1)^2)} + 3$$

in the viewing rectangle that is two times the EQUAL viewing rectangle. (Compare this with the default viewing rectangle.)

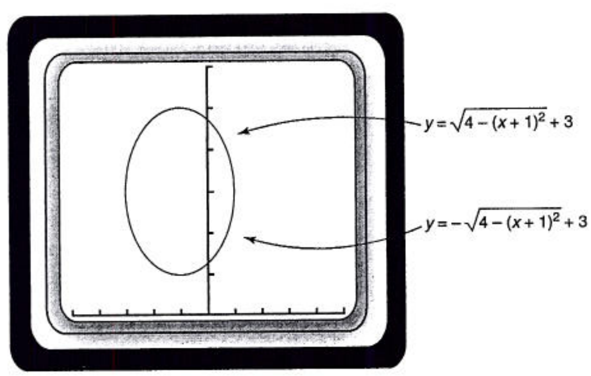


FIGURE 16 Graph of $(x + 1)^2 + (y - 3)^2 = 4$, XSCL = 2, YSCL = 2

To display the circle $Ax^2 + Ay^2 + Dx + Ey + F = 0$ on your graphing calculator, use the quadratic formula to solve for y and GRAPH

$$y = \frac{-E \pm \sqrt{E^2 - 4A(Ax^2 + Dx + F)}}{2A}.$$

For example, we display in Figure 17

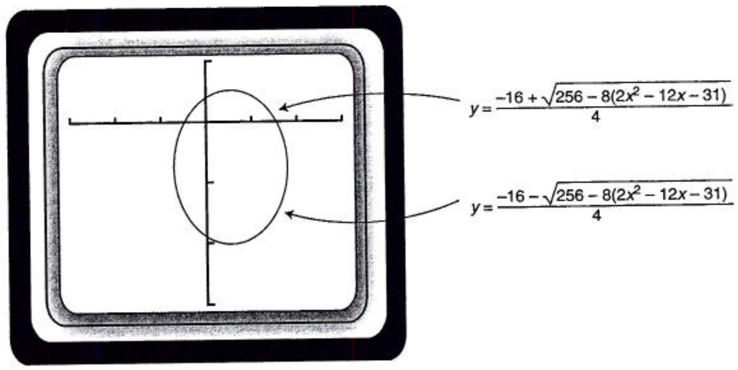


FIGURE 17 Graph of $2x^2 + 2y^2 - 12x + 16y - 31 = 0$, XSCL = 4, YSCL = 4

$$2x^2 + 2y^2 - 12x + 16y - 31 = 0$$

That is, we GRAPH

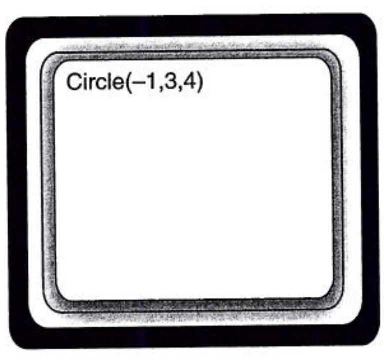
$$y = \frac{-16 + \sqrt{256 - 8(2x^2 - 12x - 31)}}{4}$$

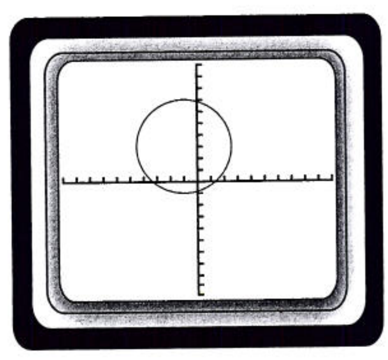
and

$$y = \frac{-16 - \sqrt{256 - 8(2x^2 - 12x - 31)}}{4}$$

in the viewing rectangle that is four times the EQUAL viewing rectangle. (Compare this with the default viewing rectangle.)

Your calculator may have a DRAW feature which will enable you to draw a circle by entering the vertex and the radius, as illustrated below:





Graph of $(x + 1)^2 + (y - 3)^2 = 4$

Exercise Set 5.2

In Exercises 1–8, find an equation of the circle with center at (h, k) and radius r.

1. $(h, k) = (2, 3), \quad r = 2$
2. $(h, k) = (-3, 0), \quad r = 3$
3. $(h, k) = (-2, -3), \quad r = \sqrt{5}$
4. $(h, k) = (2, -4), \quad r = 4$
5. $(h, k) = (0, 0), \quad r = 3$
6. $(h, k) = (0, -3), \quad r = 2$
7. $(h, k) = (-1, 4), \quad r = 2\sqrt{2}$
8. $(h, k) = (2, 2), \quad r = 2$

In Exercises 9–16, find the center and radius of the circle with the given equation. Then, determine appropriate WINDOW values and GRAPH the circle on your graphing calculator.

9. $(x - 2)^2 + (y - 3)^2 = 16$
10. $(x + 2)^2 + y^2 = 9$
11. $(x - 2)^2 + (y + 2)^2 = 4$
12. $\left(x + \frac{1}{2}\right)^2 + (y - 2)^2 = 8$
13. $(x + 4)^2 + \left(y + \frac{3}{2}\right)^2 = 18$
14. $x^2 + (y - 2)^2 = 4$
15. $\left(x - \frac{1}{3}\right)^2 + y^2 = -\frac{1}{9}$
16. $(x - 1)^2 + \left(y - \frac{1}{2}\right)^2 = 3$

In Exercises 17–24, write the equation of each given circle in standard form, and determine the center and radius if it exists. Then, determine appropriate WINDOW values and GRAPH the circle on your graphing calculator.

17. $x^2 + y^2 + 4x - 8y + 4 = 0$
18. $x^2 + y^2 - 2x + 6y - 15 = 0$
19. $2x^2 + 2y^2 - 6x - 10y + 6 = 0$
20. $2x^2 + 2y^2 + 8x - 12y - 8 = 0$
21. $2x^2 + 2y^2 - 4x - 5 = 0$
22. $4x^2 + 4y^2 - 2y + 7 = 0$
23. $3x^2 + 3y^2 - 12x + 18y + 15 = 0$
24. $4x^2 + 4y^2 + 4x + 4y - 4 = 0$

In Exercises 25–36, write the given equation in standard form, and determine if the graph of the equation is a circle, a point, or neither.

25. $x^2 + y^2 - 6x + 8y + 25 = 0$
26. $x^2 + y^2 + 4x + 6y + 5 = 0$
27. $x^2 + y^2 + 3x - 5y + 7 = 0$
28. $x^2 + y^2 - 4x - 6y - 13 = 0$
29. $2x^2 + 2y^2 - 12x - 4 = 0$
30. $2x^2 + 2y^2 + 4x - 4y + 25 = 0$
31. $2x^2 + 2y^2 - 6x - 4y - 2 = 0$
32. $2x^2 + 2y^2 - 10y + 6 = 0$
33. $3x^2 + 3y^2 + 12x - 4y - 20 = 0$
34. $x^2 + y^2 + x + y = 0$
35. $4x^2 + 4y^2 + 12x - 20y + 38 = 0$
36. $4x^2 + 4y^2 - 12x - 36 = 0$
37. Find the area of the circle whose equation is
$$x^2 + y^2 - 2x + 4y - 4 = 0$$
38. Find the circumference of the circle whose equation is
$$x^2 + y^2 - 6x + 8 = 0$$
39. Show that the circles whose equations are
$$x^2 + y^2 - 4x + 9y - 3 = 0$$
and
$$3x^2 + 3y^2 - 12x + 27y - 27 = 0$$
are concentric.
40. Find an equation of the circle that has its center at $(3, -1)$ and passes through the point $(-2, 2)$.
41. Find an equation of the circle that has its center at $(-5, 2)$ and passes through the point $(-3, 4)$.

42. The two points $(-2, 4)$ and $(4, 2)$ are the endpoints of the diameter of a circle. Write the equation of the circle in standard form.

43. The two points $(3, 5)$ and $(7, -3)$ are the endpoints of the diameter of a circle. Write the equation of the circle in standard form.

Exercises 44 and 45 relate to the Graphing Calculator Power User's Corner on "Graphing Circles."

44. Why are the circles not round in the default viewing rectangle? Can you always find a viewing rectangle to make a circle appear round, and if so, how? If not, why not?

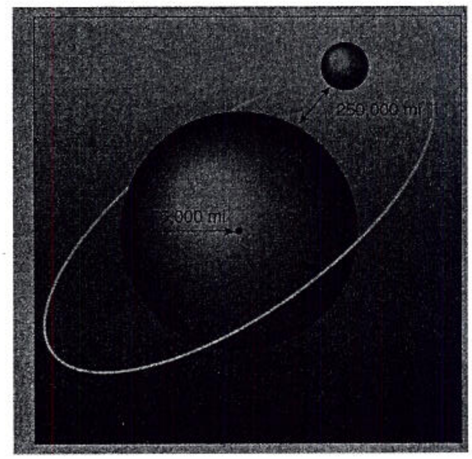
45. Why do the top and bottom halves of the circles not meet in some viewing rectangles? Can you always find a viewing rectangle to make them meet, and if so, how? If not, why not?

46. Use simultaneous equations to determine the equation of the circle passing through the points $(1, 2)$, $(-1, 2)$, and $(0, 3)$, given that the general form of a circle is

$$Ax^2 + Ay^2 + Dx + Ey + F = 0, \quad A \neq 0$$

47. Convert your solution to Exercise 46 to standard form, and state the center and radius of the circle.

48. Find the equation of the path of a satellite in a circular orbit 250,000 miles above a planet whose radius is 75,000 miles.



5.3 The Parabola

We begin our study of the parabola with the geometric definition:

Definition of a Parabola

A **parabola** is the set of all points in a plane that are equidistant from a given point and a given line, both in the plane. The given point is called the **focus,** and the given line is called the **directrix.**

In Figure 18, all points P_i on the parabola are equidistant from the focus F and the directrix D; that is, $\overline{P_iF} = \overline{P_iQ_i}$. The line through the focus that is perpendicular to the directrix is called the **axis of the parabola.** This line is also called the **axis of symmetry** since the parabola is symmetric with respect to it. The point V in Figure 18, where the parabola intersects its axis, is called the **vertex** of the parabola. The vertex is the point from which the parabola opens. Note that the vertex is the point on the parabola that is closest to the directrix.

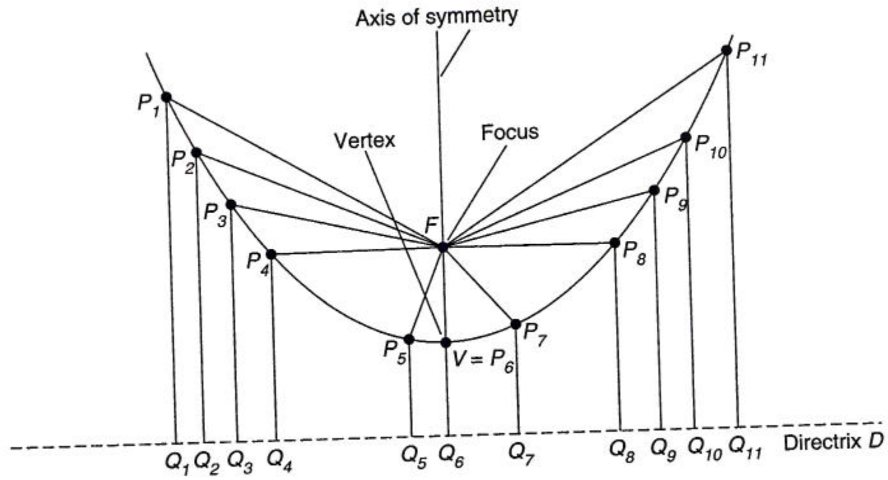


FIGURE 18 Directrix, Focus, Vertex, and Axis of a Parabola

We can apply the methods of analytic geometry to find an equation of a parabola. For a parabola with a vertical axis, choose that axis to be the y-axis, and take the origin as its vertex, as shown in Figure 19. Since the vertex is on the parabola, it is equidistant from the focus and the directrix. Thus, if the coordinates of the focus F are $(0, p)$, then the equation of the directrix is $y = -p$. We let $P(x, y)$ be any point on the parabola, and we equate the distance from P to the focus F with the distance from P to the directrix D. Using the distance formula,

$$\overline{PF} = \overline{PQ}$$
$$\sqrt{(x-0)^2 + (y-p)^2} = \sqrt{(x-x)^2 + (y+p)^2}$$

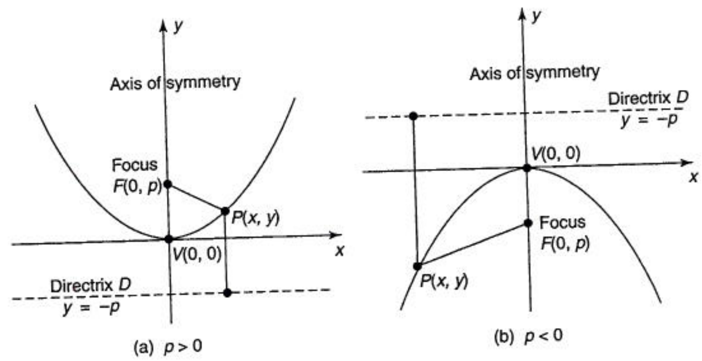


FIGURE 19 Deriving the Equation of a Parabola

Squaring both sides,

$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$
$$x^2 = 4py$$

We have obtained one form of the equation of the parabola.

Standard Form of the Equation of a Parabola with Vertical Axis and Vertex (0, 0)

$$x^2 = 4py$$

The focus is at $(0, p)$.

Conversely, it can be shown that the graph of the equation $x^2 = 4py$ is a parabola. Note that substituting $-x$ for x leaves the equation unchanged, verifying symmetry with respect to the y-axis. If $p > 0$, the parabola opens upward as shown in Figure 19(a), whereas if $p < 0$, the parabola opens downward, as shown in Figure 19(b).

EXAMPLE 1 FINDING SOME CHARACTERISTICS OF A PARABOLA

Determine the focus and directrix of the parabola $x^2 = 8y$, and sketch its graph.

SOLUTION

The equation of the parabola is of the form

$$x^2 = 4py = 8y$$

so $p = 2$. The equation of the directrix is $y = -p = -2$, and the focus is at $(0, p) = (0, 2)$. Since $p > 0$, the parabola opens upward. The graph of the parabola is shown in Figure 20.

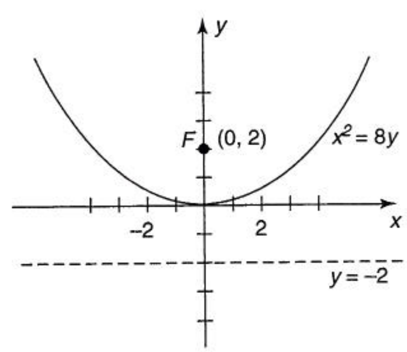


FIGURE 20 Graph of $x^2 = 8y$

✔ Progress Check

Determine the focus and directrix of the parabola $x^2 = -3y$.

Answers

focus at $(0, -\frac{3}{4})$, directrix $y = \frac{3}{4}$

EXAMPLE 2 FINDING THE EQUATION OF A PARABOLA

Find the equation of the parabola with vertex at (0, 0) and focus at $(0, -\frac{3}{2})$.

SOLUTION

Since the focus is at $(0, p)$, we have $p = -\dfrac{3}{2}$. The equation of the parabola is

$$x^2 = 4py = 4\left(-\frac{3}{2}y\right) = -6y$$

✔ Progress Check

Find the equation of the parabola with vertex at (0, 0) and focus at (0, 3).

Answer

$x^2 = 12y$

If we place the parabola as shown in Figure 21, we can proceed as we did for a parabola with a vertical axis and vertex (0, 0) to obtain the following result.

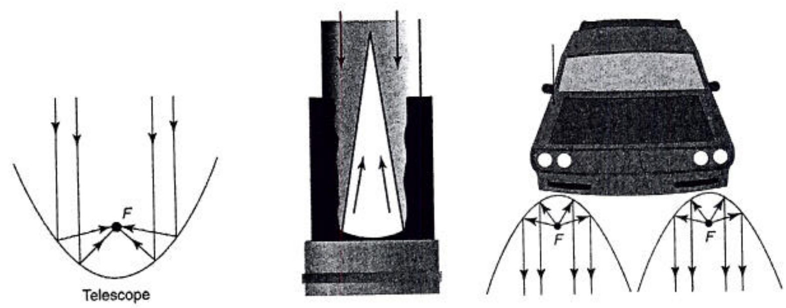
Standard Form of the Equation of a Parabola with Horizontal Axis and Vertex (0, 0)

$$y^2 = 4px$$

The focus is at $(p, 0)$.

Focus with a Parabolic Shape

The properties of the parabola are used in the design of some important devices. For example, by rotating a parabola about its axis, we obtain a *parabolic reflector*, a shape used in the headlight of an automobile. The light source (the bulb) is placed at the focus of the parabola. The headlight is coated with a reflecting material, and the rays of light bounce back in lines that are parallel to the axis of the parabola, permitting a headlight to disperse light in front of the automobile where it is needed.



A reflecting telescope reverses the use of these same properties. Here, the rays of light from a distant star, which are nearly parallel to the axis of the parabola, are reflected by the mirror to the focus. The eyepiece is placed at the focus, where the rays of light are gathered.

Note that substituting $-y$ for y leaves this equation unchanged, verifying symmetry with respect to the x-axis. If $p > 0$, the parabola opens to the right as shown in Figure 21(a); but if $p < 0$, the parabola opens to the left as shown in Figure 21(b).

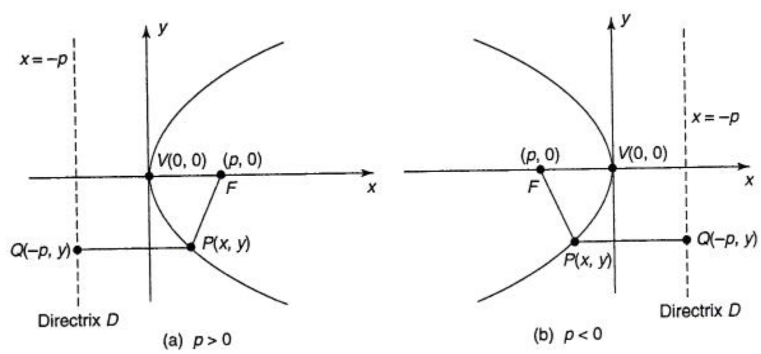


FIGURE 21 Deriving the Equation of a Parabola

EXAMPLE 3 FINDING THE EQUATION OF A PARABOLA

Find the equation of the parabola with vertex at (0, 0) and directrix $x = \frac{1}{2}$.

SOLUTION

The directrix is $x = -p$, so $p = -\frac{1}{2}$. The equation of the parabola is then

$$y^2 = 4px = -2x$$

EXAMPLE 4 FINDING THE EQUATION OF A PARABOLA

Find the equation of the parabola that has its axis as the x-axis, its vertex at (0, 0), and passes through the point (−2, 3).

SOLUTION

Since the axis of the parabola is the x-axis, the equation of the parabola is $y^2 = 4px$. The parabola passes through the point (−2, 3), so the coordinates of this point must satisfy the equation of the parabola. Thus,

$$\begin{aligned} y^2 &= 4px \\ (3)^2 &= 4p(-2) \\ 4p &= -\frac{9}{2} \end{aligned}$$

and the equation of the parabola is

$$y^2 = 4px = -\frac{9}{2}x$$

✔ Progress Check

Find the equation of the parabola that has its axis as the y-axis, its vertex at (0, 0) and passes through the point (1, −2).

Answer

$x^2 = -\frac{1}{2}y$

Vertex at (h, k)

It is also possible to determine an equation of a parabola when the vertex is at some arbitrary point (h, k). The form of the equation depends on whether the axis of the parabola is parallel to the x-axis or to the y-axis. The situations are summarized in Table 1. Note that if the point (h, k) is the origin, then $h = k = 0$, and we arrive at the equations we derived previously. Thus, in all cases, the sign of the constant p determines the direction in which the parabola opens.

Furthermore, an equation of a parabola can always be written in one of the standard forms shown in Table 1.

TABLE 1 Standard Forms of the Equation of a Parabola with Vertex (h, k)

Equation	Axis	Directix	Direction of Opening
$(x-h)^2 = 4p(y-k)$	$x = h$	$y = k - p$	Up if $p > 0$ Down if $p < 0$
$(y-k)^2 = 4p(x-h)$	$y = k$	$x = h - p$	Right if $p > 0$ Left if $p < 0$

Note that these changes in the equations of the parabola are similar to the change that occurs in the equation of the circle when the center is moved from the origin to a point (h, k). In both cases, x is replaced by $x - h$ and y is replaced by $y - k$.

EXAMPLE 5 FINDING SOME CHARACTERISTICS OF A PARABOLA

Determine the vertex, axis, and the direction in which the parabola opens.

$$\left(x - \frac{1}{2}\right)^2 = -3(y + 4)$$

SOLUTION

Comparison of the equation with the standard form

$$(x-h)^2 = 4p(y-k)$$

yields $h = \frac{1}{2}$, $k = -4$, and $p = -\frac{3}{4}$. The axis of the parabola is found by setting the square term equal to 0.

$$\left(x - \frac{1}{2}\right)^2 = 0$$

$$x = \frac{1}{2}$$

Thus, the vertex is at $(h, k) = (\frac{1}{2}, -4)$, the axis is $x = \frac{1}{2}$, and the parabola opens downward since $p < 0$.

✔ Progress Check

Determine the vertex, axis, and the direction in which the parabola opens.

$$3(y+1)^2 = 12\left(x - \frac{1}{3}\right)$$

Answers

vertex $(\frac{1}{3}, -1)$, axis $y = -1$, opens to the right

EXAMPLE 6 FINDING SOME CHARACTERISTICS OF A PARABOLA

Locate the vertex and the axis of symmetry of each of the given parabolas. Sketch the graph.

a. $x^2 + 2x - 2y - 3 = 0$ b. $y^2 - 4y + x + 1 = 0$

SOLUTION

a. We complete the square in x:

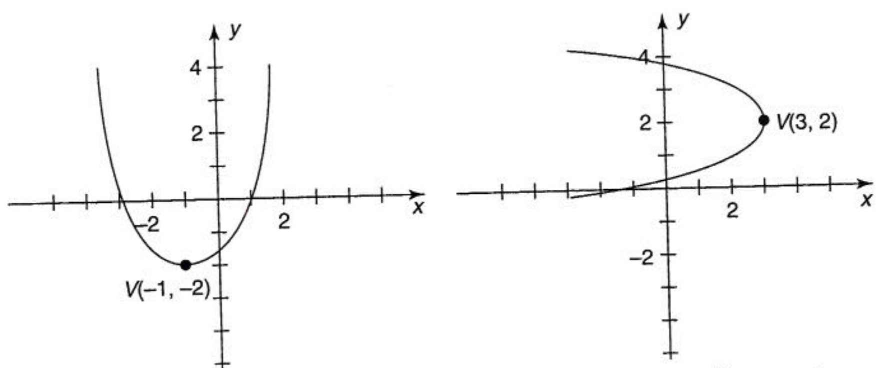
$$x^2 + 2x = 2y + 3$$
$$x^2 + 2x + 1 = 2y + 3 + 1$$
$$(x + 1)^2 = 2(y + 2)$$

The vertex of the parabola is at $(-1, -2)$ and the axis is $x = -1$ as shown in Figure 22(a).

b. We complete the square in y:

$$y^2 - 4y = -x - 1$$
$$y^2 - 4y + 4 = -x - 1 + 4$$
$$(y - 2)^2 = -(x - 3)$$

The vertex of the parabola is at $(3, 2)$ and the axis is $y = 2$ as shown in Figure 22(b).



(a) Graph of $(x + 1)^2 = 2(y + 2)$ (b) Graph of $(y - 2)^2 = -(x - 3)$

FIGURE 22 Graphs for Example 6

✔ Progress Check

Write the equation of the parabola in standard form. Locate the vertex and the axis, and sketch the graph.

a. $y^2 - 2y - 2x - 5 = 0$ b. $x^2 - 2x + 2y - 1 = 0$

Answers

a. $(y - 1)^2 = 2(x + 3)$, vertex $(-3, 1)$, axis $y = 1$. The graph is shown in Figure 23(a).

b. $(x - 1)^2 = -2(y - 1)$, vertex $(1, 1)$, axis $x = 1$. The graph is shown in Figure 23(b).

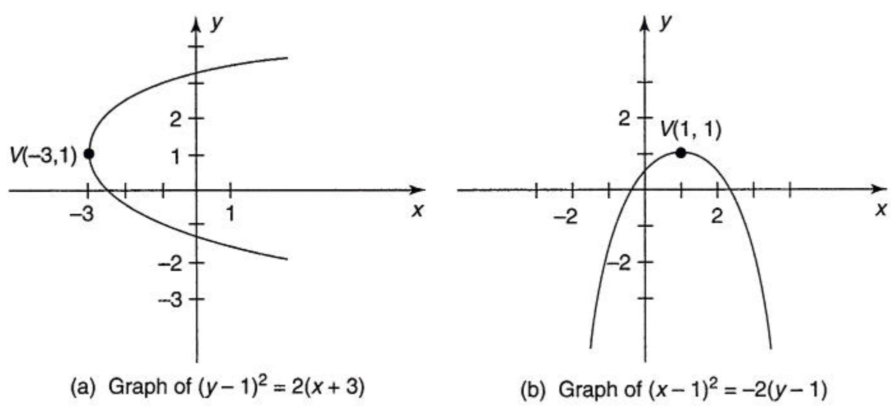


FIGURE 23 Graphs for Progress Check