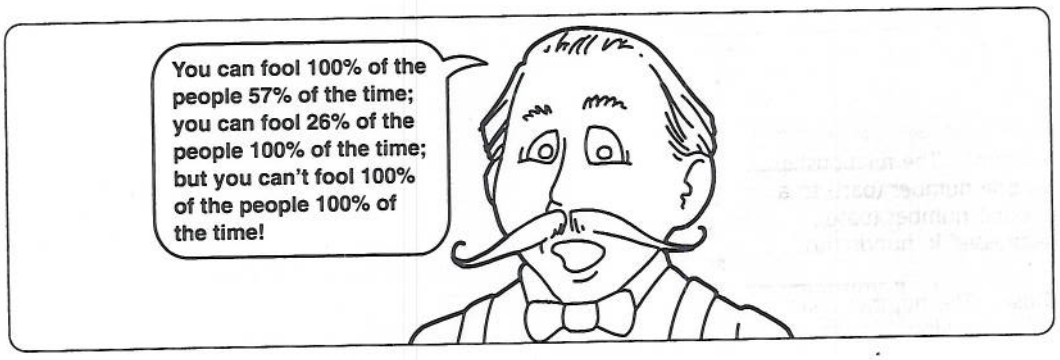
CHAPTER

4

Percent



CHAPTER OBJECTIVES

When you complete this chapter successfully, you will be able to:

1. Write fractions and decimal numbers as percents and change percents to decimal and fraction form.
2. Solve problems involving percents.

- Statistics show that 68% of students at Gigantic Urban and Rural University (GURU) change their majors at least once before graduating. What fraction of GURU students change their majors?
- Shiring Brokerage's gross earnings last year were $87,500. This year, talented financial management has brought about an 8.5% increase. Calculate the amount of increase and the new earnings.
- The Garden of Earthly Delights has discounted the price of installing a spring flower bed by 16%. If the amount of the discount is $15.68, calculate the original price and the discounted price.

As you can see, percents are an important part of everyday business. Businesspeople far removed from the accounting department—salespeople, manufacturing supervisors, and top executives—all require a knowledge of percents to talk about their operations and measure their success.

In this chapter, you will see how percents are just an extension of the fractions and decimals you have already mastered. The good news is that you probably already know more about percents than you think you do. The even better news is that, if you pay close attention to this chapter, you'll feel 100% confident about percents when you're done.

Can You:

Define percent?
Convert a decimal number to a percent?
Convert a fraction to a percent?
Change percents to decimal form?
Change percents to fraction form?
. . . If not, you need this section.

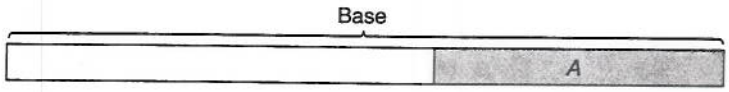
Understanding Percent

Percent The relationship of one number (part) to a second number (base), expressed in hundredths.

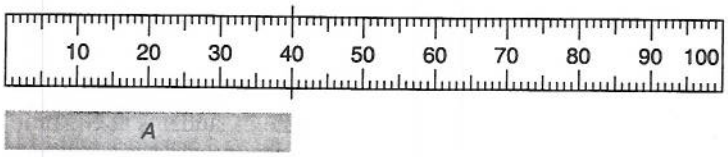
Base The number against which another number is compared in calculating a percent.

The word **percent** comes from the Latin words *per centum* meaning "by the hundred" or "for every hundred." A number expressed as a percent is being compared with a second number called the standard or **base** by dividing the base into 100 equal parts and writing the comparison number as so many hundredths of the base.

For example, what part of the base is length A?



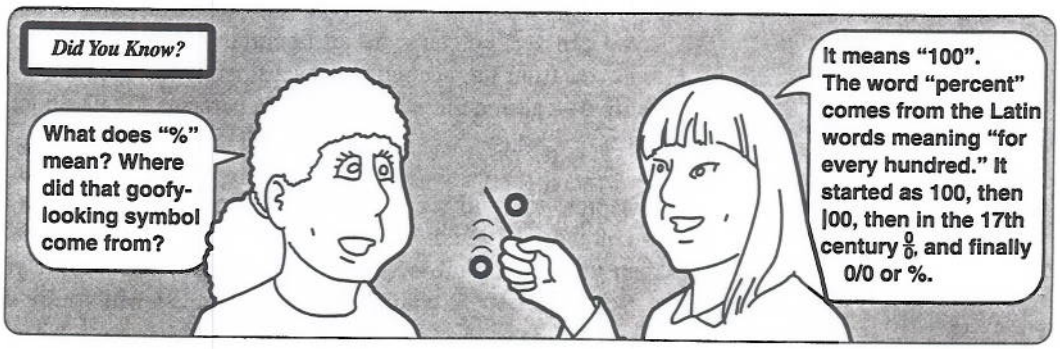
We could answer the question with a fraction or a decimal or a percent. First, divide the base into 100 equal parts. Then compare A with it.



The length of A is 40 parts out of the 100 parts that make up the base. We could write this result as

$$A = \frac{40}{100} \qquad \text{or} \qquad A = 0.40$$

To find an answer as a percent, write it as a fraction with a denominator equal to 100. Then delete the denominator and put a "%" (percent sign) after the numerator. So, in this example, A is 40% of the base.

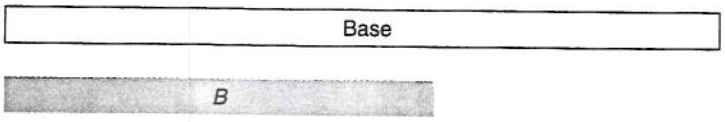


Writing 40% means 40 parts in 100 or $\frac{40}{100}$. Fractions, decimals, and percents are alternative ways to talk about a comparison of two numbers.

WORK THIS PROBLEM

The Question:

What percent of this base is length B?



✔ YOUR WORK

The Solution:

B is $\frac{60}{100}$, which is 0.60 or 60%.

You might wonder why we bother with percents if they give the same information as decimals and fractions. Businesspeople often use percents because they are a more efficient way to express a comparison such as a discount or markup rate, interest rates, tax rates, or the like. Moreover, because our number system, and our money, is based on ten and multiples of ten, it is very handy to write comparisons in hundredths or percent.

WORK THIS PROBLEM

The Question:

What part of \$1.00 is 50¢? Write your answer as a fraction, as a decimal, and as a percent.

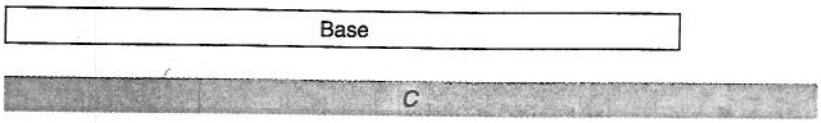
✔ YOUR WORK

The Solution:

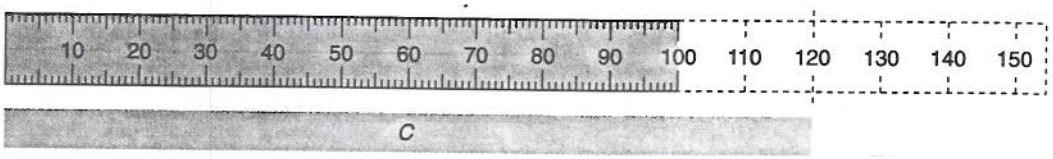
$$\frac{50¢}{100¢} = \frac{50}{100} = 0.50 = 50\%$$

Sometimes the compared number is larger than the base. An everyday example of this situation involves increased prices. For example, gasoline went from about \$0.25 per gallon in 1970 to about \$1.00 per gallon in 1990—a 300% increase.

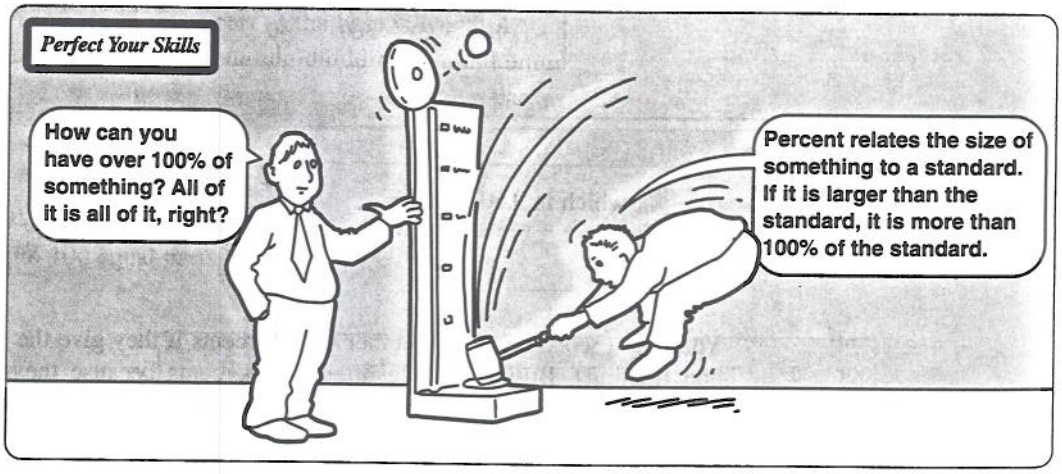
Calculating such percents is easy. For example, what percent of the base is length C?



In this case, divide the base into 100 parts and extend it in length.



The length of C is 120 parts out of the 100 parts that make up the base, so C is $\frac{120}{100}$, or 120% of the base.



Although we always think of percent as dividing the base into 100 equal parts, that does not mean that the standard length is always 100. For example, suppose you have a 10 yard length of cloth and want to know what percent you are taking if you cut off 2 yards. First, write this relationship as a fraction: $\frac{2}{10}$. Next, you must convert this fraction to a fraction with a denominator of 100. (Remember: Percent means hundredths.) Since 10 goes into 100 ten times, multiply both the numerator and denominator by 10:

$$\frac{2}{10} = \frac{2 \times 10}{10 \times 10} = \frac{20}{100}$$

The answer, then, is 20%.

WORK THIS PROBLEM

The Question:

What percent of 20 is 4?

✔ YOUR WORK

The Solution:

Because 20 goes into 100 five times, we multiply both the numerator and the denominator by 5:

$$\frac{4}{20} = \frac{4 \times 5}{20 \times 5} = \frac{20}{100} = 20\%$$

Changing a Decimal Number to a Percent

How do you change a decimal number to a percent? The procedure is simply to *multiply the decimal number by 100%.* For example,

$$\begin{aligned} 0.60 = \ & 0.60 \\ & \times 100\% \\ & 60.00\% = 60\% \end{aligned}$$

This procedure effectively *moves the decimal point two places to the right.*

Steps: Changing a Decimal Number to a Percent

STEP 1. *Move* the decimal point two places to the right. This is equivalent to multiplying by 100%.

STEP 2. *Attach* the percent sign on the right.

$$\begin{aligned} 0.375 &= 0.375 \times 100\% = 37.5\% \\ 3.4 &= 3.40 \times 100\% = 340\% \\ 0.02 &= 0.02 \times 100\% = 2\% \end{aligned}$$

Notice that any number larger than 1 is more than 100%.

YOUR TURN

WORK THIS PROBLEM

The Question:

Rewrite the following as percents.

(a) 0.70 (b) 1.25 (c) 0.001 (d) 3

✔ YOUR WORK

The Solution:

(a) $0.70 = 070.\% = 70\%$ (b) $1.25 = 125.\% = 125\%$

(c) $0.001 = 000.1\% = 0.1\%$ (d) $3 = 3.00 = 300.\% = 300\%$

Changing a Fraction to a Percent

Using this technique for changing a decimal number to a percent, we get a shortcut for writing a fraction as a percent.

Steps: Changing a Fraction to a Percent

STEP 1. *Divide* the numerator of the fraction by its denominator. Use at least two decimal places.

STEP 2. *Move* the decimal point *two digits to the right* and attach the percent sign.

For example, to change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{20}$, and $1\frac{7}{20}$ to percents,

$$\frac{1}{2} = 2\overline{)1.00} = 0.50 = 50\% \qquad \frac{3}{4} = 4\overline{)3.00} = 0.75 = 75\%$$

$$\frac{3}{20} = 20\overline{)3.00} = 0.15 = 15\% \qquad 1\frac{7}{20} = 20\overline{)27.00} = 1.35 = 135\%$$

$$\begin{array}{r} 0.15 \\ 20\overline{)3.00} \\ \underline{2\,0} \\ 1\,00 \\ \underline{1\,00} \end{array} \qquad \begin{array}{r} 1.35 \\ 20\overline{)27.00} \\ \underline{20} \\ 7\,0 \\ \underline{6\,0} \\ 1\,00 \\ \underline{1\,00} \end{array}$$

YOUR TURN

WORK THIS PROBLEM

The Question:

Calculators are currently on sale and have been reduced by $\frac{5}{16}$ of their regular price. What is $\frac{5}{16}$ as a percent?

✔ YOUR WORK

The Solution:

$$\frac{5}{16} = 16\overline{)5.0000} = 0.3125 = 31.25\%$$

$$\begin{array}{r} 0.3125 \\ 16\overline{)5.0000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \end{array}$$

Using a calculator,

$\frac{3}{20} \rightarrow 3$ [÷] 20 [=] → .15 [×] 100 [=] → 15. or 15%

$1\frac{7}{20} = \frac{27}{20} \rightarrow 27$ [÷] 20 [=] → 1.35 [×] 100 [=] → 135. or 135%

$\frac{5}{16} \rightarrow 5$ [÷] 16 [=] → 0.3125 [×] 100 [=] → 31.25 or 31.25% or $31\frac{1}{4}\%$

Some fractions cannot be converted to exact decimals. For example,

$$\frac{1}{3} = 0.333\ldots, \text{ where the 3s continue to repeat endlessly.}$$

We can round to get an approximate percent,

$$\frac{1}{3} = 0.333 = 33.3\% \text{ rounded}$$

or convert it to a fraction with 100 as denominator:

$$\frac{1}{3} = \frac{?}{100} \quad \text{gives} \quad \frac{1}{3} = \frac{1 \times 33\frac{1}{3}}{3 \times 33\frac{1}{3}} = \frac{33\frac{1}{3}}{100} = 33\tfrac{1}{3}\%$$

You may wonder why anyone would want to use percents containing a fraction. If such numbers are rounded, using them in arithmetic operations such as multiplication can give you inaccurate answers. Some interest rates, such as monthly credit card rates, use percents containing fractions. As you will see later in this chapter, percents written with fractions usually are more accurate than percents written with rounded decimals.

Fraction–Decimal–Percent Equivalents

The following equivalents are used so often that you may find it helpful to memorize them.

Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{2}$	0.50	50%	$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$
$\frac{1}{3}$	0.333	$33\frac{1}{3}\%$	$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
$\frac{2}{3}$	0.667	$66\frac{2}{3}\%$	$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
$\frac{1}{4}$	0.25	25%	$\frac{7}{8}$	0.875	$87\frac{1}{2}\%$
$\frac{3}{4}$	0.75	75%	$\frac{1}{10}$	0.10	10%
$\frac{1}{5}$	0.20	20%	$\frac{3}{10}$	0.30	30%
$\frac{2}{5}$	0.40	40%	$\frac{7}{10}$	0.70	70%
$\frac{3}{5}$	0.60	60%	$\frac{9}{10}$	0.90	90%
$\frac{4}{5}$	0.80	80%	$\frac{10}{10}$	1.00	100%
$\frac{1}{6}$	0.1667	$16\frac{2}{3}\%$	$\frac{1}{12}$	0.0833	$8\frac{1}{3}\%$
$\frac{5}{6}$	0.833	$83\frac{1}{3}\%$	$\frac{1}{16}$	0.0625	$6\frac{1}{4}\%$
			$\frac{1}{20}$	0.05	5%

WORK THIS PROBLEM

The Question:

Rewrite the following fractions as percents.

(a) $\frac{7}{5}$ (b) $\frac{2}{3}$ (c) $3\frac{1}{8}$ (d) $\frac{5}{12}$

✓ YOUR WORK

The Solution:

(a) $\frac{7}{5} = 1.4 = 1.40 = 140\%$

(b) $\frac{2}{3} = \frac{?}{100}$ $\frac{2}{3} = \frac{2 \times 33\frac{1}{3}}{3 \times 33\frac{1}{3}} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}\%$

(c) $3\frac{1}{8} = 3.125 = 312.5\%$

(d) $\frac{5}{12} = \frac{?}{100} = \frac{5 \times 8\frac{1}{3}}{12 \times 8\frac{1}{3}} = \frac{40\frac{5}{3}}{100} = \frac{41\frac{2}{3}}{100} = 41\frac{2}{3}\%$

Changing a Percent to a Decimal

In order to use percent in solving business problems, it is often necessary to change that percent to a decimal number. The procedure is to divide by 100%. For example,

$$50\% = \frac{50\%}{100\%} = \frac{50}{100} = 0.50 \qquad 100\overline{)50.0}\ \ 0.5$$

$$5\% = \frac{5\%}{100\%} = \frac{5}{100} = 0.05 \qquad 100\overline{)5.00}\ \ 0.05$$

$$0.2\% = \frac{0.2\%}{100\%} = \frac{0.2}{100} = 0.002 \qquad 100\overline{)0.200}\ \ 0.002$$

Notice that in each of these examples division by 100% has the effect of moving the decimal point two digits to the left and dropping the percent sign.

Steps: Changing a Percent to a Decimal

STEP 1. *Move* the decimal point two places to the left.

STEP 2. *Drop* the percent sign.

For example,

$$50\% = 50.\% = .50 = 0.50$$

$$5\% = 05.\% = .05 = 0.05$$

$$0.2\% = 00.2\% = .002 = 0.002$$

If a fraction is part of the percent number, it is easiest to change to a decimal number and round if necessary. If you round, be careful to include enough digits to avoid any error in your calculations—the eight digits in most calculators are plenty.

$$6\tfrac{1}{2}\% = 6.5\% = 06.5\% = 0.065$$

$$33\tfrac{1}{3}\% = 33.333\% = 0.333 \text{ rounded}$$

WORK THIS PROBLEM

The Question:

Write the following as decimal numbers. If necessary, round to three decimal digits.

(a) 4% (b) 0.5% (c) $16\frac{2}{3}\%$ (d) $79\frac{1}{4}\%$

✔ YOUR WORK

The Solution:

(a) $4\% = 04.\% = 0.04$

(b) $0.5\% = 00.5\% = 0.005$

(c) $16\frac{2}{3}\% = 16.666\% = 0.167$ rounded

(d) $79\frac{1}{4}\% = 79.25\% = 0.7925$

Changing a Percent to a Fraction

To change a percent to a fraction, just follow these simple steps.

Steps: Changing a Percent to a Fraction

STEP 1. *Divide* by 100%.

STEP 2. *Reduce* to lowest terms.

For example, convert (a) 36% (b) $12\frac{1}{2}\%$ (c) 125%.

	Step 1	*Step 2*
(a)	$36\% = \dfrac{36\%}{100\%} = \dfrac{\overset{9}{\cancel{36}}}{\underset{25}{\cancel{100}}}$	$= \dfrac{9}{25}$
(b)	$12\frac{1}{2}\% = \dfrac{12\frac{1}{2}\%}{100\%} = \dfrac{12\frac{1}{2}}{100} = \dfrac{\frac{25}{2}}{100} = \dfrac{25}{200}$	$= \dfrac{1}{8}$
(c)	$125\% = \dfrac{125\%}{100\%} = \dfrac{125}{100} = \dfrac{5}{4}$	$= 1\frac{1}{4}$

WORK THIS PROBLEM

The Question:

Convert the following to fractions.

(a) 72% (b) $16\frac{1}{2}\%$ (c) 240% (d) $7\frac{1}{2}\%$

✓ YOUR WORK

The Solution:

(a) $72\% = \dfrac{72\%}{100\%} = \dfrac{\overset{18}{\cancel{72}}}{\underset{25}{\cancel{100}}} = \dfrac{18}{25}$

(b) $16\frac{1}{2}\% = \dfrac{16\frac{1}{2}\%}{100\%} = \dfrac{16\frac{1}{2}}{100} = \dfrac{\frac{33}{2}}{100} = \dfrac{33}{200}$

(c) $240\% = \dfrac{240\%}{100\%} = \dfrac{\overset{12}{\cancel{240}}}{\underset{5}{\cancel{100}}} = \dfrac{12}{5} = 2\frac{2}{5}$

(d) $7\frac{1}{2}\% = \dfrac{7\frac{1}{2}\%}{100\%} = \dfrac{7\frac{1}{2}}{100} = \dfrac{\frac{15}{2}}{100} = \dfrac{15}{200} = \dfrac{3}{40}$

Before going on, work the problems in Section Test 4.1 to be sure you can convert decimals, fractions, and percents.

Name

Date

Course/Section

SECTION TEST 4.1 Percent

The following problems test your understanding of Section 4.1, Converting To and From Percent.

A. Write each number as a percent.

1. 0.60 = _____
2. 0.20 = _____
3. 0.45 = _____
4. 0.72 = _____
5. 0.05 = _____
6. 0.09 = _____
7. 0.7 = _____
8. 0.2 = _____
9. 1.4 = _____
10. 1.7 = _____
11. 0.025 = _____
12. 0.042 = _____
13. 0.005 = _____
14. 0.008 = _____
15. 0.258 = _____
16. 7 = _____
17. 2 = _____
18. 5.01 = _____
19. 6.05 = _____
20. 4.875 = _____
21. 3.375 = _____

B. Write each fraction as a percent.

1. $\frac{1}{4}$ = _____
2. $\frac{1}{2}$ = _____
3. $\frac{7}{8}$ = _____
4. $\frac{1}{8}$ = _____
5. $\frac{7}{20}$ = _____
6. $\frac{5}{16}$ = _____
7. $\frac{13}{20}$ = _____
8. $\frac{11}{4}$ = _____
9. $\frac{25}{8}$ = _____
10. $\frac{21}{4}$ = _____
11. $2\frac{3}{8}$ = _____
12. $1\frac{19}{20}$ = _____
13. $4\frac{13}{16}$ = _____
14. $5\frac{3}{16}$ = _____

C. Write each percent as a decimal number.

1. 9% = _____
2. 5% = _____
3. 29% = _____
4. 98% = _____
5. 6.5% = _____
6. 8.2% = _____
7. 256% = _____
8. 23.7% = _____
9. 0.05% = _____
10. 0.08% = _____
11. 85% = _____
12. 2.5% = _____
13. 605% = _____
14. 137.5% = _____

D. Write each percent as a fraction in lowest terms.

1. $5\% =$ _____
2. $8\% =$ _____
3. $24\% =$ _____
4. $75\% =$ _____
5. $60\% =$ _____
6. $20\% =$ _____
7. $250\% =$ _____
8. $0.5\% =$ _____
9. $2.5\% =$ _____
10. $1.2\% =$ _____
11. $87.5\% =$ _____
12. $7\frac{1}{2}\% =$ _____
13. $66\frac{2}{3}\% =$ _____
14. $33\frac{1}{3}\% =$ _____

E. Applied Problems.

1. Wreck-a-Mended Auto is able to restore to like-new condition $\frac{3}{4}$ of the damaged cars it receives. Express as a percent the fraction of its business this represents.

2. A surprising 32% of the buildings erected by J.R.'s Construction last year were medical offices. Express this percent as a decimal number.

3. Statistics show that 68% of students at Gigantic Urban and Rural University (GURU) change their majors at least once before graduating. What fraction of GURU students change their majors? (Be sure to express your answer in lowest terms.)

4. Watt's Electrical Service does $\frac{3}{5}$ of its work in industrial settings. Express as a percent the fraction of its work done for other businesses.

5. The Pizza Palace is proud of the fact that 75% of its customers rate the pizza as "good to excellent." Express this percent as a decimal number.

6. According to Lew's latest figures, business at his trendy restaurant is off 25%. What fraction of his business has he lost? (Be sure to express your answer in lowest terms.)

7. Sparkle Cleaning Company has discovered a way to clean floors with only $\frac{11}{16}$ as much detergent as it formerly used. Express as a percent the fraction of the former amount Sparkle now uses.

8. Wired-for-Sound Electronics sold 125% more CD players through the Internet than it did through its store. Express this percent as a decimal number.

9. The cost to Crystal's Diamonds (on the TV shopper channel) for unset diamonds has risen 130% in the last year. What fraction of the original cost does this represent? (Be sure to express your answer in lowest terms.)

10. Petal Pushers will have to raise its prices, because the cost of roses is $\frac{13}{8}$ what it was a year ago. Express the fraction of last year's price for roses to this year's as a percent.

11. The Fly-by-Night Aviary gets 45.3% of its birds from South America. Express this percent as a decimal number.

12. Big Ben's Bolts is planning to offer a new supermegabolt that is 0.05% longer than its old megabolt. What fraction longer than the old bolt will the new bolt be? (Be sure to express your answer in lowest terms.)

13. Bob's Beeflike Burgers recently switched to a roll that costs $1\frac{4}{5}$ times as much as its old roll but contains oat bran. The cost of the new roll is what percent of the cost of the old roll?

14. Female mannequins represent 58% of the stock at Maria's Mannequins. Express this percent as a decimal number.

15. Jack's Jackhammer Service was surprised to find that 37.5% of its customers want their driveways widened. What fraction of Jack's customers want wider driveways? (Be sure to express your answer in lowest terms.)

16. Bits 'n' Bytes offers a new low-price computer with $5\frac{3}{10}$ times the memory of its old low-price computer. The amount of memory of the new computer is what percent of the amount of memory of the old computer?

17. Only 5.1% of deliveries made by Zippy Delivery Service take more than 24 hours. Express this percent as a decimal number.

18. The I Scream Shop now offers an ice cream that is $7\frac{3}{4}\%$ butterfat. What fraction of this ice cream is butterfat? (Be sure to express your answer in lowest terms.)

19. Woody's Lumber Mill was shocked to find that logs now cost $12\frac{11}{32}$ times what they did a year ago. Express as a percent the rise in the price of logs.

20. Lots of Plots Realty made a tidy 287.5% profit on its sale of a plot to a condominium developer last month. Express this percent as a decimal number.

21. The Mouse Hole, a gourmet cheese shop, has done well in part because $83\frac{1}{3}\%$ of its customers buy 5 or more pounds per month. What fraction of the Mouse Hole's customers does this represent? (Be sure to express your answer in lowest terms.)

SECTION 4.2: Solving Percent Problems

Can You:

Solve a percent problem for the base?
Solve a percent problem for the percent?
Solve a percent problem for the part?
. . . If not, you need this section.

General Guidelines

Working with percents—whether in business, industry, science, or other areas—almost always involves one of three basic types of problems. All of these types of problems involve three quantities:

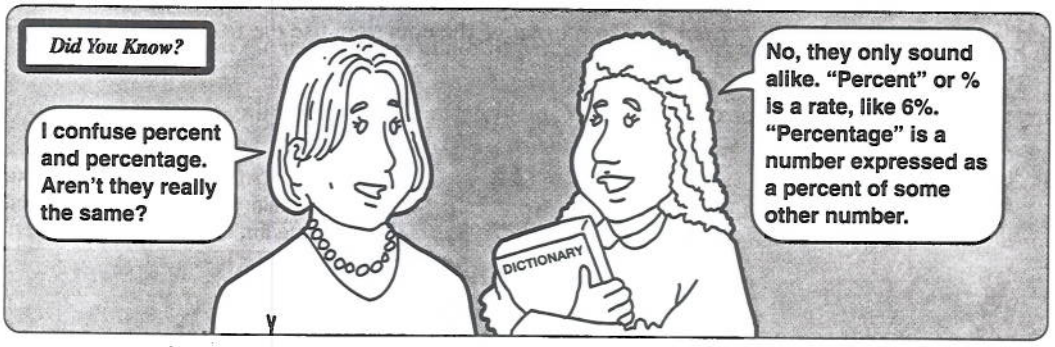
Base The number against which another number is compared in calculating a percent; in percent problems, it is sometimes called the *total* because it represents the whole.

Part The amount being compared with the base; in percent problems, it is sometimes called the *percentage*.

Percent The relationship of one number (part) to a second number (base), expressed in hundredths; in percent problems, it is sometimes called the *rate*.

1. The **base** (sometimes called the *total*)—the amount used for a comparison
2. The **part** (sometimes called the *percentage*)—the amount being compared with the base
3. The **percent** (sometimes called the *rate*)—the relationship of the part to the base

Each of the three basic percent problems involves finding one of these three quantities when the other two are known.



Solving Percent Problems

You can solve each of the three types of percent problems using five steps.

Steps: Solving Percent Problems

STEP 1. *Translate* the problem sentence into a math statement.

STEP 2. *Label* the quantities by type (percent, base, and part).

STEP 3. *Rearrange* the equation so that the unknown quantity is alone on the left of the equal sign and the other quantities are on the right. Use the following boxed Equation Finder.

STEP 4. *Solve* the problem by doing arithmetic.

STEP 5. *Check* your answer.

Before you try to use these steps, you should know a bit more about each.

Step 1. To *translate* the problem sentence, look for certain words and phrases that appear in most percent problems. They are signals alerting you to the mathematical operations to be done. The most common signal words in percent problems are

Signal Words	Translate as
is, is equal to, equals, will be	=
of	×

Use a □, letter of the alphabet, or ? for the unknown quantity you are asked to find. For example, the question

30% of what number is 15? should be translated

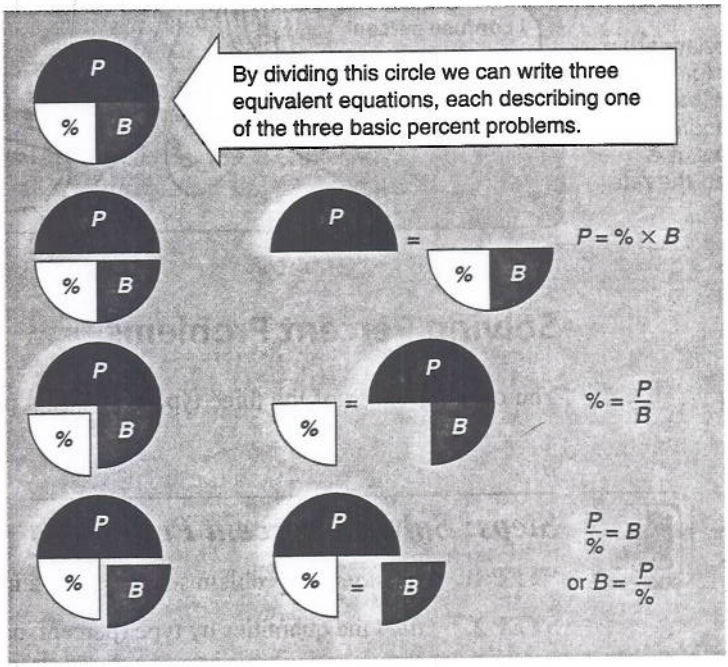
30% × ? = 15 or 30% × ? = 15

In this case, 30% is the *percent;* ? (the unknown quantity) is the *base;* and 15 is the part of the *total.*

Step 2. To *label* the parts, mark the base with a *B.* Identify the part with a *P.* Label the percent with a %. In our example, the result is

$$30\% \times ? = 15$$
$$\% \times B = P$$

Step 3. To *rearrange* the equation, use the following Equation Finder.



The three equations

$$P = \% \times B, \quad \% = \frac{P}{B}, \quad \text{and} \quad B = \frac{P}{\%}$$ are all equivalent.

In our example, the appropriate equation is $B = \frac{P}{\%}$. Therefore the equation $30\% \times B = 15$ becomes

$$B = \frac{P}{\%} = \frac{15}{30\%}$$

Step 4. To *solve* the problem, without the use of a calculator, you *must* rewrite all percents as fractions or decimals. Only then can you use them in a multiplication. In our example, $30\% = 0.30$, so

$$B = \frac{15}{0.30} = 50$$

Step 5. To *check* your answer, put the number you have found back into the original problem or equation to see if it makes sense. If possible, use the answer to calculate one of the other numbers in the equation as a check.

$$30\% \text{ of } 50 = 15$$
$$.30 \times 50 = 15$$

Most business calculators have a percent key that automatically converts a percent to a decimal number. If you have a calculator with a percent key, use the following sequence of keystrokes to solve this problem using a calculator.

15 [÷] 30 [%] ⟶ 50.

More instruction on the use of a calculator in percent calculations will be found in Appendix A.

Now let's look carefully at each type of problem. We'll explain each, give examples, show you how to solve them, and work through a few together.

P-Type Problems

P-Type Problems Percent problems that ask you to find the part, given the percent and the base.

Problems that ask you to find the part or percentage are called ***P*-type problems**. Such problems are usually stated in the form: "Find 30% of 50", or "What is 30% of 50?" The real question is

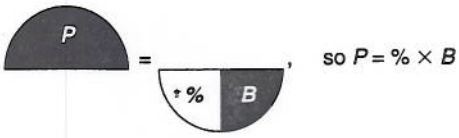
30% of 50 is what number?

Step 1. *Translate:* $30\% \times 50 = ?$

Step 2. *Label:* $\% \times B = P$

Step 3. *Rearrange:* $? = 30\% \times 50$

Remember from the Equation Finder that



Step 4. *Solve:* Before we give you the solution to this problem, you should try to solve it yourself. What answer did you get? The most common answers are 1500 and 15.

If you said 1500, you forgot our warning in step 4 and multiplied the percent without converting it to a decimal or fraction. Keep in mind that $30 \times 50 = 1500$, but $30\% \times 50$ is *not* 1500. To find the correct answer, convert 30% to a decimal first.

$$30\% = 30.\% = 0.30$$
$$30\% \times 50 = 0.30 \times 50 = 15$$

Step 5. *Check:* $30\% \times 50 = 15$
$0.30 \times 50 = 15$

With a calculator, use the following sequence of keystrokes to solve this problem.

50 [×] 30 [%] → 15.

Notice that when you use the percent key on a calculator, the base number usually is entered first: $30\% \times 50$ is entered as $50 \times 30\%$. Check your calculator for the correct sequence.

WORK THIS PROBLEM

The Question:

Solve the following problems.

(a) Find $8\frac{1}{2}\%$ of 160.
(b) Find 2% of 140.
(c) 35% of 20 = ?
(d) $7\frac{1}{4}\%$ of \$1000 = ?
(e) 120% of 15 is what number?
(f) What is $5\frac{1}{3}\%$ of 3.3?

✔ YOUR WORK

The Solution:

(a) ***Step 1.*** *Translate:* $8\frac{1}{2}\% \times 160 = ?$

Step 2. *Label:* $\% \times B = P$

Step 3. *Rearrange:* $P = 8\frac{1}{2}\% \times 160$

Step 4. *Solve:* $8\frac{1}{2}\% = 8.5\% = 08.5\% = 0.085$

$P = 0.085 \times 160 = 13.6$

Step 5. *Check:* $8\frac{1}{2}\% \times 160 = 13.6$
$0.085 \times 160 = 13.6$

(b) ***Step 1.*** $2\% \times 140 = ?$

Step 2. $\% \times B = P$

Step 3. $P = 2\% \times 140$

Step 4. $2\% = 02.\% = 0.02$

$P = 0.02 \times 140 = 2.8$

Step 5. $2\% \times 140 = 2.8$
$0.02 \times 140 = 2.8$

(c) ***Step 1.*** $35\% \times 20 = ?$

Step 2. $\% \times B = P$

Step 3. $P = 35\% \times 20$

Step 4. $35\% = 35.\% = 0.35$

$P = 0.35 \times 20 = 7$

Step 5. $35\% \times 20 = 7$
$0.35 \times 20 = 7$

(d) ***Step 1.*** $7\frac{1}{4}\% \times \$1000 = ?$

Step 2. $\% \times B = P$

Step 3. $P = 7\frac{1}{4}\% \times \$1000$

Step 4. $7\frac{1}{4}\% = 7.25\% = 07.25\% = 0.0725$

$P = 0.0725 \times \$1000 = \$72.50$

Step 5. $7\frac{1}{4}\% \times \$1000 = \$72.50$
$0.0725 \times \$1000 = \$72.50$

(e) ***Step 1.*** $120\% \times 15 = ?$

Step 2. $\% \times B = P$

Step 3. $P = 120\% \times 15$

Step 4. $120\% = 120.\% = 1.20$

$P = 1.2 \times 15 = 18$

Step 5. $120\% \times 15 = 18$
$1.20 \times 15 = 18$

(f) ***Step 1.*** $5\frac{1}{3}\% \times 3.3 = ?$

Step 2. $\% \times B = P$

Step 3. $P = 5\frac{1}{3}\% \times 3.3$

Step 4. Rounding $5\frac{1}{3}\%$ to a decimal number will result in an approximate answer.

$5\frac{1}{3}\% = 5.33\% = 05.33\% = 0.0533$ rounded

$P = 0.0533 \times 3.3 = 0.17589 = 0.176$ rounded

More decimal places in the rounded decimal number will result in greater accuracy. But to get the most accurate, we use fractions, not decimals.

$$5\tfrac{1}{3}\% = \frac{5\frac{1}{3}}{100} = \frac{\frac{16}{3}}{100} = \frac{16}{300}$$

$$P = \frac{16}{300} \times 3.3 = \frac{16}{300} \times \frac{33}{10} = \frac{528}{3000} = 0.176$$

Step 5. $5\frac{1}{3}\% \times 3.3 = 0.176$
$0.0533 \times 3.3 = 0.176$ rounded

Using a calculator,

(a) 160 [×] 8.5 [%] → 13.6

(b) 15 [×] 120 [%] → 18.

(f) 3.3 [×] 5.33 [%] → 0.17584 or 0.176 rounded.

Notice in all these calculations that when using a calculator, you enter the base number B into the calculator **first,** then multiply by the percent number.

Rounding $5\frac{1}{3}$ to 5.33 gives an approximate answer.

How to Misuse Percent

1. In general you cannot add, subtract, multiply, or divide percent numbers. Percent helps you compare two numbers. You cannot use it in normal arithmetic operations.

   For example, if 60% of class 1 earned A grades and 50% of class 2 earned A grades, what was the total percent of A grades for the two classes?

   The answer is that you cannot tell unless you know the number of students in each class.

2. In advertisements designed to trap the unwary, you might hear that "children had 23% fewer cavities when they used . . ." or "50% more doctors use. . . ."

   Fewer than what? Fewer than the worst dental health group the advertiser could find? Fewer than the national average?

   More than what? More than a year ago? More than nurses? More than other adults? More than infants?

   There must be some reference or base given in order for the percent number to have any meaning at all.

   BEWARE of people who misuse percent!

A great many business problems are *P*-type problems.

YOUR TURN

WORK THIS PROBLEM

The Question:

A reasonable budget allows 25% of net income for housing. How much should be allowed for housing if the net monthly income is \$2989.51? Round your answer to the nearest cent.

✓ YOUR WORK

The Solution:

Step 1. $25\% \times \$2989.51 = ?$

Step 2. $\% \times ? = P$

Step 3. $P = 25\% \times \$2989.51$

Step 4. $25\% = .25\% = 0.25$

$P = 0.25 \times \$2989.51 = \$747.3775 = \$747.38$ rounded

Step 5. $25\% \times \$2989.51 = \$747.38$

$0.25 \times \$2989.51 = \$747.38$ rounded

%-Type Problems Percent problems that ask you to find the percent, given the part and the base.

%-Type Problems

Problems that ask you to find the percent are called **%-type problems.** Such problems are usually stated in the form: "Find what percent 7 is of 16," or "7 is what percent of 16?" The real question is

What percent of 16 is 7?

Step 1. *Translate:* $?\% \times 16 = 7$

Step 2. *Label:* $\% \times B = P$ All of the problem statements are equivalent to this equation.

Step 3. *Rearrange:* Remember the Equation Finder:

so $?\% = \frac{7}{16}$ 16 is the base and 7 is the part of the base being described.

Step 4. *Solve:* $?\% = \frac{7}{16} = 16\overline{)7.0000} = 0.4375 = 43.75\%$

$$\begin{array}{r} 0.4375 \\ 16\overline{)7.0000} \\ \underline{64} \\ 60 \\ \underline{48} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \end{array}$$

Step 5. *Check:* $43.75\% \times 16 = ?$
$0.4375 \times 16 = 7$

The solution to a %-type problem is always a fraction or decimal number that you must convert to a percent. If you had trouble converting $\frac{7}{16}$ to a percent, you should review this process in Section 4.1.

WORK THIS PROBLEM

The Question:

Solve the following.

(a) What percent of 40 is 16?
(b) Find what percent of 25 is 65.
(c) \$6.50 is what percent of \$18.00?
(d) What percent of 2 is 3.5?
(e) $10\frac{2}{5}$ is what percent of 2.6?

✓ YOUR WORK

The Solution:

(a) ***Step 1.*** $?\% \times 40 = 16$

Step 2. $\% \times B = P$

Step 3. $\% = \dfrac{P}{B}$ (per the Equation Finder), so $?\% = \dfrac{16}{40}$

Step 4. $?\% = \dfrac{16}{40} = 0.40 = 40\%$

Step 5. $40\% \times 40 = 16$
$0.40 \times 40 = 16$

(b) ***Step 1.*** $?\% \times 25 = 65$

Step 2. $\% \times B = P$

Step 3. $\% = \dfrac{P}{B}$, so $?\% = \dfrac{65}{25}$

Step 4. $?\% = \dfrac{65}{25} = 2.60 = 260\%$

Step 5. $260\% \times 25 = 65$
$2.60 \times 25 = 65$

The most difficult part of this problem is in deciding whether the percent needed is found from $\frac{65}{25}$ or $\frac{25}{65}$. There is no magic to it. If you read the problem very carefully, you will see that it speaks of 65 as a part "of 25." The base is 25. The part is 65.

(c) ***Step 1.*** $\$6.50 = ?\% \times \$18.00$

Step 2. $P = \% \times B$

Step 3. $\% = \dfrac{P}{B}$, so $?\% = \dfrac{6.50}{18.00}$